

Gravitational Waves Astronomy: a cornerstone for gravitational theories

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Abstract Realizing a gravitational wave (GW) astronomy in next years is a great challenge for the scientific community. By giving a significant amount of new information, GWs will be a cornerstone for a better understanding of gravitational physics. In this paper we re-discuss that the GW astronomy will permit to solve a captivating issue of gravitation. In fact, it will be the definitive test for Einstein's general relativity (GR), or, alternatively, a strong endorsement for extended theories of gravity (ETG).

1 Introduction

The scientific community hopes in a first direct detection of GWs in next years [1]. The realization of a GW astronomy, by giving a significant amount of new information, will be a cornerstone for a better understanding of gravitational physics. In fact, the discovery of GW emission by the compact binary system PSR1913+16, composed by two Neutron Stars [2], has been, for physicists working in this field, the ultimate thrust allowing to reach the extremely sophisticated technology needed for investigating in this field of research. In this paper we re-discuss that the GW astronomy will permit to solve a captivating issue of gravitation. In fact, it will be the definitive test for Einstein's GR, or, alternatively, a strong endorsement for ETG [3].

Although Einstein's GR [4] achieved great success (see for example the opinion of Landau who says that GR is, together with quantum field theory, the best scientific theory of all [5]) and withstood many experimental tests, it also displayed many shortcomings and flaws which today make theoreticians question whether it is the definitive theory of gravity, see [6]-[19] and references within. As distinct from other field theories, like the electromagnetic theory, GR is very difficult to quantize. This fact rules out the possibility of treating gravitation like other quantum theories,

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and precludes the unification of gravity with other interactions. At the present time, it is not possible to realize a consistent quantum gravity theory which leads to the unification of gravitation with the other forces. From an historical point of view, Einstein believed that, in the path to unification of theories, quantum mechanics had to be subjected to a more general deterministic theory, which he called *generalized theory of gravitation*, but he did not obtain the final equations of such a theory (see for example the biography of Einstein which has been written by Pais [20]). At present, this point of view is partially retrieved by some theorists, starting from the Nobel Laureate G. 't Hooft [21].

One can define *ETG* those semi-classical theories where the Lagrangian is modified, in respect of the standard Einstein-Hilbert gravitational Lagrangian [5], adding high-order terms in the curvature invariants (terms like R^2 , $R^{\alpha\beta}R_{\alpha\beta}$, $R^{\alpha\beta\gamma\delta}R_{\alpha\beta\gamma\delta}$, $R\Box R$, $R\Box^k R$) or terms with scalar fields non-minimally coupled to geometry (terms like $\phi^2 R$) [6]-[19]. In general, one has to emphasize that terms like those are present in all the approaches to the problem of unification between gravity and other interactions. Additionally, from a cosmological point of view, such modifications of GR generate inflationary frameworks which are very important as they solve many problems of the standard universe model (see [22] for a review).

In the general context of cosmological evidence, there are also other considerations which suggest an extension of GR. As a matter of fact, the accelerated expansion of the universe, which is observed today, implies that cosmological dynamics is dominated by the so called Dark Energy, which gives a large negative pressure. This is the standard picture, in which this new ingredient is considered as a source on the right-hand side of the field equations. It should be some form of un-clustered, non-zero vacuum energy which, together with the clustered Dark Matter, drives the global dynamics. This is the so called “concordance model” (Λ CDM) which gives, in agreement with the CMBR, LSS and SNeIa data, a good picture of the observed Universe today, but presents several shortcomings such as the well known “coincidence” and “cosmological constant” problems [23]. An alternative approach is changing the left-hand side of the field equations, to see if the observed cosmic dynamics can be achieved by extending General Relativity [6]-[19]. In this different context, it is not required to find candidates for Dark Energy and Dark Matter, that, till now, have not been found; only the “observed” ingredients, which are curvature and baryonic matter, have to be taken into account. Considering this point of view, one can think that gravity is different at various scales and there is room for alternative theories [6]-[19]. In principle, the most popular Dark Energy and Dark Matter models can be achieved considering $f(R)$ theories of gravity, where R is the Ricci curvature scalar, and/or scalar-sensor gravity (STG) [6]-[19]. In this picture, even the sensitive detectors for gravitational waves (GWs), like bars and interferometers, whose data analysis recently started [1], could, in principle, be important to confirm or rule out the physical consistency of GR or of any other theory of gravitation. This is because, in the context of ETG, some differences between GR and the others theories can be pointed out starting from the linearized theory of gravity, see [3] and [24]-[28].

Now, let us consider this issue in more detail.

2 Using gravitational waves to discriminate

GWs are a consequence of Einstein's GR [4], which presuppose GWs to be ripples in the space-time curvature travelling at light speed [29, 30]. Only asymmetric astrophysics sources can emit GWs. The most efficient are coalescing binaries systems, while a single rotating pulsar can rely only on spherical asymmetries, usually very small. Supernovae could have relevant asymmetries, being potential sources[1].

The most important cosmological source of GWs is, in principle, the so called stochastic background of GWs which, together with the Cosmic Background Radiation (CBR), would carry, if detected, a huge amount of information on the early stages of the Universe evolution [25], [31]-[35]. The existence of a relic stochastic background of GWs is a consequence of general assumptions. Essentially it derives from a mixing between basic principles of classical theories of gravity and of quantum field theory [31]-[34]. The model derives from the inflationary scenario for the early universe [22], which is tuned in a good way with the WMAP data on the CBR (in particular exponential inflation and spectral index ≈ 1 [36]. The GWs perturbations arise from the uncertainty principle and the spectrum of relic GWs is generated from the adiabatically-amplified zero-point fluctuations [31]-[34]. The analysis has been recently generalized to ETG in [25] and [35].

In 1957, F.A.E. Pirani, who was a member of the Bondi's research group, proposed the geodesic deviation equation as a tool for designing a practical GW detector [37]. Pirani showed that if a GW propagates in a spatial region where two test masses are present, the effect is to drive the masses to have oscillations.

In 1959, Joseph Weber studied a detector that, in principle, might be able to measure displacements smaller than the size of the nucleus [38]. He developed an experiment using a large suspended bar of aluminium, with a high resonant Q at a frequency of about 1 kHz . Then, in 1960, he tried to test the general relativistic prediction of GWs from strong gravity collisions [39] and, in 1969, he claimed evidence for observation of gravitational waves (based on coincident signals) from two bars separated by 1000 km [40]. He also proposed the idea of doing an experiment to detect gravitational waves using laser interferometers [40]. In fact, all the modern detectors can be considered like being originated from early Weber's ideas [1].

In recent papers [26, 27] it has been shown that GWs from ETG generate different oscillations of test masses, with respect to GWs from standard GR. Thus, an accurate analysis of such a motion can be used in order to discriminate among various theories.

In general, GWs manifest them-self by exerting tidal forces on the test-masses which are the mirror and the beam-splitter in the case of an interferometer [1].

Working with $G = 1$, $c = 1$ and $\hbar = 1$ (natural units), the line element for a GW arising from standard GR and propagating in the z direction is [3, 28, 41]

$$ds^2 = dt^2 - dz^2 - (1 + h_+)dx^2 - (1 - h_+)dy^2 - 2h_{\times}dxdy, \quad (1)$$

where $h_+(t-z)$ and $h_\times(t-z)$ are the weak perturbations due to the $+$ and the \times polarizations which are expressed in terms of synchronous coordinates in the transverse-traceless (TT) gauge [3, 28, 41].

In the case of standard GR the motion of test masses, due to GWs and analysed in the gauge of the local observer, is well known [41]. By putting the beam-splitter in the origin of the coordinate system, the components of the separation vector are the mirror's coordinates. At first order in h_+ , the displacements of the mirror due by the $+$ polarization of a GW propagating in the z direction are given by [41]:

$$\delta x_M(t) = \frac{1}{2}x_{M0}h_+(t) \quad (2)$$

and

$$\delta y_M(t) = -\frac{1}{2}y_{M0}h_+(t), \quad (3)$$

where x_{M0} and y_{M0} are the initial (unperturbed) coordinates of the mirror. The \times polarization generates an analogous motion for test masses which are rotated of 45-degree with respect the z axis [41].

In all ETG a third *massive* polarization of GWs is present [3], [24]-[27], which is usually labelled with Φ , and the line element for such a third polarization can be always put in a conformally flat form in both of the cases of STG and $f(R)$ theories [24]-[27]:

$$ds^2 = [1 + \Phi(t - v_G z)](-dt^2 + dz^2 + dx^2 + dy^2). \quad (4)$$

v_G in (4) is the particle's velocity (the group velocity in terms of a wave-packet [3], [24]-[27]). In the case of STG the third mode can be massless. In that case $v_G = 1$ and, at first order in Φ , the displacements of the mirror due to these massless scalar GWs are given by [26]

$$\delta x_M(t) = \frac{1}{2}x_{M0}\Phi(t) \quad (5)$$

and

$$\delta y_M(t) = \frac{1}{2}y_{M0}\Phi(t). \quad (6)$$

In the case of massive scalar GWs and of $f(R)$ theories it is [26, 27]

$$\delta x_M(t) = \frac{1}{2}x_{M0}\Phi(t)$$

$$\delta y_M(t) = \frac{1}{2}y_{M0}\Phi(t) \quad (7)$$

$$\delta z_M(t) = -\frac{1}{2}m^2 z_{M0}\Psi(t),$$

where [26, 27]

$$\ddot{\Psi}(t) \equiv \Phi(t). \quad (8)$$

Note: the most general definition is $\psi(t - v_G z) + a(t - v_G z) + b$, but one assumes only small variations of the positions of the test masses, thus $a = b = 0$ [26, 27].

Then, in the case of massive GWs a longitudinal component is present because of the presence of a small mass m [26, 27]. As the interpretation of Φ is in terms of a wave-packet, solution of the Klein-Gordon equation [26, 27]

$$\square\Phi = m^2\Phi, \quad (9)$$

it is also

$$\psi(t - v_G z) = -\frac{1}{\omega^2}\Phi(t - v_G z). \quad (10)$$

Thus, if advanced projects on the detection of GWs will improve their sensitivity allowing to perform a GWs astronomy (this is due because signals from GWs are quite weak) [1], one will only have to look which is the motion of the mirror in respect to the beam splitter of an interferometer in the locally inertial coordinate system in order to understand which is the correct theory of gravity. If such a motion will be governed only by Eqs. (2) and (3) we will conclude that GR is the ultimate theory of gravity. If the motion of the mirror is governed also by Eqs. (5) and (6), in addition to the motion arising from Eqs. (2) and (3), we will conclude that massless STG is the correct theory of gravitation. Finally, if the motion of the mirror is governed also by Eqs. (7) in addition to the ordinary motion of Eqs. (2) and (3), we will conclude that the correct theory of gravity will be massive STG which is equivalent to $f(R)$ theories. Even if such signals will be quite weak, a consistent GWs astronomy will permit to understand which is the direction of the propagating GW by using coincidences between various detectors and to compute a hypothetical group velocity v_G by using delay times, thus, all the quantities of the above equations could be, in principle, determined.

3 Conclusion remarks

We re-discussed that the GW astronomy will permit to solve a captivating issue of gravitation. If advanced projects on the detection of GWs will improve their sensitivity allowing to perform a GWs astronomy, such a GWs astronomy will be the definitive test for Einstein's GR, or, alternatively, a strong endorsement for ETG. In fact, a careful analysis of the motion of the mirror of the interferometer with respect to the beam splitter will permit to discriminate among GR and ETG.

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