Comparative Study of Fuzzy Information Processing in Type-2 Fuzzy Systems

Oscar Castillo and Patricia Melin

Abstract. Fuzzy information processing in type-2 fuzzy systems has been implemented in most cases based on the Karnik and Mendel (KM) and Wu-Mendel (WM) approaches. However, both of these approaches are time consuming for most real-world applications, in particular for control problems. For this reason, a more efficient method based on evolutionary algorithms has been proposed by Castillo and Melin (CM). This method is based on directly obtaining the type reduced results by using an evolutionnary algorithm (EA). The basic idea is that with an EA the upper and lower membership functions in the output can be obtained directly based on experimental data available for a particular problem. A comparative study (in control applications) of the three methods, based on accuracy and efficiency is presented, and the CM method is shown to outperform both the KM and WM methods in efficiency while accuracy produced by this method is comparable.

Keywords: Intelligent Control, Type-2 Fuzzy Logic, Interval Fuzzy Logic, Hybrid Intelligent Systems, Evolutionary Algorithm, Hardware Implementation, Fuzzy Controllers, Type Reduction.

1 Introduction

Uncertainty affects decision-making and appears in a number of different forms. The concept of information is fully connected with the concept of uncertainty. The most fundamental aspect of this connection is that the uncertainty involved in any problem-solving situation is a result of some information deficiency, which may be incomplete, imprecise, fragmentary, not fully reliable, vague, contradictory, or deficient in some other way. Uncertainty is an attribute of information (Zadeh 2005). The general framework of fuzzy reasoning allows handling much of this uncertainty and fuzzy systems that employ type-1 fuzzy sets represent uncertainty by numbers in the range [0, 1]. When a phenomenon is uncertain, like a measurement, it is difficult to determine its exact value, and of course type-1 fuzzy sets

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W. Pedrycz and S.-M. Chen (Eds.): Granular Computing and Intell. Sys., ISRL 13, pp. 75–93.
springerlink.com

make more sense than using sets (Zadeh 1975). However, it is not reasonable to use an accurate membership function for something uncertain, so in this case what we need is higher order fuzzy sets, those which are able to handle these uncertainties, like the so called type-2 fuzzy sets (Mendel 2004) (Mizumoto and Tanaka 1976). So, the amount of uncertainty can be managed by using type-2 fuzzy logic because it offers better capabilities to handle linguistic uncertainties by modeling vagueness and unreliability of information (Wagenknecht and Hartmann 1988).

Recently, we have seen the use of type-2 fuzzy sets in Fuzzy Logic Systems (FLS) in different areas of application (Castillo and Melin 2008). A novel approach for realizing the vision of ambient intelligence in ubiquitous computing environments (UCEs), is based on intelligent agents that use type-2 fuzzy systems which are able to handle the different sources of uncertainty in UCEs to give a good response (Doctor et al. 2005). There are also papers with emphasis on the implementation of type-2 FLS (Karnik et al. 1999) and in others, it is explained how type-2 fuzzy sets let us model the effects of uncertainties in rule-base FLS (Mendel and John 2002). In industry, type-2 fuzzy logic and neural networks was used in the control of non-linear dynamic plants (Melin and Castillo 2004); also we can find studies in the field of mobile robots (Astudillo et al. 2006) (Hagras 2004). In this paper we deal with the application of interval type-2 fuzzy control to non-linear dynamic systems. It is a well known fact, that in the control of real systems, the instrumentation elements (instrumentation amplifier, sensors, digital to analog, analog to digital converters, etc.) introduce some sort of unpredictable values in the information that has been collected (Castillo and Melin 2001). The controllers designed under idealized conditions tend to behave in an inappropriate manner (Castillo and Melin 2003). For this reason, type-2 fuzzy controllers, which can cope better with uncertainty, may have better performance under non-ideal conditions (Castillo and Melin 2004).

Fuzzy information processing in interval type-2 fuzzy systems has been implemented in most cases based on the Karnik and Mendel (KM) and Wu-Mendel (WM) approaches (Karnik and Mendel 2001). However, both of these approaches are time consuming for most real-world applications, in particular for control problems (Coupland and John 2008) (Starzewski 2009) (Martinez et al. 2009). For this reason, a more efficient method based on evolutionary algorithms (Sepulveda et al. 2007) has been proposed by Castillo and Melin (CM). This method is based on directly obtaining the type reduced results by searching the space of possible results using an evolutionary algorithm (Montiel et al. 2007). The basic idea is that with an EA the upper and lower membership functions in the output can be obtained directly based on experimental data for a particular problem. In this paper, a comparative study (in control applications) of the three methods, based on accuracy and efficiency is presented. The CM method is shown to outperform both the KM and WM methods in efficiency while accuracy is comparable. This fact makes the CM method a good choice for real-world control applications in which efficiency is of fundamental importance.

2 Fuzzy Logic Systems

In this section, a brief overview of type-1 and type-2 fuzzy systems is presented. This overview is considered to be necessary to understand the basic concepts needed to develop the methods and algorithms presented later in the paper.

2.1 Type-1 Fuzzy Logic Systems

Soft computing techniques have become an important research topic, which can be applied in the design of intelligent controllers, which utilize the human experience in a more natural form than the conventional mathematical approach (Zadeh 1971) (Zadeh 1973). A FLS, described completely in terms of type-1 fuzzy sets is called a type-1 fuzzy logic system (type-1 FLS). In this paper, the fuzzy controller has two input variables, which are the error $e(t)$ and the change of error $\Delta e(t)$,

$$e(t) = r(t) - y(t) \tag{1}$$

$$\Delta e(t) = e(t) - e(t-1) \tag{2}$$

The control system can be represented as in Figure 1.

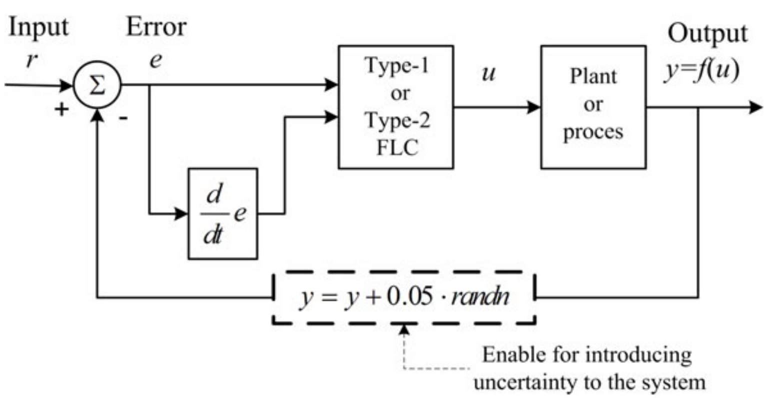

Fig. 1 System used for obtaining the experimental results.

2.2 Type-2 Fuzzy Logic Systems

If for a type-1 membership function, as in Figure 2, we blur its values to the left and to the right, as illustrated in Figure 3, then a type-2 membership function is obtained. In this case, for a specific value x', the membership function (u'), takes on different values, which are not all weighted the same, so we can characterize them by a distribution of membership values.

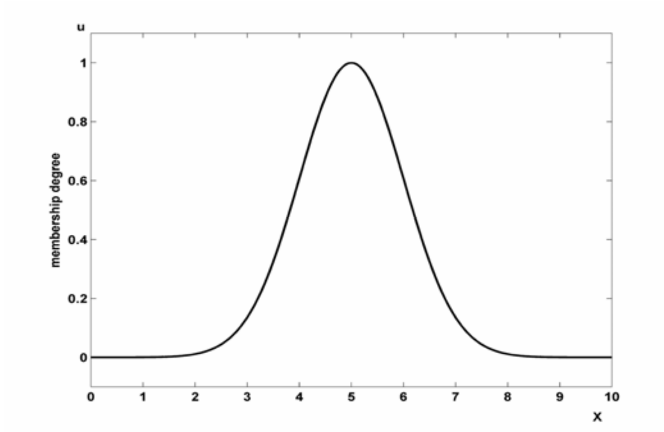

Fig. 2 Type-1 membership function.

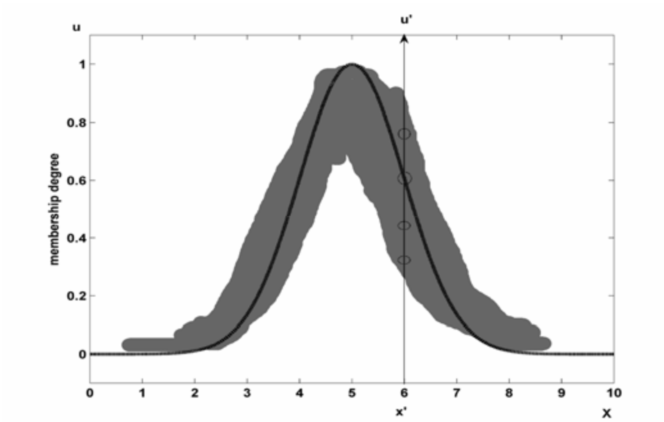

Fig. 3 Blurred type-1 membership function.

A type-2 fuzzy set $\widetilde{A}$, is characterized by the membership function (Mendel 2001) (Mendel and Mouzouris 1999):

$$\widetilde{A} = \{((x,u), \mu_{\widetilde{A}}(x,u)) \mid \forall x \in X, \forall u \in J_x \subseteq [0,1]\} \tag{3}$$

in which $0 \leq \mu_{\widetilde{A}}(x,u) \leq 1$. Another expression for $\widetilde{A}$ reads as,

$$\widetilde{A} = \int_{x \in X} \int_{u \in J_x} \mu_{\widetilde{A}}(x,u) / (x,u) \qquad J_x \subseteq [0,1] \tag{4}$$

Where $\int\int$ denotes the union over all admissible input variables *x* and *u*. For discrete universes of discourse, the symbol $\int$ is replaced by $\sum$ (Mendel and

John 2002). In fact $J_x \subseteq [0,1]$ represents the primary membership of *x*, and $\mu_{\tilde{A}}(x,u)$ is a type-1 fuzzy set known as the secondary set. Hence, a type-2 membership grade can be any subset in [0,1], the primary membership, and corresponding to each primary membership, there is a secondary membership (which can also be in [0,1]) that defines the possibilities for the primary membership. Uncertainty is represented by a region, which is called the footprint of uncertainty (FOU). When $\mu_{\tilde{A}}(x,u) = 1, \forall u \in J_x \subseteq [0,1]$ we have an interval type-2 membership function, as shown in Figure 4. The uniform shading for the FOU represents the entire interval type-2 fuzzy set and it can be described in terms of an upper membership function $\bar{\mu}_{\tilde{A}}(x)$ and a lower membership function $\underline{\mu}_{\tilde{A}}(x)$.

A FLS described using at least one type-2 fuzzy set is called a type-2 FLS. Type-1 FLSs are unable to directly handle rule uncertainties, because they use type-1 fuzzy sets that are certain (Castro et al. 2009). On the other hand, type-2 FLSs, are very useful in circumstances where it is difficult to determine an exact membership function, and there are measurement uncertainties (Mendel 2001) (Li and Zhang 2006).

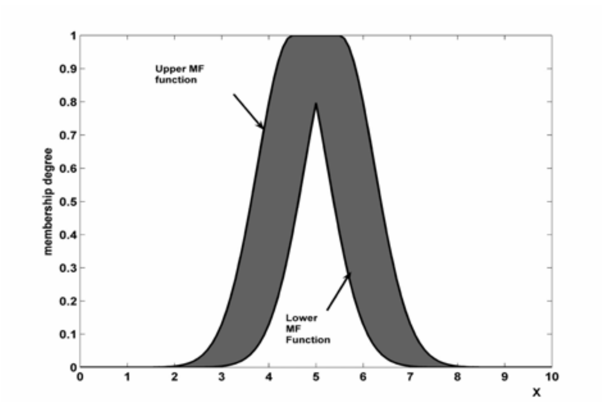

Fig. 4 Interval type-2 membership function.

A type-2 FLS is again characterized by IF-THEN rules, but its antecedent or consequent sets are now of type-2. Similar to a type-1 FLS, a type-2 FLS includes a fuzzifier, a rule base, fuzzy inference engine, and an output processor, as we can see in Figure 5. The output processor includes type-reducer and defuzzifier; it generates a type-1 fuzzy set output (type-reducer) or a crisp number (defuzzifier) (Karnik and Mendel 2001).

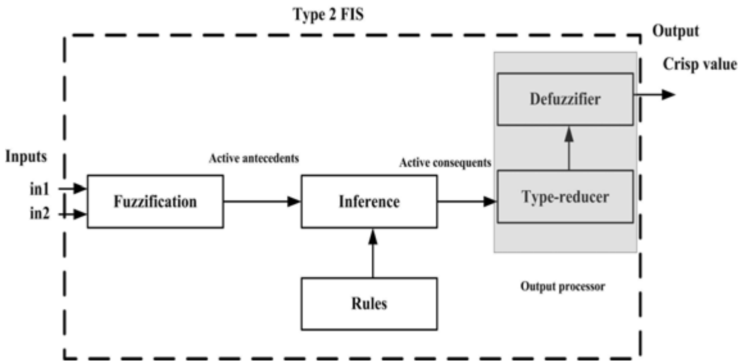

Fig. 5 Type-2 Fuzzy Logic System

2.2.1 Fuzzifier

The fuzzifier maps a point $\mathbf{x}=(x_1,\ldots,x_p)^T \in X_1 x X_2 x \ldots x X_p \equiv \mathbf{X}$ into a type-2 fuzzy set $\tilde{A}_x$ in **X**, interval type-2 fuzzy sets in this case. We will use type-2 singleton fuzzifier, in a singleton fuzzification, the input fuzzy set has only a single point with nonzero membership (Mendel 2001). $\tilde{A}_x$ is a type-2 fuzzy singleton if $\mu_{\tilde{A}_x}(x)=1/1$ for **x=x'** and $\mu_{\tilde{A}_x}(x)=1/0$ for all other **x≠x'**.

2.2.2 Rules

The structure of rules in a type-1 FLS and a type-2 FLS is the same, but in the latter the antecedents and the consequents will be represented by type-2 fuzzy sets. So for a type-2 FLS with p inputs $x_1 \in X_1,\ldots,x_p \in X_p$ and one output $y \in Y$, Multiple Input Single Output (MISO), if we assume there are M rules, the lth rule in the type-2 FLS can be written as follows (Mendel 2001):

$$R^l\text{: IF } x_1 \text{ is } \tilde{F}_1^l \text{ and } \cdots \text{and } x_p \text{ is } \tilde{F}_p^l \text{, THEN } y \text{ is } \tilde{G}^l \qquad l=1,\ldots,M \tag{5}$$

2.2.3 Inference

In the type-2 FLS, the inference engine combines rules and gives a mapping from input type-2 fuzzy sets to output type-2 fuzzy sets. It is necessary to compute the join ⊔, (unions) and the meet ⊓ (intersections), as well as use the extended sup-star compositions (sup star compositions) of type-2 relations. If $\tilde{F}^l{}_1 \times \cdots \times \tilde{F}^l{}_p = \tilde{A}^l$, expression (5) can be re-written as

$$R^l : \tilde{F}^l{}_1 \times \cdots \times \tilde{F}^l{}_p \rightarrow \tilde{G}^l = \tilde{A}^l \rightarrow \tilde{G}^l \qquad l=1,\ldots,M \tag{6}$$

R^l is described by the membership function $\mu_{R^l}(\mathbf{x}, y) = \mu_{R^l}(x_1,\ldots,x_p, y)$, where

$$\mu_{R^l}(\mathbf{x}, y) = \mu_{\tilde{A}^l \to \tilde{G}^l}(\mathbf{x}, y) \tag{7}$$

can be written as (Mendel 2001):

$$\mu_{R^l}(\mathbf{x}, y) = \mu_{\tilde{A}^l \to \tilde{G}^l}(\mathbf{x}, y) = \mu_{\tilde{F}_1^l}(x_1) \sqcap \cdots \sqcap \mu_{\tilde{F}_p^l}(x_p) \sqcap \mu_{\tilde{G}^l}(y)$$
$$= [\sqcap_{i=1}^{p} \mu_{\tilde{F}^l_i}(x_i)] \sqcap \mu_{\tilde{G}^l}(y) \tag{8}$$

In general, the p-dimensional input to R^l is given by the type-2 fuzzy set $\tilde{A}_x$ whose membership function is

$$\mu_{\tilde{A}_x}(\mathbf{x}) = \mu_{\tilde{x}_1}(x_1) \sqcap \cdots \sqcap \mu_{\tilde{x}p}(x_p) = \sqcap_{i=1}^{p} \mu_{\tilde{x}i}(x_i) \tag{9}$$

where $\tilde{X}_i (i = 1,..., p)$ are the labels of the fuzzy sets describing the inputs. Each rule R^l determines a type-2 fuzzy set $\tilde{B}^l = \tilde{A}_x \circ R^l$ such that:

$$\mu_{\tilde{B}^l}(y) = \mu_{\tilde{A}_x \circ R^l} = \sqcup_{x \in \mathbf{X}} \left[\mu_{\tilde{A}_x}(\mathbf{x}) \sqcap \mu_{R^l}(\mathbf{x}, y) \right] \qquad y \in Y \;\; l=1,\ldots,M \tag{10}$$

This equation is the input/output relation in Figure 5 between the type-2 fuzzy set that activates one rule in the inference engine and the type-2 fuzzy set at the output of that engine. In the FLS we used interval type-2 fuzzy sets and meet under product t-norm, so the result of the input and antecedent operations, which are contained in the firing set $\sqcap_{i=1}^{p} \mu_{\tilde{F}_{i_i}}(x'_i \equiv F^l(\mathbf{x}')$, is an interval type-1 set (Mendel 2001),

$$F^l(\mathbf{x}') = \left[\underline{f}^l(\mathbf{x}'), \overline{f}^l(\mathbf{x}') \right] \equiv \left[\underline{f}^l, \overline{f}^l \right] \tag{11}$$

where

$$\underline{f}^l(\mathbf{x}') = \underline{\mu}_{\tilde{F}_1^l}(x_1') * \cdots * \underline{\mu}_{\tilde{F}_p^l}(x_p') \tag{12}$$

$$\overline{f}^l(\mathbf{x}') = \overline{\mu}_{\tilde{F}_1^l}(x_1') * \cdots * \overline{\mu}_{\tilde{F}_p^l}(x_p') \tag{13}$$

where * is the product operation.

2.2.4 Type Reducer

The type-reducer generates a type-1 fuzzy set output, which is then converted in a crisp output through the defuzzifier. This type-1 fuzzy set is also an interval set, for the case of our FLS we used center of sets (cos) type reduction, $Y_{\cos}$ which is expressed as (Mendel 2001):

$$Y_{\cos}(\mathbf{x}) = [y_l, y_r] = \int_{y^1 \in [y_l^1, y_r^1]} \cdots \int_{y^M \in [y_l^M, y_r^M]} \int_{f^1 \in [\underline{f}^1, \overline{f}^1]} \cdots \int_{f^M \in [\underline{f}^M, \overline{f}^M]} 1 \Big/ \frac{\sum_{i=1}^{M} f^i y^i}{\sum_{i=1}^{M} f^i} \quad (14)$$

this interval set is determined by its two end points, y_l and y_r, which corresponds to the centroid of the type-2 interval consequent set $\tilde{G}^i$,

$$C_{\tilde{G}^i} = \int_{\theta_1 \in J_{y1}} \cdots \int_{\theta_N \in J_{yN}} 1 \Big/ \frac{\sum_{i=1}^{N} y_i \theta_i}{\sum_{i=1}^{N} \theta_i} = [y_l^{\,i}, y_r^{\,i}] \quad (15)$$

before the computation of $Y_{\cos}(\mathbf{x})$, we must evaluate equation (15), and its two end points, y_l and y_r. If the values of f_i and y_i that are associated with y_l are denoted f_l^i and y_l^i, respectively, and the values of f_i and y_i that are associated with y_r are denoted f_r^i and y_r^i, respectively, from 14, we have

$$y_l = \frac{\sum_{i=1}^{M} f_l^{\,i} y_l^{\,i}}{\sum_{i=1}^{M} f_l^{\,i}} \quad (16)$$

$$y_r = \frac{\sum_{i=1}^{M} f_r^{\,i} y_r^{\,i}}{\sum_{i=1}^{M} f_r^{\,i}} \quad (17)$$

2.2.5 Defuzzifier

From the type-reducer we obtain an interval set $Y_{\cos}$, to defuzzify it we use the average of y_l and y_r, so the defuzzified output of an interval singleton type-2 FLS is (Mendel 2001)

$$y(\mathbf{x}) = \frac{y_l + y_r}{2} \quad (18)$$

3 Average Type-2 FIS (CM Method)

In cases where the performance of an IT2FIS is important, especially in real time applications, an option to avoid the computational delay of type-reduction, is the Wu-Mendel method (Mendel 2001), which is based on the computation of inner and outer bound sets. Another option to improve computing speed in an IT2FIS, is to use the average of two type-1 FIS method (CM method), which was proposed for systems where the type-2 MFs of the inputs and output, have no uncertainty in the mean or center; it is achieved by substituting the IT2FIS with two type-1 FIS, located adequately at the upper and lower footprint of uncertainty (FOU) of the type-2 MFs (Sepulveda et al. 2007).

For the average (CM) method the fuzzification, the inference and the defuzzification stages for each FIS remain identical, the difference is at the output because the crisp value is calculated by taking the arithmetic average of the crisp output of each type-1 FIS, as it is shown in Figure 6, using the height method to calculate the defuzzified crisp output. In the average (CM) method, to achieve the defuzzification, one type-1 FIS is used for the upper bound of uncertainty, and the second FIS for the lower bound of uncertainty. So, as it was explained in Section 2, the defuzzification of a type-1 FIS is used in the average (CM) method and it is illustrated in Figure 6.

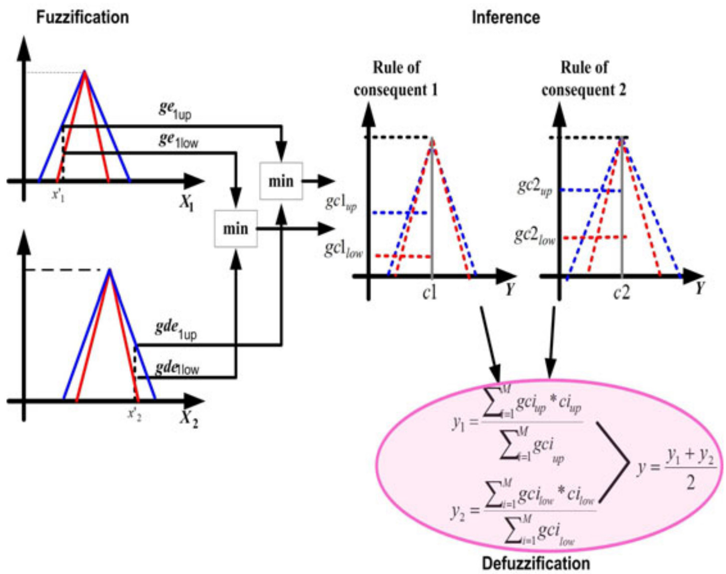

Fig. 6 The fuzzification, the inference and the defuzzification stages in the average (CM) method uses two type-1 FIS.

4 Experimental Results for Intelligent Control

The experimental results are presented here to show a comparison in the system's response in a feedback controller when using a type-1 FLC or a type-2 FLC. A set of five experiments is described in this section. The first two experiments were performed in ideal conditions, i.e., without any kind of disturbance. In the last three experiments, Gaussian noise was added to the feedback loop with the purpose of simulating, in a global way, the effects of uncertainty from several sources. Figure 1 shows the feedback control system that was used for obtaining the simulation results. The complete system was simulated, and the controller was designed to follow the input as closely as possible. The plant is a nonlinear system modeled with equation:

$$y(i) = 0.2 \cdot y(i-3) \cdot 0.07y(i-2) + 0.9 \cdot y(i-1) + 0.05u(i-1) + 0.5 \cdot u(i-2) \quad (19)$$

To illustrate the dynamics of the system, two different inputs are applied, first the input of equation is given by:

$$u(i) = \begin{cases} 0 & 1 \le i < 5 \\ .1 & 5 \le i < 10 \\ .5 & 10 \le i < 15 \\ 1 & 15 \le i < 20 \\ .5 & 20 \le i < 25 \\ 1 & 25 \le i < 30 \\ 0 & 30 \le i < 35 \\ 1.47 & 35 \le i < 40 \end{cases} \quad (20)$$

Now, for a slightly different input given by equation:

$$u(i) = \begin{cases} 0 & 1 \le i < 5 \\ .1 & 5 \le i < 10 \\ .5 & 10 \le i < 15 \\ 1 & 15 \le i < 20 \\ .5 & 20 \le i < 25 \\ 1 & 25 \le i < 30 \\ 0 & 30 \le i < 35 \\ 1.4 & 35 \le i < 40 \end{cases} \quad (21)$$

Going back to the control problem, this system given by equation (19) was used in Figure 1, under the name of plant or process, in this figure we can see that the controller's output is applied directly to the plant's input. Since we are interested in comparing the performance between type-1 and type-2 FLC systems, the controller was tested in two ways:

1. Considering the system as ideal. We have not introduced in the modules of the control system any source of uncertainty (experiments 1 and 2).
2. Simulation of the effects of uncertain modules (subsystems) response introducing some uncertainty (experiments 3, 4 and 5).

For both cases, as it is shown in Figure 1, the system's output is directly connected to the summing junction, but in the second case, the uncertainty was simulated introducing random noise with normal distribution (the dashed square in Figure 1). We added noise to the system's output $y(i)$ using a function "randn", which generates random numbers with a Gaussian distribution. The signal and the added noise in turn, were obtained by using the expression (22), the result $y(i)$ was introduced to the summing junction of the controller system. Note that in expression (22) we are using the value 0.05, for experiments 3 and 4, but in the set of tests for experiment 5, we varied this value to obtain different SNR values.

$$y(i) = y(i) + 0.05 \cdot randn \tag{22}$$

The system was tested using as input, a unit step sequence free of noise, $r(i)$. For evaluating the system's response and comparing between type 1 and type 2 fuzzy controllers, the performance criteria of Integral of Squared Error (ISE), Integral of Absolute Value of Error (IAE), and Integral of Time per Absolute Value of Error (ITAE) were used. In Table 3, we summarize the values obtained in an ideal system for each criterion considering 400 units of time. For calculating ITAE a sampling time of $T_s = 0.1$ sec. was considered. In Experiment 5, we tested the systems, type-1 and type-2 FLCs, introducing different values of noise η. This was done by modifying the signal to noise ratio SNR (Proakis and Manolakis 1996),

$$SNR = \frac{\sum |s|^2}{\sum |\eta|^2} = \frac{P_{signal}}{P_{noise}} \tag{23}$$

Because many signals have a very wide dynamic range, SNRs are usually expressed in the logarithmic decibel scale in SNR(db),

$$SNR(db) = 10 \log_{10} \left(\frac{P_{signal}}{P_{noise}} \right) \tag{24}$$

In Table 4, we show, for different values of SNR(db), the behavior of the errors ISE, IAE, ITAE for type-1 and type-2 FLCs. In all the cases the results for type-2 FLC are better than type-1 FLC. In the type-1 FLC, Gaussian membership functions (Gaussian MFs) for the inputs and for the output were used. A Gaussian MF is specified by two parameters {c,σ}:

$$\mu_A(x) = e^{-\frac{1}{2}\left(\frac{x-c}{\sigma}\right)^2} \tag{25}$$

c represents the MFs center and σ determines the spread of the MFs.

For each of the inputs of the type-1 FLC, three Gaussian MFs were defined as: negative, zero, positive. The universe of discourse for these membership functions is in the range [-10 10]. For the output of the type-1 FLC, we have five Gaussian MFs denoted by NG, N, Z, P and PG. Table 1 illustrates the characteristics of the MFs of the inputs and output of the type-1 FLC.

Table 1 Characteristics of the Inputs and Output of the Type-1 FLC.

Variable	Term	Center c	Standard deviation σ
Input e	negative	-10	4.2466
	zero	0	4.2466
	positive	10	4.2466
Input Δe	Negative	-10	4.2466
	Zero	0	4.2466
	positive	10	4.2466
Output *cde*	NG	-10	2.1233
	N	-5	2.1233
	Z	0	2.1233
	P	5	2.1233
	PG	10	2.1233

In experiments 2, 4, and 5, for the type-2 FLC, as in type-1 FLC, we also selected Gaussian MFs for the inputs and for the output, but in this case we have interval type-2 Gaussian MFs with a fixed center, c, and some spread σ, i.e.,

$$\mu_A(x) = e^{-\frac{1}{2}\left(\frac{x-c}{\sigma}\right)^2} \tag{26}$$

In terms of the upper and lower membership functions, we have for $\overline{\mu}_{\tilde{A}}(x)$,

$$\overline{\mu}_{\tilde{A}}(x) = N(c, \sigma_2; x) \tag{27}$$

and for the lower membership function $\underline{\mu}_{\tilde{A}}(x)$,

$$\underline{\mu}_{\tilde{A}}(x) = N(c, \sigma_1; x) \tag{28}$$

where $N(c, \sigma_2, x) \equiv e^{-\frac{1}{2}\left(\frac{x-c}{\sigma_2}\right)^2}$, and $N(c, \sigma_1, x) \equiv e^{-\frac{1}{2}\left(\frac{x-c}{\sigma_1}\right)^2}$, (Mendel 2001). Hence, in the type-2 FLC, for each input we defined three-interval type-2 fuzzy Gaussian MFs: negative, zero, positive in the interval [-10 10], as illustrated in Figures 7 and 8. For computing the output we have five interval type-2 fuzzy Gaussian MFs, which are NG, N, Z, P and PG, in the interval [-10 10], as can be seen in Figure 9. Table 2 shows the characteristics of the inputs and output of the type-2 FLC.

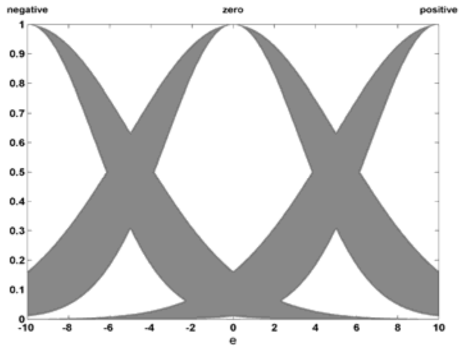

Fig. 7 Input e membership functions for the type-2 FLC.

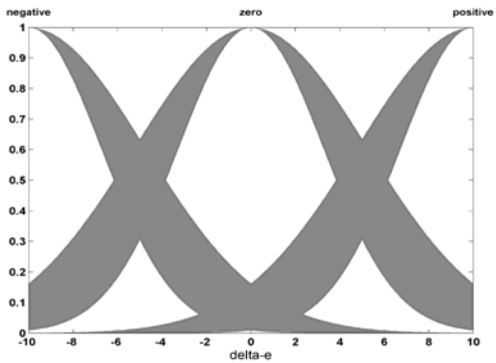

Fig. 8 Input Δe membership functions for the type-2 FLC.

In all experiments, we have a dash-dot line for illustrating the system's response and behavior of type-1 FLC, in the same sense, a continuous line for type-2 FLC. The reference *r* is shown with a dot line.

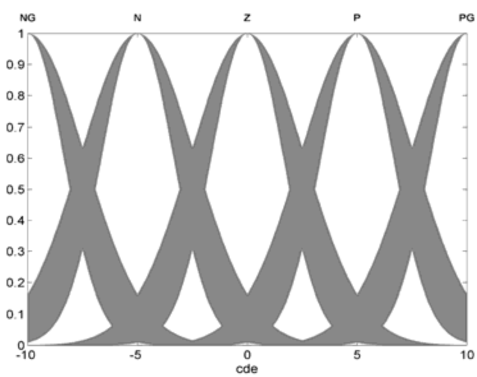

Fig. 9 Output cde membership functions for the type-2 FLC.

Table 2 Input and Output Parameters of the Type-2 FLC.

Variable	Term	Center c	Standard deviation σ_1	Standard deviation σ_2
Input e	negative	-10	5.2466	3.2466
	zero	0	5.2466	3.2466
	positive	10	5.2466	3.2466
Input Δe	Negative	-10	5.2466	3.2466
	Zero	0	5.2466	3.2466
	positive	10	5.2466	3.2466
Output *cde*	NG	-10	2.6233	1.6233
	N	-5	2.6233	1.6233
	Z	0	2.6233	1.6233
	P	5	2.6233	1.6233
	PG	10	2.6233	1.6233

Experiment 1: Simulation of an ideal system with a type-1 FLC.

In this experiment, uncertainty data was not added to the system, and the system response produced a settling time of about 140 units of time; i.e., the system tends to stabilize with time and the output will follow accurately the input. In Table 3, we listed the values of ISE, IAE, and ITAE for this experiment.

Table 3 Performance Criteria for Type-1 and Type-2 Fuzzy Controllers for 20 dB Signal to Noise Ratio (After 200 Samples).

Performance Criteria	**Type-1 FLC**		**Type-2 FLC**	
	Ideal System	Syst. with uncertainty	Ideal System	Syst. with uncertainty
ISE	7.65	19.4	6.8	18.3
IAE	17.68	49.5	16.4	44.8
ITAE	62.46	444.2	56.39	402.9

Experiment 2: Simulation of an ideal system using the type-2 FLC.

Here, the same test conditions of Experiment 1 were used, but in this case, we implemented the controller with type-2 fuzzy logic. The corresponding performance criteria are listed in Table 3. We can observe that when using a type-2 FLC we obtained the lower errors.

Experiment 3: System with uncertainty using a type-1 FLC.

In this case, expression (25) was used to simulate the effects of uncertainty introduced to the system by transducers, amplifiers, and any other element that in real world applications affects expected values. In this experiment the noise level was assumed to be in the range of 20 dB of SNR ratio.

Experiment 4: System with uncertainty using a type-2 FLC.

In this experiment, uncertainty was introduced in the system, in the same way as in Experiment 3. In this case, a type-2 FLC was used and the results obtained with a type-1 FLC (Experiment 3) were improved.

Experiment 5. Varying the Signal to Noise Ratio (SNR) in type-1 and type-2 FLCs.

To test the robustness of the type-1 and type-2 FLCs, we repeated experiments 3 and 4 giving different noise levels, going from 30 db to 8 db of SNR ratio in each experiment. In Table 4, we summarized the values for ISE, IAE, and ITAE considering 200 units of time with a P_{signal} of 22.98 dB in all cases. As it can be seen in Table 4, in presence of different noise levels, the behavior of type-2 FLC is in general better than type-1 FLC.

Table 4 Behavior of theType-1 and Type-2 Fuzzy Logic Controllers after Variation of Signal to Noise Ratio (Values Obtained for 200 Samples).

Noise variation				Type-1 FLC			Type-2 FLC		
SNR (dB)	SNR	Sum-Noise	Sum-Noise (dB)	ISE	IAE	ITAE	ISE	IAE	ITAE
8	6.4	187.42	22.72	321.1	198.1	2234.1	299.4	194.1	2023.1
10	10.05	119.2	20.762	178.1	148.4	1599.4	168.7	142.2	1413.5
12	15.86	75.56	18.783	104.7	114.5	1193.8	102.1	108.8	1057.7
14	25.13	47.702	16.785	64.1	90.5	915.5	63.7	84.8	814.6
16	39.88	30.062	14.78	40.9	72.8	710.9	40.6	67.3	637.8
18	63.21	18.967	12.78	27.4	59.6	559.1	26.6	54.2	504.4
20	100.04	11.984	10.78	19.4	49.5	444.2	18.3	44.8	402.9
22	158.54	7.56	8.78	14.7	42	356.9	13.2	37.8	324.6
24	251.3	4.77	6.78	11.9	36.2	289	10.3	32.5	264.2
26	398.2	3.01	4.78	10.1	31.9	236.7	8.5	28.6	217.3
28	631.5	1.89	2.78	9.1	28.5	196.3	7.5	25.5	180.7
30	1008	1.19	0.78	8.5	25.9	164.9	7	23.3	152.6

From Table 4, considering two examples, the extreme cases; we have for an SNR ratio of 8 dB, in type-1 FLC the following performance values ISE=321.1, IAE=198.1, ITAE=2234.1; and for the same case, in type-2 FLC, we have ISE=299.4, IAE=194.1, ITAE=2023.1. For 30 db of SNR ratio, we have for the type-1 FLC, ISE=8.5, IAE=25.9, ITAE=164.9, and for the type-2 FLC, ISE=7, IAE=23.3, ITAE=152.6. These values indicate a better performance of the type-2 FLC than type-1 FLC, because they are a representation of the errors, and as the error increases the performance of the system goes down.

Finally in Table 5 we show the values obtained in the optimization process of the optimal parameters for the MFs after 30 tests of: the variance, the Standard deviation, best ISE value, average ISE obtained with the optimized interval type-2 FLC, and with the average of two optimized type-1 FLCs (CM method).

Table 5 Comparison of the Variance, Standard Deviation, Best ISE value, ISE average, obtained with the Optimized Interval Type-2 FLC and the optimized Average of two Type-1 FLCs

Parameters	Type-2 FLC (WM Method)	Average of two Type-1 FLCs (CM)
Search Interval	2.74 to 5.75	2.74 to 5.75
Best ISE value	4.3014	4.1950
ISE Average	4.4005	4.3460
Standard deviation	0.1653	0.1424
Variance	0.0273	0.0203

In order to know which system behaves in a better way in the experiments, where uncertainty was simulated through different noise levels, first we compare the values of the ISE, IAE and ITAE errors obtained with the optimized parameters of the MFs of the interval type-2 FLC and the average of the two type-1 FLCs. The second comparison is made with the values of standard deviation and the variance obtained in each optimization process to get the optimal parameters of the MFs for the minimal ISE, IAE and ITAE errors. Figure 10 shows a comparison between the ISE values of the type-2 FLC based on the WM method (ISE T2) and the ISE of the type-2 FLC based on CM method (ISE PROM), which uses the average of two type-1 fuzzy systems. In this case, the ISE values are consistently lower for the CM method.

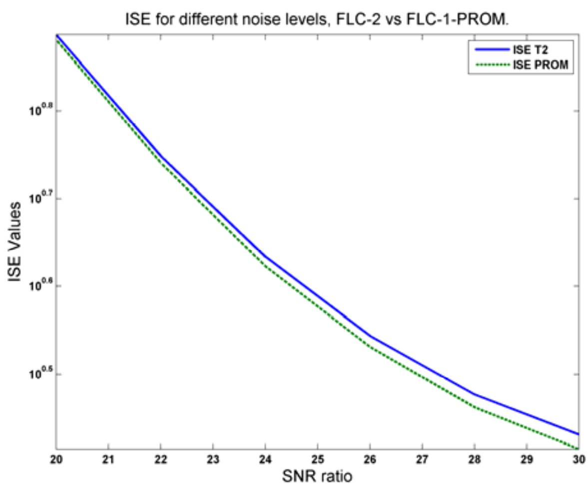

Fig. 10 Comparison of the ISE errors of optimized interval type-2 FLC and the optimized average of two type-1 FLCs, for different noise levels.

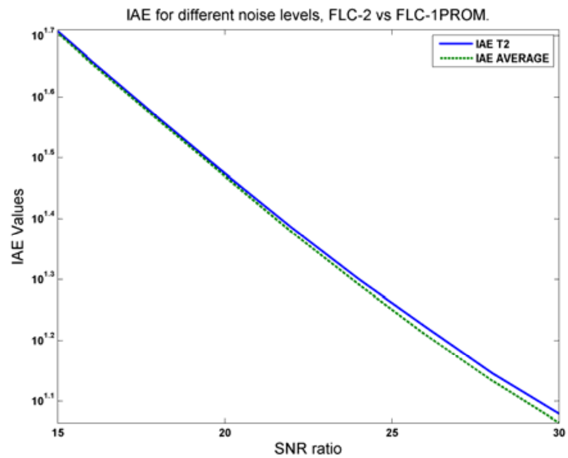

Fig. 11 Comparison of the IA errors of optimized interval type-2 FLC and the optimized average of two type-1 FLC, for different noise levels.

We can see in Tables 4 and 5 that with the average of two type-1 FLC optimized under certain FOU, it was obtained a minimum advantage in the values of ISE, IAE and ITAE errors than with the interval type-2 FLC optimized under the same conditions than the average of two type-1 FLC. In Figures 10, 11 and 12 it is shown that this advantage is notorious for low noise level.

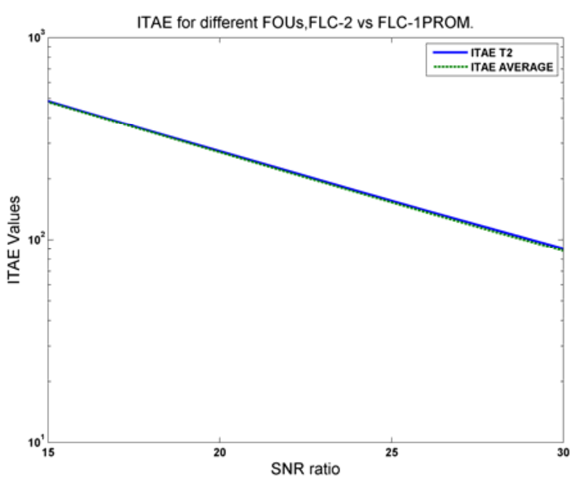

Fig. 12 Comparison of the ITAE errors of optimized interval type-2 FLC and the optimized average of two type-1 FLC, for different noise levels. Practically they behave in the same manner.

In this paper, an improved type-2 inference engine with the CM method was proposed and implemented into an FPGA. The type-2 engine process all the rules in parallel providing high speed computations, the processing time of the whole inference engine is just one clock cycle, approximately 0.02 microseconds for the Spartan 3 FPGA (Montiel et al. 2008). The processing time of a type-2 system implemented with the type-1 inference engine will not grow up since both inference engines (of the two type-1 fuzzy systems) are connected in parallel, hence the processing time remains almost the same for this stage. On the other hand, using KM or WM the times required for type-2 processing would be at least 1000 times more than with the CM method. This makes the proposed CM method of fundamental importance for real-world type-2 fuzzy logic applications, in particular for intelligent control.

5 Conclusions

We have presented the study of the controllers' design for nonlinear control systems using type-1 and type-2 fuzzy logic. We presented five experiments where we simulated the systems' responses with and without uncertainty presence. In the experiments, a quantification of errors was achieved and documented in detail for different criteria such as ISE, IAE, and ITAE. It was also shown that the lower overshoot and the best settling times were obtained using a type-2 FLC. Based on the experimental results, we can say that the best results are obtained using type-2 fuzzy systems. A comparative study of the three methods, based on accuracy and efficiency is presented, and the CM is shown to outperform both the KM and WM methods in efficiency while accuracy is comparable. In our opinion, this is because the lower and upper membership functions' estimations, of the outputs, are more easily found by directly obtaining them using an optimization method, like an evolutionary algorithm. This fact makes the CM method a good choice for real-world control applications in which efficiency is of fundamental importance.

References

Astudillo, L., Castillo, O., Aguilar, L.T.: Intelligent control of an autonomous mobile robot using type-2 fuzzy logic. In: Proceedings of the International Conference on Artificial Intelligence, Las Vegas Nevada (2006)

Castillo, O., Melin, P.: Soft computing for control of non-linear dynamical systems. Springer, Heidelberg (2001)

Castillo, O., Melin, P.: Soft computing and fractal theory for intelligent manufacturing. Springer, Heidelberg (2003)

Castillo, O., Melin, P.: A new approach for plant monitoring using type-2 fuzzy logic and fractal theory. Int. J. Gen. Syst. 33, 305–319 (2004)

Castillo, O., Melin, P.: Type-2 fuzzy logic: theory and applications. Springer, Heidelberg (2008)

Castro, J.R., Castillo, O., Melin, P., Rodriguez-Diaz, A.: A hybrid learning algorithm for a class of interval type-2 fuzzy neural networks. Inf. Sci. 179, 2175–2193 (2009)

Coupland, S., John, R.I.: New geometric inference techniques for type-2 fuzzy sets. Int. J. Approx. Rea. 49, 198–211 (2008)

Doctor, F., Hagras, H., Callaghan, V.: A type-2 fuzzy embedded agent to realize ambient intelligence in ubiquitous computing environments. Inf. Sci. 171, 309–334 (2005)

Hagras, H.: Hierarchical type-2 fuzzy logic control architecture for autonomous mobile robots. IEEE Trans. on Fuzzy Sys. 12, 524–539 (2004)

Karnik, N.N., Mendel, J.M., Liang, Q.: Type-2 fuzzy logic systems. IEEE Trans. Fuzzy Syst. 7, 643–658 (1999)

Karnik, N.N., Mendel, J.M.: Operations on type-2 fuzzy sets. Fuzzy Sets Syst. 122, 327–348 (2001)

Karnik, N.N., Mendel, J.M.: Centroid of a type-2 fuzzy set. Inf. Sci. 132, 195–220 (2001)

Li, S., Zhang, X.: Fuzzy logic controller with interval-valued inference for distributed parameter system. Int. J. Innovat. Comput. Inf. Control 2, 1197–1206 (2006)

Martinez, R., Castillo, O., Aguilar, L.T.: Optimization of interval type-2 fuzzy logic controllers for a perturbed autonomous wheeled mobile robot using Genetic Algorithms. Inf.Sci. 179, 2158–2174 (2009)

Melin, P., Castillo, O.: A new method for adaptive control of non-linear plants using type-2 fuzzy logic and neural networks. Int. J. Gen. Syst. 33, 289–304 (2004)

Mendel, J.M.: Uncertain rule-based fuzzy logic systems: introduction and new directions. Prentice Hall, New Jersey (2001)

Mendel, J.M.: Computing derivatives in interval type-2 fuzzy logic systems. IEEE Trans. Fuzzy Syst. 12, 84–98 (2004)

Mendel, J.M., John, R.I.: Type-2 Fuzzy Sets Made Simple. IEEE Trans. Fuzzy Syst. 10, 117–127 (2002)

Mendel, J.M., Mouzouris, G.C.: Type-2 fuzzy logic systems. IEEE Trans. Fuzzy Syst. 7, 643–658 (1999)

Mizumoto, M., Tanaka, K.: Some properties of fuzzy sets of type-2. Inform Control 31, 312–340 (1976)

Montiel, O., Castillo, O., Melin, P., Rodriguez-Diaz, A., Sepulveda, R.: Human evolutionary model: a new approach to optimization. Inf. Sci. 177, 2075–2098 (2007)

Montiel, O., Maldonado, Y., Sepulveda, R., Castillo, O.: Simple tuned fuzzy controller embedded into an FPGA. In: Proceedings of the 2008 NAFIPS Conference, New York, USA (2008)

Proakis, J.G., Manolakis, D.G.: Digital signal processing principles, algorithms and applications. Prentice Hall, New Jersey (1996)

Sepulveda, R., Castillo, O., Melin, P., Rodriguez-Diaz, A., Montiel, O.: Experimental study of intelligent controllers under uncertainty using type-1 and type-2 fuzzy logic. Inf. Sci. 177, 2023–2048 (2007)

Starczewski, J.T.: Efficient triangular type-2 fuzzy logic systems. Int. J. of Approx Rea. 50, 799–811 (2009)

Wagenknecht, M., Hartmann, K.: Application of fuzzy sets of type 2 to the solution of fuzzy equations systems. Fuzzy Sets Syst. 25, 183–190 (1988)

Zadeh, L.A.: Similarity relations and fuzzy ordering. Inf. Sci. 3, 177–206 (1971)

Zadeh, L.A.: Outline of a new approach to the analysis of complex systems and decision processes. IEEE Trans. Syst. Man and Cyber. 3, 28–44 (1973)

Zadeh, L.A.: The concept of a linguistic variable and its application to approximate reasoning. Part 1. Inf. Sci. 8, 199–249 (1975)

Zadeh, L.A.: Toward a generalized theory of uncertainty (GTU)-an outline. Inf. Sci. 172, 1–40 (2005)