Type-2 Fuzzy Similarity in Partial Truth and Intuitionistic Reasoning

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Abstract. Representing and manipulating the vague concepts of partially true knowledge pose a major challenge to the development of machine intelligence. In particular, the issue of how to extract approximate facts from vague and partially true statements has received considerable attention in the field of fuzzy information processing. However, vagueness is often due to a lack of available information, making it impossible to satisfactorily evaluate membership. Atanassov (1996) demonstrated the feasibility of mapping intuitionistic fuzzy sets to historical fuzzy sets. Intuitionistic fuzzy sets are isomorphic to interval valued fuzzy sets, while interval valued fuzzy sets have been regarded as unique value among type-2 fuzzy sets. This study presents a theoretical method to represent and manipulate partially true knowledge. The proposed method is based on the measurement of similarity among type-2 fuzzy sets, which are used directly to handle rule uncertainty that type-1 fuzzy sets are unable to deal with. Moreover, the switching relationship between type-2 fuzzy sets and intuitionist fuzzy sets is defined axiomatically. Results of this study demonstrate the effectiveness of the proposed theoretical method in pattern recognition and reasoning with regard to medical diagnosis.

Keywords: Type-2 fuzzy sets, Intuitionistic fuzzy sets, Fuzzy similarity, Partial truth.

1 Introduction

In order to distinguish between similar entities or groups of entities in daily life, one must determine the degree of similarity between them. Fuzzy set theory was developed by Zadeh, and ushered in an era of research into the measurement of similarity between fuzzy sets. These fuzzy set-based developments are applicable in data preprocessing, data mining for identifying dependency relationships between concepts, inference reasoning (Li et al. 2002, Tianjiang et al. 2002, Li et al. 2005, Song 2008, Fu X and Shen 2010, Petry and Yager 2010), and pattern

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recognition (Dengfeg and Chuntian 2002, Mitchell 2003, Zhizhen and Pengfei 2003, Hung and Yang 2004, Tang 2008, Sledge et al. 2010). Among other recently developed ways of measuring similarity, intuitionistic fuzzy sets are regarded as a distinct category. For examples of intuitionistic fuzzy sets, please refer to Chen (1995, 1997), Hong and Kim (1999) and Fan and Zhangyan (2001), Xu (2007), Mushrif and Ray (2009), and Chaira (2010).

In this chapter, we analyze existing measures of similarity among type-2 fuzzy sets based on counter-intuitive examples of partially true knowledge. We axiomatically define how type-2 fuzzy sets and intuitionistic fuzzy sets are related. Finally, we illustrate the usefulness of our proposed conversion in the application to reasoning in medical diagnosis.

1.1 Partially Truth

Reasoning systems are altered to handle incomplete or partially true knowledge by splitting each partially true statement into two components: a proposition component; and an associated truth-value component (Dubois and Prade 1980). The truth-value component provides an effective means of modifying the significance of the original proposition. A proposition such as "x is F" is expressed as "x is F is τ", where τ denotes a linguistic value of partial truth qualification, defined as the degree of compatibility of the situation with the proposition "x is F". Therefore, "x is F" denotes a proposition component, and a linguistic truth value, τ, denotes an associated truth-value component. Equivalent statements in natural language include:

" 'David is healthy' is quite true."

" 'The speed is moderate' is absolutely true."

The unit interval [0, 1] is taken as a set of partially true values. Any vague definition related to truth can be represented by a fuzzy set on [0, 1].

A simple vague proposition about the truth-value, such as "This truth value represents 'mostly true'", can be translated into a rule in the form of a general fuzzy set:

"Truth value is mostly true"

$$= \sum_{x \in X} \mu_{mostly\ true}(x)/x$$

$$= \frac{0.4}{0.65} + \frac{0.5}{0.7} + \frac{0.75}{0.75} + \frac{1}{0.8} + \frac{0.75}{0.9} + \frac{0.26}{0.95}.$$

The value of $\mu_{mostly\ true}(x)$ does not change the meaning of the proposition, but represents a subjective opinion concerning the meaning of the proposition. That is, when $\mu_{mostly\ true}(x) = 0$, the truth value certainly differs from x, and when $\mu_{mostly\ true}(x) = 1$, the truth value equals x. Notably, $\mu_{mostly\ true}(x)$ reveals the uncertainty of the original knowledge.

Assuming that this partial truth qualification is local rather than absolute, Bellman and Zadeh obtained a true statement based on a partially true statement, and derived the corresponding fuzzy set as a representation of such a statement (Zadeh 1979, Bellman and Zadeh 1977). Baldwin proposed implied statements, consisting of a fuzzy truth value restricted to a Lukasiewicz logical implication related to a fuzzy truth space (Baldwin 1979). Based on set-theoretical considerations, that study also obtained constraints to fuzzy truth values on truth value restrictions from conditional fuzzy linguistic statements, by applying an inverse procedure to modify the truth functions.

Accordingly, Raha and Ray proposed a theoretical method for reasoning with a partial truth value associated with a vague sentence (Raha and Ray 1999, 1997, 2000). The partial truth values were defined by fuzzy sets on the universe of discourse [0,1], which is a unit interval. This vague proposition is presented as a possibility distribution, in which each possibility distribution is assigned to and manipulated by a fuzzy set/relation.

In contrast with the above approaches, the theoretical method developed in this study attempts to eliminate the deficiencies involved in the representation of partially true knowledge. Despite associating and manipulating the partial truth value according to the proposition, previous methods have denoted and estimated the corresponding fuzzy set and qualification of partial truth, separately. In other words, set-theoretical considerations cannot be used to derive partially true knowledge, as long as partially true statements are not associated with the existing proposition. The proposed theoretical method used to represent and manipulate such partially true knowledge is therefore based on the type-2 fuzzy set theory.

1.2 Type-2 Fuzzy Sets

Type-2 fuzzy sets were initially defined by Zadeh (1979), and characterized by a fuzzy membership. The membership value for each element of this set is a fuzzy set in [0,1], whereas the membership grade of a type-1 fuzzy set is a numeric value in [0,1]. To clarify the above statement, the fuzzy set '*tall*' is represented as:

$$tall = \frac{0.95}{Michael} + \frac{0.4}{Danny} + \frac{0.6}{Robert}.$$

Conversely, the interpretation of type-2 fuzzy sets is

$$tall = \frac{High}{Michael} + \frac{Low}{Danny} + \frac{Medium}{Robert},$$

where membership functions of High, Low, and Medium, themselves are fuzzy sets. The former set is measured by one condition for one element, while the latter set is evaluated by several conditions for one element. Type-2 fuzzy sets are useful when the exact membership function for a type-1 fuzzy set cannot be easily determined. For this reason, Ttype-2 fuzzy sets are advantageous for the incorporation of uncertainty.

According to Mendel (2000, 2006), type-2 fuzzy sets are defined as follows. For simplicity, the universe of discourse is assumed to be a finite set, although the definition is also applicable for infinite universes of discourse.

Definition 1 (Mendel 2000, 2006)**.** Type-2 fuzzy set, $\tilde{A}$, is characterized by a type-2 membership function, $\mu_{\tilde{A},}(x)$, where X is the universal set, $x \in X$ and $u \in J_x \subseteq [0,1]$, J_x is the possible membership collection for every x. Meanwhile, the amplitude of the secondary membership function is called a secondary grade, and $f_x(u)$ is a secondary grade. That is,

$$\tilde{A} = \left\{\left(x, \mu_{\tilde{A},}(x)\right) \middle| \forall x \in X\right\},$$

or, as

$$\tilde{A} = \sum_{x \in X} \mu_{\tilde{A}}(x)/x = \sum_{x \in X} [\sum_{x \in J_x} f_x(u)/u]/x.$$

1.3 Intuitionistic Fuzzy Sets

Assume that X denotes the universe of discourse, $X = \{x_1, x_2, \cdots, x_n\}$. Ordinary fuzzy set theory lacks an effective means of incorporating that hesitation into the degree of membership. Atanassov (1986) developed intuitionistic fuzzy sets, along with the ability to model hesitation and uncertainty by using an additional degree. Each intuitionistic fuzzy set $\breve{A}$ allots a membership degree $\mu_{\breve{A}}(x)$ and a non-membership degree $v_{\breve{A}}(x)$ to each element x of the universe X, note that $\mu_{\breve{A}}(x) \in [0,1]$, $v_{\breve{A}}(x) \in [0,1]$ and $\mu_{\breve{A}}(x) + v_{\breve{A}}(x) \leq 1$. The value $\pi(x) = 1 - (\mu_{\breve{A}}(x) + v_{\breve{A}}(x))$ is called the hesitation part, which is the hesitancy degree of whether x belongs to $\breve{A}$. The set of all the intuitionistic fuzzy sets in X is representing as $IFS(X)$.

Definition 2 (Atanassov 1986)**.** When the universe of discourse X is discrete, an intuitionistic fuzzy set $\breve{A}$ is denoted as follows:

$$\breve{A} = \textstyle\sum_{i=1}^{n} [x, \mu_{\breve{A}}(x), v_{\breve{A}}(x)],\ \forall x_i \in X.$$

For the sake of simplicity, the universe of discourse is assumed to be a finite set, although the definition can be applied for infinite sets.

The following properties are expressed for all $\breve{A}$ and $\breve{B}$ belonging to IFSs(X) in (Pappis and Karacpailidis 1993),

(a). $\breve{A} \leq \breve{B}$ *if and only if* $\mu_{\breve{A}}(x) \leq \mu_{\breve{B}}(x)$ *and* $v_{\breve{A}}(x) \geq v_{\breve{B}}(x)$ *for all* $x \in X$.

(b). $\breve{A} = \breve{B}$ *if and only if* $\breve{A} \leq \breve{B}$ *and* $\breve{A} \geq \breve{B}$.

(c). $\breve{A}_c = \sum_{i=1}^{n} [x, v_{\breve{A}}(x), \mu_{\breve{A}}(x)],\ \forall x_i \in X$.

1.4 Type-2 Fuzzy Similarity

In our study, the universe of discourse denotes a finite set, the type-2 fuzzy set $\tilde{A}$ is expressed as:

$$\tilde{A} = \sum_{x\in X}[\sum_{x\in J_x} f_x(u)/u]/x = \sum_{i=1}^{N}[\sum_{x\in J_{x_i}} f_x(u)/u]/x_i$$
$$= [\sum_{k=1}^{M_1} f_{x_1}(u_{1k})/u_{1k}]/x_1 + \cdots + [\sum_{k=1}^{M_N} f_{x_N}(u_{Nk})/u_{Nk}]/x_N.$$

Assume that x has been incorporated into N values, with each value u discretized into M_i values. Many choices are possible for these secondary membership functions. The secondary membership function can be treated as a type-1 membership function along each x. Hence, the similarity between type-2 fuzzy sets is

$$\tilde{S}(\tilde{A},\tilde{B}) = 1/N\sum_{i=1}^{N} S(\mu_{\tilde{A}}(x_i),\mu_{\tilde{B}}(x_i)),$$

where $\tilde{S}(\cdot)$ can be any traditional similarity index for the general fuzzy sets. Note that, $\mu_{\tilde{A}}(x_i)$ and $\mu_{\tilde{B}}(x_i)$ are two secondary membership functions. For instance, the proposed similarity methods of Rahaet al. (2002) and Pappiset al. (1993) are:

$$S(A,B) = \sum_{x\in X}\{\mu_A(x)\cdot\mu_B(x)\}/\sum_{x\in X} max\{\mu_A(x),\mu_B(x)\}^2, \quad (1)$$

or

$$S(A,B) = |A||B|\cos(\theta)/(\max(|A|^2,|B|^2)),$$

where A and B are two type-1 fuzzy sets; $|A|$ is the length of the vector A, and $\cos(\theta)$ is the cosine of the angle between the two vectors. An important consideration is to select the similarity index of type-2 fuzzy sets such that the index exhibits the properties of similarity. Expression(1) is adopted in this study, and the similarity index of type-2 fuzzy sets is formulated as follows:

$$\tilde{S}(\tilde{A},\tilde{B}) = 1/N\sum_{i=1}^{N} S(\mu_{\tilde{A}}(x_i),\mu_{\tilde{B}}(x_i))$$
$$= 1/N\sum_{i=1}^{N}\frac{\sum_u\{f_{x_i}(u),g_{x_i}(u)\}}{\sum_u[\max\{f_{x_i}(u),g_{x_i}(u)\}^2]}, \quad (2)$$

where $x_i \in X$ and $u \in J_x$. In addition, when we defined $\sum_u[\max\{f_{x_i}(u), g_{x_i}(u)\}^2=0$, then $S(\mu_{\tilde{A}}(x_i), \mu_{\tilde{B}}(x_i)) = 1$, that is, $\tilde{S}(\tilde{A}, \tilde{B}) = 1$.

Notably, because the primary membership values may not be the same for a specific value of x, that is J_x on $\tilde{A}$ and J_x on $\tilde{B}$ cannot be exactly computed in some cases. Because the notion of a zero membership value generalizes in fuzzy set theory to the situation in which a non-zero membership is not clearly stated, this concept has also been executed in this study. The value '0' in the secondary grade denotes this item as useless in the deterministic process. The value '0' is applied as the appended secondary grade to those missing items in the minus between J_x on $\tilde{A}$ and J_x on $\tilde{B}$ for the computation of similarity.

2 Reasoning with Type-2 Similarity

A type-1 fuzzy inference engine normally combines rules and provides a mapping from input type-1 fuzzy sets to output type-1 fuzzy sets. Additionally, the inference process is very similar in the case of type-2 inference. The inference engine combines rules and provides a mapping from input type-2 fuzzy sets to output type-2 fuzzy sets. Another difference is in the defuzzification. In the type-2 cases, the output sets are type-2; the extended defuzzification operation in the type-2 case gives the type-1 fuzzy sets as the output. This operation is called a "type reducer", and the type-1 fuzzy set is obtained as a "type reduced set", which may then be defuzzified to obtain a single crisp number.

This study presents a new reasoning method, involving the measure of similarity between type-2 fuzzy sets as an inference methodology. Consider two type-2 fuzzy sets $\tilde{A}$ and $\tilde{A}'$ defined in the same universe of discourse X. Another two type-2 fuzzy sets $\tilde{B}$ and $\tilde{B}'$ are defined over the same universe of discourse Y. Two corresponding linguistic variables x and y are also defined, and the typical propositions is presented as:

$$\text{Rule: IF } x \text{ is } \tilde{A} \text{ then } y \text{ is } \tilde{B}$$

$$\text{Fact: } x \text{ is } \tilde{A}'$$

$$\Rightarrow \text{Conclusion: } y \text{ is } \tilde{B}'$$

Let $\tilde{S}(\tilde{A}, \tilde{A}')$ denote the measure of similarity between two type-2 fuzzy sets $\tilde{A}$ and $\tilde{A}'$. Existing methods use the measure of similarity to directly compute the inference without considering the induced relationship. In the proposed method, the authors translate the conditional statement into a fuzzy relationship. The similarity between the fact and the antecedent of the rule is used to modify the derived relationship. That is, every change in the conditional premise and in the fact is incorporated into the induced fuzzy relationship. Accordingly, a conclusion can be derived using the sub-projection operation. Thus, the conclusion is influenced by the modification of the fact and the antecedent of the rule fired.

Two modification procedures are proposed to modify the derived relationship. They are listed as follows:

$$\text{expansion form: } \mu_{\tilde{Q}'}(x,y) = m_1(\mu_{\tilde{Q}}(x,y), \tilde{S}(\tilde{A},\tilde{A}'))$$
$$= \sum_{u \in J_{x,y}} (\frac{f_{x,y}(u)}{\tilde{S}(\tilde{A},\tilde{A}')})/(\frac{u}{\tilde{S}(\tilde{A},\tilde{A}')}),$$

and

$$\text{reduction form: } \mu_{\tilde{Q}'}(x,y) = m_2(\mu_{\tilde{Q}}(x,y), \tilde{S}(\tilde{A},\tilde{A}'))$$
$$= \sum_{u \in J_{x,y}} (f_{x,y}(u) \cdot \tilde{S}(\tilde{A},\tilde{A}'))/(u \cdot \tilde{S}(\tilde{A},\tilde{A}')),$$

where $\tilde{Q}$ is a fuzzy relation in the Cartesian product space $X \times Y$, $m_1(\cdot)$ and $m_2(\cdot)$are two modification functions for the expansion and reduction forms; and $f_{x,y}(u)$ is the secondary grade of the fuzzy relationship. In this study, the proposed method focused on the significant difference between $\tilde{A}$ and $\tilde{A}'$ to make the conclusion $\tilde{B}'$ less specific, and then choosing the expansion form. Hence, with a decrease in similarity, occurring at a significant difference between $\tilde{A}$ and $\tilde{A}'$, the inferred conclusion would be close to *Y*. Conversely, when $\tilde{A} = \tilde{A}'$ the inferred conclusion is obtained as $\tilde{B} = \tilde{B}'$. Notably, when $\tilde{S}(\tilde{A},\tilde{A}') = 0$, nothing can be concluded when $\tilde{A}$ and $\tilde{A}$ are dissimilar, and $\tilde{B} = \tilde{B}^c$ is obtained.

Subsequently, assume that k linguistic variables $x_1, \ldots, x_k$ defined on the universes of discourses $X_1, \ldots, X_k$. These typical propositions are listed:

$$\text{Rule: IF } x \text{ is } \tilde{A}_1 \text{ and } \ldots \text{and } x \text{ is } \tilde{A}_k \text{ then } y \text{ is } \tilde{B}$$
$$\text{Fact: IF } x \text{ is } \tilde{A}'_1 \text{ and } \ldots \text{and } x \text{ is } \tilde{A}'_k$$
$$\Longrightarrow \text{Conclusion: } y \text{ is } \tilde{B}'$$

3 Truth-Qualified Proposition

Accordingly, the partially truth-qualified statements of the form illustrated as follows:

" ' David is healthy' is quite true," or
" 'The temperature is moderate' is mostly true."

Simple statements are of the general propositional form,

"*x* is *F*; *t* is *Q*"

where *x* and *t* are two linguistic variables, and *t* denotes the truth value. *F* represents the vague descriptions of the object *x*, and *Q* denotes the truth of proposition "*x* is *F*". Restated, *F* and *Q* are type-1 fuzzy sets. Consequently, the previous general propositional form can be translated into a type-2 fuzzy statement,

"*x* is *F*"

where

$$\tilde{F} = \sum \mu_{\tilde{F}}(x)/x = \sum [Q_x(u)/u]/x, \quad u\epsilon J_x \subseteq U = [0,1].$$

Notably, a secondary grade, $Q_x(u)$, is applied to state the truth value.

The partially truth-qualified statement is represented as the type-2 fuzzy set. Consequently, the truth value of a composite proposition is computed as follows:

$$(x \text{ is } F; t \text{ is } Q) \wedge (x \text{ is } G; t \text{ is } R)$$
$$= (x \text{ is } F) \wedge (x \text{ is } G) \Longrightarrow \mu_{\tilde{F}}(x) \cap \mu_{\tilde{G}}(x),$$

$$(x \text{ is } F; t \text{ is } Q) \vee (x \text{ is } G; t \text{ is } R)$$
$$= (x \text{ is } F) \vee (x \text{ is } G) \Longrightarrow \mu_{\tilde{F}}(x) \cup \mu_{\tilde{G}}(x),$$

and

$$\neg(x \text{ is } F; t \text{ is } Q) = (x \text{ is } F^c) \Longrightarrow \mu_{\tilde{F}^c}(x).$$

Accordingly, the deductive processes are introduced on the similarity measure among type-2 fuzzy sets. In the following, k linguistic variables $x_1, \dots, x_k$ are defined on the universe of discourses $X_1, \dots, X_k$. t denotes as the truth of the proposition. These typical propositions listed as:

$$rule\text{: if } x_1 \text{ is } A_1 \text{ and } \dots \text{ and } x_k \text{ is } A_k \text{ then } y \text{ is } B; t \text{ is } C_{true}$$
$$fact\text{: if } x_1 \text{ is } A'_1 \text{ and } \dots \text{ and } x_k \text{ is } A'_k \;; t \text{ is } C'_{true}$$
$$\Longrightarrow conclusion\text{: } y \text{ is } B';\ t \text{ is } C''_{true}.$$

The partially true proposition is represented by the statement of type-2 fuzzy sets,

$$rule\text{: if } x_1 \text{ is } \tilde{A}_1 \text{ and } \dots \text{ and } x_k \text{ is } \tilde{A}_k \text{ then } y\ is\ \tilde{B}$$
$$fact\text{: if } x_1 \text{ is } \tilde{A}'_1 \text{ and } \dots \text{ and } x_k \text{ is } \tilde{A}'_k$$
$$\Longrightarrow conclusion\text{: } y \text{ is } \tilde{B}',$$

where $i = 1, \dots, k$. Secondary grades represent as $f_{C_{true}}(u)$, $f_{C'_{true}}(u)$ and $f_{C''_{true}}(u)$, respectively. That is, these type-2 fuzzy sets are

$$\tilde{A}_i = \sum\nolimits_{x_i \epsilon X_i} [\sum\nolimits_{u \epsilon J_{x_i}} f_{C_{true}}(u)/u]/x_i,$$

$$\tilde{A}'_i = \sum\nolimits_{x_i \epsilon X_i} [\sum\nolimits_{u \epsilon J_{x_i}} f_{C'_{true}}(u)/u]/x_i,$$

$$\tilde{B} = \sum\nolimits_{y \epsilon Y} [\sum\nolimits_{w \epsilon J_y} f_{C_{true}}(w)/w]/y,$$

$$\tilde{B}' = \sum\nolimits_{y \epsilon Y} [\sum\nolimits_{w \epsilon J_y} f_{C''_{true}}(w)/w]/y.$$

The case of truth qualification proposition is shown in the following example.

Example 1*:* Herein, an example of the proposition from (Raha and Ray 1999), "It is almost fairly_true that people will feel not so uncomfortable, when it is true that humidity is moderate," is referenced from the general knowledge that "It is fairly_true when humidity is high and people feel uncomfortable". Accordingly, the conclusion of the form is derived with the proposed method. Here, the following is assumed.

$\tilde{A}$=(humidity is) high,

$\tilde{B}$=(human's tolerance is) uncomfortable,

C_{true}=*farily_true*,

$\widetilde{A}'$=(humidity is) moderate,

C'_{true}=*true*.

Consequently, the propositions are listed as follows.

rule: if humidity is *high* then tolerance is *uncomfortable*, the truth is *fairly_true.*

fact: if humidity is *moderate*, the truth is *true*.

Then, the purpose is to represent the inexact concepts in the propositions as type-2 fuzzy sets based on an appropriate universe of discourses. Let the universes of discourse be denoted as follows,

$$\text{Percentile humidity} \in [0,1],$$

$$\text{Percentile tolerance index} \in [0,1],$$

$$\text{Truth value} \in [0,1].$$

Thus, at any particular time, the humidity of air is normalized in the choice of universe. Similarity, tolerance index '1.0' means "feeling comfortable"; anything less than '1.0' means "partially comfortable", and '0.0' means "absolutely uncomfortable". Hence, the definitions of type-2 fuzzy sets are listed as follows,

$$high = \frac{0.3/0.25}{0.25} + \frac{0.6/0.5}{0.25} + \frac{0.8/0.75}{0.75} + \frac{1/1}{1},$$

$$uncomfortable = \frac{0.9/0.8}{0.0} + \frac{0.7/0.65}{0.125} + \frac{0.6/0.55}{0.25} + \frac{0.4/0.35}{0.5} + \frac{0.3/0.2}{0.75} + \frac{0.2/0.1}{1.0},$$

$$moderate = \frac{0.65/0.5}{0.25} + \frac{0.9/0.75}{0.5} + \frac{1/1}{0.75} + \frac{0.9/0.75}{1}.$$

Accordingly, the similarity is given by

$$\tilde{S}(high, moderate) = \frac{1}{4}\left(\frac{0.3 \cdot 0.5 + 0.65 \cdot 0.5}{0.5^2 + 0.65^2} + \frac{0.6 \cdot 0.5 + 0.5 \cdot 0.9}{0.6^2 + 0.9^2} + \frac{0.8 \cdot 0.5 + 0.5 \cdot 0.1}{0.8^2 + 1^2} + \frac{1 \cdot 0.5 + 0.9 \cdot 0.5}{1^2 + 0.9^2}\right) = 0.61,$$

and, the fuzzy relation $\tilde{Q}$ computed as follows:

$$\tilde{Q}=\frac{\frac{0.3}{0.5}}{(0.25,0.0)}+\frac{\frac{0.3}{0.5}}{(0.25,0.125)}+\frac{\frac{0.3}{0.5}}{(0.25,0.25)}+\frac{\frac{0.3}{0.5}}{(0.25,0.5)}+\frac{\frac{0.3}{0.5}}{(0.25,0.75)}+\frac{\frac{0.2}{0.1}}{(0.25,1.0)}+\frac{\frac{0.6}{0.5}}{(0.5,0.0)}+\frac{\frac{0.6}{0.5}}{(0.5,0.125)}+\frac{\frac{0.6}{0.5}}{(0.5,0.25)}+\frac{\frac{0.4}{0.35}}{(0.5,0.5)}+\frac{\frac{0.3}{0.2}}{(0.5,0.75)}+\frac{\frac{0.2}{0.1}}{(0.5,1.0)}+\frac{\frac{0.8}{0.75}}{(0.75,0.0)}+\frac{\frac{0.7}{0.65}}{(0.75,0.125)}+\frac{\frac{0.6}{0.55}}{(0.75,0.25)}+\frac{\frac{0.4}{0.35}}{(0.75,0.5)}+\frac{\frac{0.3}{0.2}}{(0.75,0.75)}+\frac{\frac{0.2}{0.1}}{(0.75,1.0)}+\frac{\frac{0.9}{0.8}}{(1.0,0.0)}+\frac{\frac{0.7}{0.65}}{(1.0,0.125)}+\frac{\frac{0.6}{0.55}}{(1.0,0.25)}+\frac{\frac{0.4}{0.35}}{(1.0,0.5)}+\frac{\frac{0.3}{0.2}}{(1.0,0.75)}+\frac{\frac{0.2}{0.1}}{(1.0,1.0)}.$$

Furthermore, the relation $\tilde{Q}'$ is adjusted by

$$\tilde{Q}'=\frac{\frac{0.49}{0.82}}{(0.25,0.0)}+\frac{\frac{0.49}{0.82}}{(0.25,0.125)}+\frac{\frac{0.49}{0.82}}{(0.25,0.25)}+\frac{\frac{0.49}{0.57}}{(0.25,0.5)}+\frac{\frac{0.49}{0.33}}{(0.25,0.75)}+\frac{\frac{0.33}{0.16}}{(0.25,1.0)}+\frac{\frac{0.98}{0.82}}{(0.5,0.0)}+\frac{\frac{0.98}{0.82}}{(0.5,0.125)}+\frac{\frac{0.98}{0.82}}{(0.5,0.25)}+\frac{\frac{0.66}{0.57}}{(0.5,0.5)}+\frac{\frac{0.49}{0.33}}{(0.5,0.75)}+\frac{\frac{0.33}{0.16}}{(0.5,1.0)}+\frac{\frac{1}{1}}{(0.75,0.0)}+\frac{\frac{1}{1}}{(0.75,0.125)}+\frac{\frac{0.98}{0.9}}{(0.75,0.25)}+\frac{\frac{0.66}{0.57}}{(0.75,0.5)}+\frac{\frac{0.49}{0.33}}{(0.75,0.75)}+\frac{\frac{0.33}{0.16}}{(0.75,1.0)}+\frac{\frac{1}{1}}{(1.0,0.0)}+\frac{\frac{1}{1}}{(1.0,0.125)}+\frac{\frac{0.98}{0.9}}{(1.0,0.25)}+\frac{\frac{0.66}{0.57}}{(1.0,0.5)}+\frac{\frac{0.49}{0.33}}{(1.0,0.75)}+\frac{\frac{0.33}{0.16}}{(1.0,1.0)}.$$

Consequently, $\tilde{Q}'$ is projected to obtain the conclusion according to

$$\tilde{B}'=\frac{\left(\frac{0.49}{0.82}\right)\cup\left(\frac{0.98}{0.82}\right)\cup\left(\frac{1}{1}\right)\cup\left(\frac{1}{1}\right)}{0.0}+\frac{\left(\frac{0.49}{0.82}\right)\cup\left(\frac{0.98}{0.82}\right)\cup\left(\frac{1}{1}\right)\cup\left(\frac{1}{1}\right)}{0.125}+\frac{\left(\frac{0.49}{0.82}\right)\cup\left(\frac{0.98}{0.82}\right)\cup\left(\frac{0.98}{0.9}\right)\cup\left(\frac{0.98}{0.9}\right)}{0.25}+\frac{\left(\frac{0.49}{0.57}\right)\cup\left(\frac{0.66}{0.57}\right)\cup\left(\frac{0.66}{0.57}\right)\cup\left(\frac{0.66}{0.57}\right)}{0.5}+\frac{\left(\frac{0.49}{0.33}\right)\cup\left(\frac{0.49}{0.33}\right)\cup\left(\frac{0.49}{0.33}\right)\cup\left(\frac{0.49}{0.33}\right)}{0.75}+\frac{\left(\frac{0.33}{0.16}\right)\cup\left(\frac{0.33}{0.16}\right)\cup\left(\frac{0.33}{0.16}\right)\cup\left(\frac{0.33}{0.16}\right)}{1.0}$$

$$=\frac{(\frac{1}{1})}{0.0}+\frac{(\frac{1}{1})}{0.125}+\frac{(\frac{0.98}{0.9})}{0.25}+\frac{(\frac{0.66}{0.57})}{0.5}+\frac{(\frac{0.49}{0.33})}{0.75}+\frac{(\frac{0.33}{0.16})}{0.125}.$$

Hence, the conclusion describes the tolerance when the humidity is moderate and the truth condition is true. According to the derivation, when the humidity is moderate, the people will feel less uncomfortable due to the secondary grades rising after the tolerance index “0.25”. Moreover, this case was also applied by the ordinary fuzzy implication in Pappis and Karacapilidis (1993), wherein the results were obtained as

$$B' = \frac{1}{0.0} + \frac{1}{0.125} + \frac{0.775}{0.25} + \frac{0.543}{0.5} + \frac{0.31}{0.75} + \frac{0.07}{1.0},$$

and the truth function was given as

$$truth = \frac{0.25}{0.75} + \frac{0.5}{0.8} + \frac{0.75}{0.85} + \frac{1}{0.9} + \frac{0.95}{0.95} + \frac{0.9}{1.0}.$$

The corresponding results show that the general fuzzy implication is not sufficient to handle the fuzzy proposition with the truth function, because the truth function is independent of the fuzzy processing, and the results of the truth function are hard to associate to the original proposition. Conversely, the partial truth statements are associated with the proposition in our proposed method; the set-theoretic considerations can be used to derive partial true knowledge.

4 The Relationship between Intuitionistic Fuzzy Sets and Type-2 Fuzzy Sets

Atanassov (1996) associated a mapping from $IFSs(X)$ to $FS_2(X)$, where the set of all intuitionistic fuzzy sets and type-2 fuzzy sets in X are representing as $IFS(X)$ and $FS_2(X)$. That study also defined the following operator:

$$\check{A} = \{< x, \mu_A(x_i), v_A(x_i) > | x \in X\} \rightarrow$$
$$f_\alpha(\check{A}) = \{< x, \mu_A(x_i) + \alpha\pi_A(x_i), 1 - \mu_A(x_i) - \alpha\pi_A(x_i) > | x \in X\},$$

where $f_\alpha: IFSs(X) \rightarrow FSs(X)$ (Atanassov 1986). The operator f_α coincides with the operator D_α given in Atanassov. However, the limitations of above the equation are listed as follows,

(a). The equation considers only the membership degree and omits the imperfect information (non-membership degree).

(b). The operator cannot handle the reverse switching. i.e. from fuzzy sets to intuitionistic fuzzy sets.

(c). Considering the complementation of FSs such as that of Sugeno (1977) or Yager (1979), the sum of membership and non-membership from one, the result is a negative number. Therefore, the elementary intuitionism condition given by Atanassov is not satisfied.

In addition for each element $x \in X$, the type-2 fuzzy set can model hesitation and additional uncertainties by using additional degrees. For instance, the polarizing concepts, i.e., more/less, optimistic/pessimistic, membership/non-membership, can be inferred by the secondary grades. Restated, the secondary grades are defined to determine the magnitudes, allowing us to weight the degree of intuitionism of an intuitionistic fuzzy set. Thus, to eliminate the above limitations, the relationship is introduced as follows.

Definition 3. Let $\breve{A} \in IFSs(X)$ define as follows,

$$\breve{A} = \sum_{i=1}^{n} [x_i, \mu_A(x_i), v_A(x_i)],$$

where $x_i \in X$. Then, the association from $IFSs(X)$ to $FS_2(X)$ is defined as

$$\tilde{A} = \sum_{i=1}^{n} [1/(\mu_A(x_i) + p\pi_A(x_i)) + 0/(1 - v_A(x_i) + p\pi_A(x_i))]\,/x_i, \quad (3)$$

The secondary grade in (3) represents the indeterminacy index of membership/non-membership degree, which models the un-hesitancy of determining the extent to which an object satisfies a specific property. Restated, 1/0 in secondary grades implies the total certain/uncertain with respect to membership/non-membership herein. Additionally, according to P1 (properties of IFSs), the definition of non-membership degree of an element x is $v_{\breve{A}}(x_i) \geq v_{\breve{B}}(x_i) \Leftrightarrow \breve{A} \leq \breve{B}$. This notion contradicts a natural generalization of a standard fuzzy set of the containment statement of Zadeh, $v_{\breve{A}}(x_i) \leq v_{\breve{B}}(x_i) \Leftrightarrow \breve{A} \leq \breve{B}$. Thus, the non-membership value in (3) is obtained by subtracting the $v_{\breve{B}}(x_i)$ from one.

Accordingly, the following proposition is proven to validate the relationship between T2FS and IFS. To simply the proof, intuitionistic fuzzy set refers to the extension of the fuzzy sets. Namely, the membership and non-membership degrees are added an equal to one.

Proposition 1. Let $\breve{A}, \breve{B} \in IFSs(X)$, denote that the switch from $IFSs(X)$ to $FS_2(X)$ are validated, then $\breve{A} \leq \breve{B}$ if and only if $\tilde{A} \leq \tilde{B}$.

Proof. Assume that two IFSs $\breve{A}$ and $\breve{B}$ are defined as follows:

$$\breve{A} = \sum_{i=1}^{n} [x_i, \mu_{\breve{A}}(x_i), v_{\breve{A}}(x_i)],$$

and

$$\breve{B} = \sum_{i=1}^{n} [x_i, \mu_{\breve{B}}(x_i), v_{\breve{B}}(x_i)],$$

where $\forall x_i \in X$.

Thus, two corresponding type-2 fuzzy sets are defined as:

$$\tilde{A} = \sum_{i=1}^{n} [1/(\mu_{\breve{A}}(x_i) + p\pi_{\breve{A}}(x_i)) + 0/(1 - v_{\breve{A}}(x_i) + p\pi_{\breve{A}}(x_i))]\,/x_i\,,$$

and

$$\tilde{B} = \sum_{i=1}^{n} [1/(\mu_{\breve{B}}(x_i) + p\pi_{\breve{B}}(x_i)) + 0/(1 - v_{\breve{B}}(x_i) + p\pi_{\breve{B}}(x_i))]\,/x_i.$$

Hence, assume that $\breve{A} \leq \breve{B}$, then

$$\mu_A(x_i) \leq \mu_B(x_i) \text{ and } v_A(x_i) \geq v_B(x_i)$$
$$\Rightarrow \mu_A(x_i) - v_A(x_i) \leq \mu_B(x_i) - v_B(x_i).$$

On the contrary, assume that $\tilde{A} \leq \tilde{B}$, that is

$$\mu_{\check{A}}(x_i) + p\pi_{\check{A}}(x_i) \leq \mu_{\check{B}}(x_i) + p\pi_{\check{B}}(x_i),$$

and

$$1 - v_{\check{A}}(x_i) + p\pi_{\check{A}}(x_i) \leq 1 - v_{\check{B}}(x_i) + p\pi_{\check{B}}(x_i),$$
$$\Rightarrow \mu_{\check{A}}(x_i) - v_{\check{A}}(x_i) \leq \mu_{\check{B}}(x_i) - v_{\check{B}}(x_i).$$

Hence, the proposition is proven. □

Furthermore, it is known that if A is a fuzzy set on a referential X and $c: [0,1] \rightarrow [0,1]$ is a fuzzy complement, the set

$$A = \sum_{i=1}^{n} [x_i, \mu_A(x_i), c(\mu_A(x_i))] \quad , \tag{4}$$

defined as an intuitionistic fuzzy set (De et al. 2001). However, according to our previous statement, non-membership value is not a natural generalization of a standard FS. Besides, if we take Sugeno's negation (Sugeno and Terano 1977)

$$c_\lambda(x) = \frac{1 - x}{1 + \lambda x}, \text{with } -1 < \lambda < 0,$$

or Yager's negation (1979)

$$c_\lambda(x) = (1 - x^\omega)^{1/\omega}, \text{with } 1 < \omega,$$

as a fuzzy complement, (3) is not an intuitionistic fuzzy set, because $\mu_A(x_i) + c(\mu_A(x_i)) > 1$ and therefore, $\pi_A(x_i) < 0$. Hence, for the purpose to clear state the relation of type-2 fuzzy sets and intuitionistic fuzzy sets, the reverse relationship is defined as follows:

Definition 4. Let $\tilde{A} \in FS_2(X)$ define as follows,

$$\tilde{A} = \sum_{i=1}^{n} [1/\mu_1(x_i) + 0/\mu_2(x_i)] / x_i,$$

where $x_i \in X$. Then, the one way transforms from type-2 fuzzy set to intuitionistic fuzzy set is defined as:

$$\check{A} = \sum_{i=1}^{n} [x_i, \mu_1(x_i) - p\pi_{\check{A}}(x_i), 1 - \mu_2(x_i) - p\pi_{\check{A}}(x_i)], \tag{5}$$

where

$$\pi_{\check{A}}(x_i) = \begin{cases} \dfrac{\mu_1(x_i) - \mu_2(x_i)}{2p - 1}, & if\ \mu_1(x_i) > \mu_2(x_i), \\ \dfrac{\mu_2(x_i) - \mu_1(x_i)}{1 - 2p}, & if\ \mu_1(x_i) \leq \mu_2(x_i), \end{cases}$$

and $p \in [0,1]$.

Proposition 2. According to (5), then membership degree, non-membership degree and hesitation part are summed to one.
Proof. We intend to obtain

$$\mu_1(x_i) - p\pi_{\check{A}}(x_i) + 1 - \mu_2(x_i) - p\pi_{\check{A}}(x_i) + \pi_{\check{A}}(x_i) = 1.$$

$$\text{Thus,} \because \mu_1(x_i) - \mu_2(x_i) + 1 + (1-2p)\pi_{\check{A}}(x_i)$$
$$= \mu_1(x_i) - \mu_2(x_i) + 1 + (1-2p)\frac{(\mu_1(x_i) - \mu_2(x_i))}{2p-1}.$$

Accordingly, if $\mu_1(x_i) > \mu_2(x_i)$, and then the above equation is summarized as

$$\mu_1(x_i) - \mu_2(x_i) + 1 + (1-2p)\frac{\mu_1(x_i) - \mu_2(x_i)}{2p-1}$$
$$\Rightarrow \mu_1(x_i) - \mu_2(x_i) + 1 - \left(\mu_1(x_i) - \mu_2(x_i)\right) = 1.$$

$\mu_1(x_i) \leq \mu_2(x_i)$ denotes the same. Thus, the proposition has been proved. □

Proposition 3. Let $\tilde{A}$ is defined as follows:

$$\tilde{A} = \sum_{i=1}^{n} [1/\mu_{\tilde{A}}(x_i) + 0/c(\mu_{\tilde{A}}(x_i))]\,/x_i,$$

and the fuzzy complement is Sugeno's negation. Assume that the conversion from $FS_2(X)$ to $IFSs(X)$ are validated, then

$$\mu_A(x_i) - p\pi_{\check{A}}(x_i) + (1 - c(\mu_A(x_i))) - p\pi_{\check{A}}(x_i) \leq 1, \tag{6}$$

when $-1 < \lambda < 0$.

Proof. Accordingly, to the above equation, it means that we need to approve.

$$\mu_A(x_i) - p\pi_{\check{A}}(x_i) - c(\mu_A(x_i)) - p\pi_{\check{A}}(x_i) \leq 0.$$

$$\because\ p \in [0,1], \text{and } -p\pi_{\check{A}}(x_i) \text{ is always negative}$$
$$\therefore \text{we only need to approve } \mu_A(x_i) - c(\mu_A(x_i)) \leq 0,$$

According to Sugeno and Terano (1977), $-1 < \lambda < 0$ is derived to obtain the bigger output then input values in Sugeno's class. Thus, the above equation is always true when $-1 < \lambda < 0$.

Conversely, assume that $\mu_A(x_i) + (1 - c(\mu_A(x_i))) \leq 1$, then we intend to apply the Sugeno's negation, $\mu_A(x_i) + (1 - \frac{1-\mu_A(x_i)}{1+\lambda\mu_A(x_i)}) \leq 1$.

$$\Rightarrow \frac{\mu_A(x_i) + \lambda(\mu_A(x_i))^2 - 1 + \mu_A(x_i)}{1 + \lambda\mu_A(x_i)} \leq 0.$$

$$\because 1 + \lambda\mu_A(x_i) \geq 0, \text{then}$$

$$\therefore\ 2\mu_A(x_i) + \lambda(\mu_A(x_i))^2 - 1 \leq 0. \tag{7}$$

If we can limit λ in $-1 < \lambda < 0$, then we can approve the Expression(7).

Thus, the proposition is proven. □

5 Application to Medical Diagnosis

A medical knowledge base focuses on how to properly diagnose *D* for a patient *T* with given values of symptoms *S*. Therefore, this section introduces a method for handling medical diagnostic problems based on the type-2 similarity. In a given pathology, assume that *S* denotes a set of symptoms, *D* denotes a set of diagnoses, and *T* denotes a set of patients. Analogous to De *et al.* notation of "Intuitionistic Medical Knowledge" (De et al. 2001), the authors defined "Type-2 Similarity Medical Knowledge" as a reasoning process from the set of symptoms *S* to the set of diagnoses *D*.

Consider four patients Al, Bob, Joe and Ted, i.e. *T*={Al, Bob, Joe, Ted}. Their symptoms are high temperature, headache, stomach pain, cough and chest pain, i.e. *S*={Temperature, Headache, Stomach pain, Cough, Chest pain}. The set of diagnosis is defined, i.e. *D*={Viral Fever, Malaria, Typhoid, Stomach problem, Heart problem}. Tables 2 and 3 summarize the intuitionistic fuzzy relations $T \rightarrow S$ and $S \rightarrow D$.

Hence, attempts to calculate for each patient t_j of his symptoms from a set of symptoms s_i characteristic of each diagnosis d_k. The reasoning process is as follows. (I) switch the acquired medical knowledge base from $IFS(X)$ to $FS_2(X)$, (II) to calculate the similarity of symptoms s_i between each patient t_j and each diagnosis d_k, where $i = 1, \cdots, 5, j = 1, \cdots, 4$ and $k = 1, \cdots, 5$, (III) to determine higher similarities, implying a proper diagnosis. The relationships of $T \rightarrow S$, $D \rightarrow S$, that is, the mapping from $IFS(\mathrm{X})$ to $FS_2(\mathrm{X})$ is switched and listed as follows (for the sake of simplicity, take *Al* and *Temperature* for example):

$$Al = \frac{\frac{1}{0.8 + p \cdot 0.1} + \frac{0}{0.9 + p \cdot 0.1}}{Temperature} + \frac{\frac{1}{0.6 + p \cdot 0.3} + \frac{0}{0.9 + p \cdot 0.3}}{Headache} + \frac{\frac{1}{0.2} + \frac{0}{0.2}}{Stomach\ pain} + \frac{\frac{1}{0.6 + p \cdot 0.3} + \frac{0}{0.9 + p \cdot 0.3}}{Cough} + \frac{\frac{1}{0.1 + p \cdot 0.3} + \frac{0}{0.4 + p \cdot 0.3}}{Chest\ pain}$$

and

$$Viral\ fever = \frac{\frac{1}{0.4 + p \cdot 0.6} + \frac{0}{1 + p \cdot 0.6}}{Temperature} + \frac{\frac{1}{0.3 + p \cdot 0.2} + \frac{0}{0.5 + p \cdot 0.2}}{Headache} + \frac{\frac{1}{0.1 + p \cdot 0.2} + \frac{0}{0.3 + p \cdot 0.2}}{Stomach\ pain} + \frac{\frac{1}{0.4 + p \cdot 0.3} + \frac{0}{0.7 + p \cdot 0.3}}{Cough} + \frac{\frac{1}{0.1 + p \cdot 0.2} + \frac{0}{0.3 + p \cdot 0.2}}{Chest\ problem}.$$

Notably, $p \in [0,1]$. Tables 4 and 5 list the similarity measure (2) for each patient from the considered set of possible diagnoses for $p = 0, 1$, respectively. In Table 6, Szmidt and Kacprzyk diagnosed by estimating the distances of three parameters: membership function, non-membership and the hesitation margin for all patient symptoms. That study also developed a geometrical interpretation to evaluate the similarity between IFSs, as well as properly diagnose in Table 7. De *et al.* defined the "Intuitionistic Medical Knowledge" to discuss how symptoms and diagnosis are related. Table 8 summarizes those results. The formal fuzzy similarity

$$S(A,B) = \frac{\sum_n \{1 - |\mu_A(x) - \mu_B(x)|\}}{n}$$

is applied in the medical diagnosis shown in Table 9 (Raha et al. 2002).

Table 10 displays all of the results from the above-mentioned methods. Medical software or computer systems assist physicians in patient care and facilitate the diagnosis of complex medical conditions. Determining which method can facilitate an exact diagnosis is extremely difficult. According to Table 10, Bob obviously suffers from stomach problems (all methods agree) and Joe is inflicted with typhoid (five out of the six methods agree). Above results demonstrate that the proposed theoretical method can facilitate the diagnosis. As for diagnoses of viral fever and malaria, our results indicate that these two diagnoses have difficulty in accuracy (nearly half of the methods approve one or the other diagnosis); in addition, these two symptoms are involved with each other. The proposed method differs from other methods in this respect. Moreover, the proposed method can be used as a facilitator. Distinct diagnosis results can be obtained by type-2 fuzzy sets, including a high effectiveness in handling imprecise and imperfect information than intuitionistic fuzzy sets can.

Table 1 Symmetric discrimination measures ('*' marks as the recognizing result)

	P_1	P_2	P_3
Dengfeng's Method [26]	0.78	0.8	0.85*
Mitchell's Method [27]	0.54	0.54	0.61*
Vlachos's Method [28]	0.4492	0.3487	0.2480*
Proposed Method	2.0169	2.0585	2.1253*

Table 2 Symptoms characteristic for the patients considered

	Temperature	*Headache*	*Stomach Pain*	*Cough*	*Chest pain*
Al	(0.8,0.1)	(0.6,0.1)	(0.2,0.8)	(0.6,0.1)	(0.1,0.6)
Bob	(0.0,0.8)	(0.4,0.4)	(0.6,0.1)	(0.1,0.7)	(0.1,0.8)
Joe	(0.8,0.1)	(0.8,0.1)	(0.0,0.6)	(0.2,0.7)	(0.0,0.5)
Ted	(0.6,0.1)	(0.5,0.4)	(0.3,0.4)	(0.7,0.2)	(0.3,0.4)

Table 3 Symptoms characteristic for the diagnoses considered

	Viral fever	*Malaria*	*Typhoid*	*Stomach problem*	*Chest problem*
Temperature	(0.4,0.0)	(0.7,0.0)	(0.3,0.3)	(0.1,0.7)	(0.1,0.8)
Headache	(0.3,0.5)	(0.2,0.6)	(0.6,0.1)	(0.2,0.4)	(0.0,0.8)
Stomach pain	(0.1,0.7)	(0.0,0.9)	(0.2,0.7)	(0.8,0.0)	(0.2,0.8)
Cough	(0.4,0.3)	(0.7,0.0)	(0.2,0.6)	(0.2,0.7)	(0.2,0.8)
Chest pain	(0.1,0.7)	(0.1,0.8)	(0.1,0.9)	(0.2,0.7)	(0.8,0.1)

Table 4 Result is measured by type-2 similarity of $p = 0$ ('*' marks as the diagnosis result)

	Viral fever	*Malaria*	*Typhoid*	*Stomach problem*	*Chest problem*
Al	0.68*	0.59	0.47	0.41	0.38
Bob	0.47	0.41	0.51	0.75*	0.42
Joe	0.59	0.44	0.62*	0.50	0.36
Ted	0.65*	0.56	0.51	0.48	0.32

Table 5 Result is measured by type-2 similarity of $p = 1$ ('*' marks as the diagnosis result)

	Viral fever	*Malaria*	*Typhoid*	*Stomach problem*	*Chest problem*
Al	0.71*	0.69	0.46	0.47	0.46
Bob	0.54	0.51	0.53	0.83*	0.44
Joe	0.63	0.48	0.65	0.66*	0.45
Ted	0.77*	0.62	0.55	0.56	0.4

Table 6 Result is measured by Szmidt et al. in (Szmidt et al., 2001) ('*' marks as the diagnosis result)

	Viral fever	*Malaria*	*Typhoid*	*Stomach problem*	*Chest problem*
Al	0.29	0.25*	0.32	0.53	0.58
Bob	0.43	0.56	0.33	0.14*	0.46
Joe	0.36	0.41	0.32*	0.52	0.57
Ted	0.25*	0.29	0.35	0.43	0.5

Table 7 Result is measured by Szmidt et al. in (Szmidt et al., 2004). ('*' marks as the diagnosis result)

	Viral fever	*Malaria*	*Typhoid*	*Stomach problem*	*Chest problem*
Al	0.75*	1.19	1.31	3.27	∞
Bob	2.1	3.73	1.1	0.35*	∞
Joe	0.87	1.52	0.46*	2.61	∞
Ted	0.95	0.77*	1.67	∞	2.56

Table 8 Results measured by De et al in (De et al., 2001). ('*' marks as the diagnosis result)

	Viral fever	*Malaria*	*Typhoid*	*Stomach problem*	*Chest problem*
Al	0.35	0.68*	0.57	0.04	0.08
Bob	0.2	0.08	0.32	0.57*	0.04
Joe	0.35	0.68*	0.57	0.04	0.05
Ted	0.32	0.68*	0.44	0.18	0.18

Table 9 Results measured by the formal fuzzy similarity. ('*' marks as the diagnosis result)

	Viral fever	*Malaria*	*Typhoid*	*Stomach problem*	*Chest problem*
Al	0.8	0.84*	0.82	0.56	0.52
Bob	0.74	0.58	0.8	0.86*	0.66
Joe	0.74	0.74	0.8*	0.54	0.5
Ted	0.78	0.82*	0.76	0.62	0.58

Table 10 All the considered results

	$p = 0$	$p = 1$	Szmidt in (2001)	Szmidt in (2004)	De in (2001)	Fuzzy similarity
Al	*Viral fever*	*Viral fever*	*Malaria*	*Viral fever*	*Malaria*	*Malaria*
Bob	*Stomach problem*	*Stomach problem*	*Stomach problem*	*Stomach problem*	*Stomach problem*	*Stomach problem*
Joe	*Typhoid*	*Typhoid*	*Malaria*	*Typhoid*	*Typhoid*	*Typhoid*
Ted	*Viral fever*	*Viral fever*	*Malaria*	*Malaria*	*Malaria*	*Malaria*

6 Conclusions

This study represents a new direction in approximate reasoning based on vague knowledge, which is associated with partial or incomplete truth values. The proposed theoretical method can handle vague quantities by converting this partial truth-value into a precisely quantified statement based on the type-2 fuzzy inference system. The membership functions of type-2 fuzzy sets have more parameters than those of type-1 fuzzy sets, providing a greater degree of design freedom. Therefore, type-2 fuzzy sets may outperform type-1 fuzzy sets, particularly in uncertain environments. Moreover, the proposed method can perform reasoning with incomplete knowledge; helping to yield meaningful resolutions using fuzzy sentential logic, and systematically compute uncertainty. Furthermore, a mutual switch between type-2 fuzzy sets and intuitionistic fuzzy sets was defined, and a medical diagnosis was generalized through switching and reasoning according to type-2 similarity. Consequently, easy comprehension and axiomatic definitions are provided during the switching process. Importantly, the proposed method makes it possible to extend the usage of type-2 fuzzy sets and renews the relationship between type-2 fuzzy sets and intuitionistic fuzzy sets.

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