Chapter 4
Proportions

The simplest quantity that can be measured from an image showing different facies or lithotypes is the percentage of space taken up by each facies. So the first step in a plurigaussian study is to calculate the proportions from the experimental data. In the stationary case, we only need calculate the percentage of all the data that belong to each facies to get the proportions. However in most practical cases, the geology is more complicated. For example, petroleum reservoirs are not stationary in the vertical direction because of cyclic changes during their deposition. Vertical proportion curves were designed to quantify these changes. We will show how to compute and interpret these curves. In more complicated cases the proportions vary laterally as well. In that case we build up a 3D matrix of proportion curves. Although plurigaussian simulations were first developed for the oil industry, they are now used in mining as well. Proportion curves and 3D proportion matrices are also used there to estimate the proportions of each rock type.

The first section in this chapter describes vertical proportion curves for the simplest case where the reservoir or deposit is stationary horizontally. These curves summarise the vertical variability in the proportions. After that we treat the more general case of non-stationarity where we use 3D proportion matrices. One key factor when calculating vertical proportion curves (VPC, for short) is the choice of the reference level. An example is presented to illustrate the impact of inappropriate choices on VPC.

How to Calculate Vertical Proportion Curves

Vertical proportion curves first proposed by Matheron et al. (1987) are a simple tool for quantifying the evolution in the amount of each facies or lithotype present as a function of depth. They are computed along lines parallel to the chosen reference level (generally a chrono-stratigraphic marker). The results are presented as a graph showing the proportion of each facies at each level. We illustrate this procedure using a simple example.

Figure 4.1 shows five fictive wells each containing five core sections of equal length. Three lithotypes which we call sandstone, shaly-sandstone and shale, have

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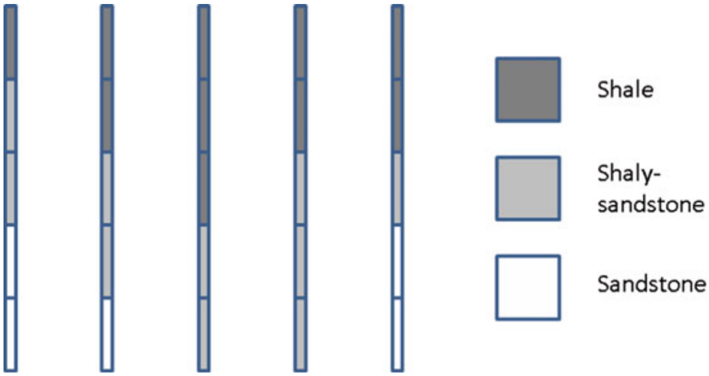

Fig. 4.1 Five drill-holes each containing five core sections of equal length

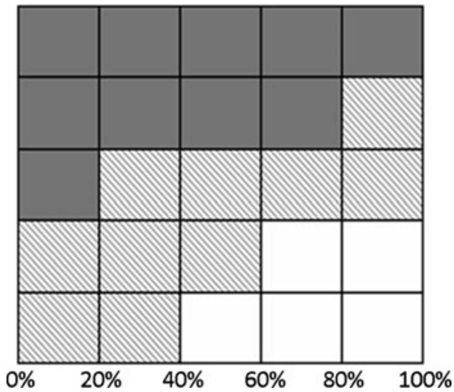

Fig. 4.2 Vertical Proportion Curves showing the proportion of each facies per level

been recorded. Looking at this figure, we see that there is more shale at the top of the wells than at the bottom. In fact, all five core sections in the top row are shale (i.e. 100% shale). In the second row there is 80% shale and 20% shaly-sandstone. At the bottom there is 40% shaly-sandstone and 60% sandstone, but no shale.

These proportions are presented graphically in Fig. 4.2. The order that the facies are arranged in is important. It must reflect the evolution seen in the geology. In this case the interpretation is simple: the grain size at the top of the wells is much smaller than at the bottom. This indicates sedimentological evolution from a high energy medium at the bottom to a low energy one at the top.

Example 1: The Ravenscar Sequence

The Ravenscar sequence which outcrops in cliff faces near Scarborough in North Yorkshire, consists of about 200 m of siliclastic sediments dating from the Middle

Jurassic era. The Cleveland basin to which it belongs is a progradation of a deltaic system with marine influences. Figure 4.3 (taken from Beucher et al. 2006) shows a conceptual model of it.

The Ravenscar sequence consists of seven units. From the top down, these are Scarborough, Gristhorpe, Millepore, Sycarham, Cloughton, Ellerbeck and Saltwick. Each had a distinctly different depositional environment. For example, the Saltwick unit was laid down in a continental environment whereas the Ellerbeck unit is characterised by steady changes in the sea level. Figures 4.4 and 4.5 (from Beucher et al. 2006) show the vertical proportion curves for these two units. The three main channel producing episodes are clearly visible in the VPC for the Saltwick unit. In contrast to this, the VPC for the Ellerbeck unit shows regular changes from an open

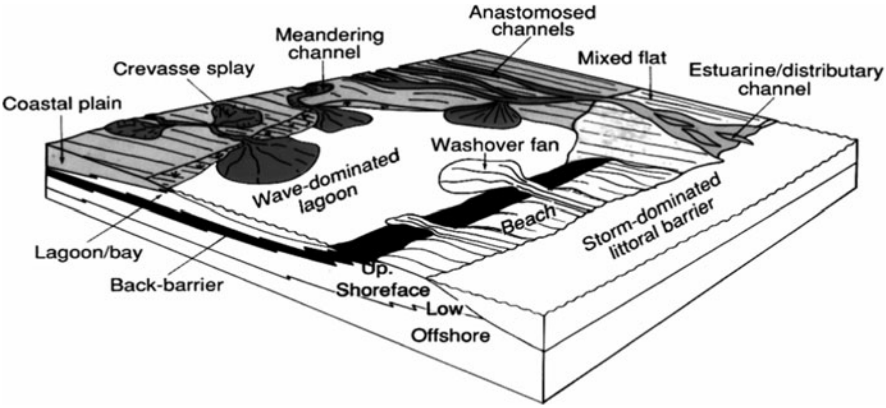

Fig. 4.3 Conceptual model of the deposition

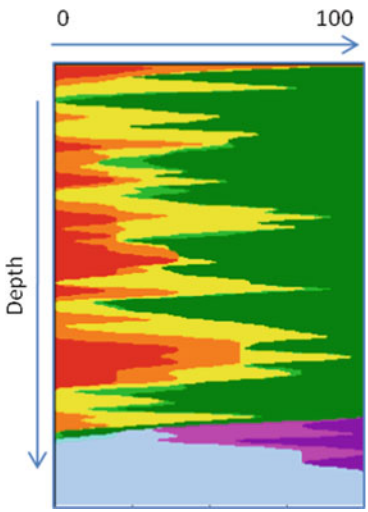

Fig. 4.4 Vertical proportion curves for the Saltwick unit of the Ravenscar sequence

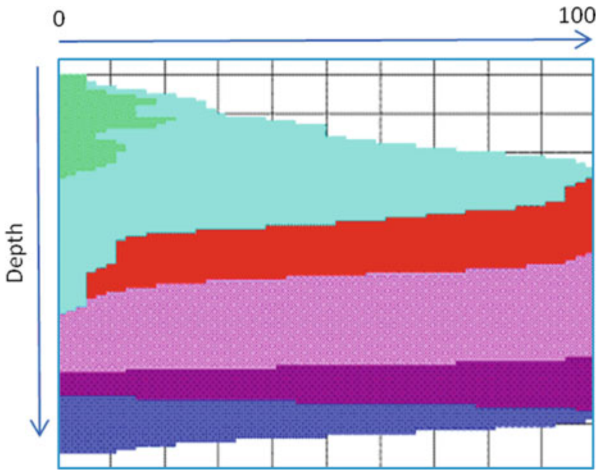

Fig. 4.5 Vertical proportion curves for the Ellerbeck unit of the Ravenscar sequence

marine environment (marine mudstones and marls) at the bottom of the unit to continental facies at the top. Intermediate lithounits include argillaceous sandstones and transgressive sandstone corresponding to the shoreface and clean sandstone from the foreshore and upper shoreface. The top of the unit is characterised by a lagoon environment with a floodplain, continental mudstone and argillaceous sands.

Example 2: Facies with Contrasting Anisotropies in a Gold Deposit

The Lupin mine in Northwest Canada is a good example of a stratiform banded iron formation (BIF) hosted gold deposit. Some of the gold is uniformly disseminated in thin, laterally continuous units of sulphur-rich BIF while the rest is contained in steeply inclined quartz veins that have overprinted the BIF. See Kerswill et al. (1996) for more information. The two gold-bearing lithofacies and the barren background are present in the schematic vertical cross-section of the orebody shown in Fig. 4.6 from Roth et al. (1998).

Figure 4.7 shows the experimental VPC computed from Fig. 4.6 with the proportion of quartz vein in dark grey and that of sulphide BIF in light grey. The proportion of quartz vein changes gradually, reaching a maximum near the middle of the cross-section. In contrast to this, the proportion of sulphide BIF is erratic.

Before this VPC can be used to calculate the thresholds we have to decide whether these peaks are representative of genuine geological features in the deposit or whether they are due to statistical fluctuations. Looking back at Fig. 4.4, we see that same problem arose with the VPC for the Saltwick unit of the Ravenscar sequence. If the peaks are a real feature they should be modelled. Otherwise we should smooth them. Alternatively, the reference level has to be changed. This decision can only be made after discussion between the geologist or the engineer

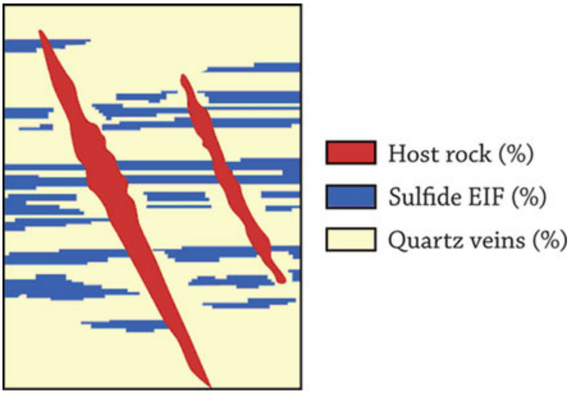

Fig. 4.6 Schematic vertical cross-section of the Lupin orebody showing two mineralised lithofacies with contrasting anisotropies on a waste background

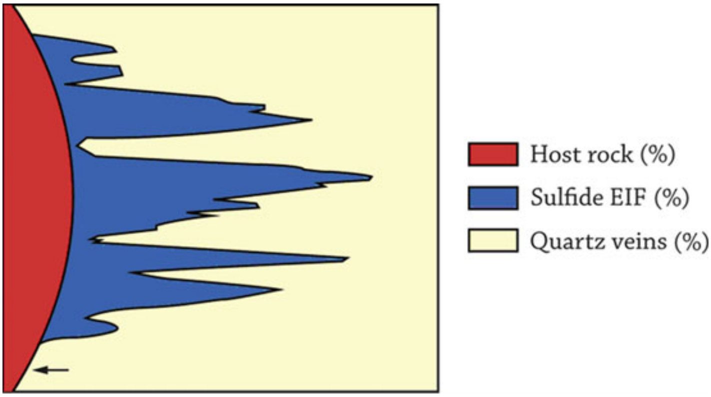

Fig. 4.7 The corresponding vertical proportion curves

and the geostatistician. In both of these cases it was decided that the peaks represented real features of the deposit/reservoir that should be reproduced in the simulations. So the thresholds were set to generate more BIF (or more channels as the case may be) at the corresponding depths.

Horizontal Non-stationarity

The vertical proportion curves presented in the previous section were computed assuming that the deposit/reservoir was at least horizontally stationary. In many cases, this is not true. The proportions vary laterally from one area to another. The Millpore and Gristhorpe units of the Ravescar sequence in Yorkshire are a typical example of this. The depositional environment varied from marine to littoral deposits. It is characterised by marine mudstone, wash-over argillaceous sandstones through to deltaic environment with a floodplain and fluvial and deltaic channels. Beucher et al. (2006) constructed a 3D matrix of proportion curves to model the evolution in the proportions both vertically and laterally. Their first steps was to group wells locally and compute vertical proportion curves for each set. The percentage of each of eight lithotypes was then kriged at the nodes of a regular grid and the kriged estimates were recombined to give vertical proportion curves. As the proportions were kriged individually, they do not add up to 100% at each grid node. So the values were rescaled to guarantee this. Figure 4.8 shows nine cells from the 3D proportion matrix.

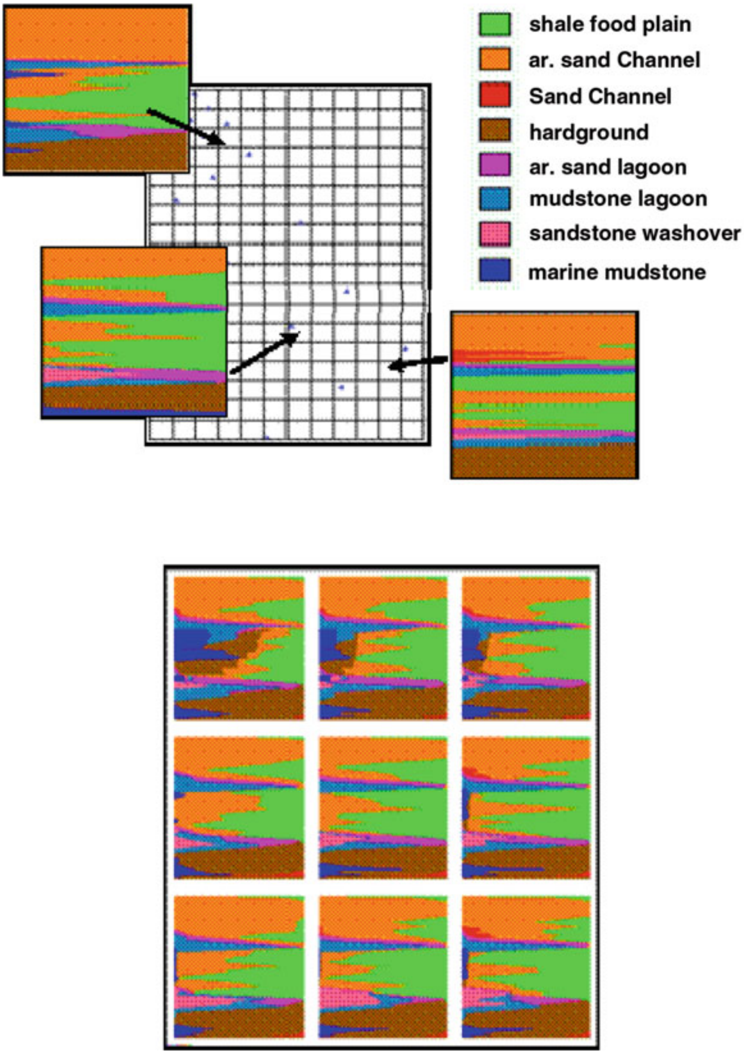

Fig. 4.8 Kriging VPC onto a regular grid (above). Representing nine cells from the 3D proportion matrix (below)

Having to rescale the proportions to make them sum to 100% is not entirely satisfactory. One could envisage using cokriging, but this would result in difficulties setting up the structural model (i.e. the variograms and the cross-variograms). A more fundamental reason for using a simple technique like kriging is that it is then easier to incorporate additional information, for example from seismic data. Moulière et al. 1997; Moulière 1998 considered a case where a 2D seismic attribute gave the cumulative thickness of one lithotype (shaly sandstone) for each unit in the formation. So it was important to incorporate this constraint into the proportion curves. She proposed two methods for doing this: one based on cokriging and the other using kriging. She showed that the simulations obtained after taking account of the available seismic information were much more realistic than those based on data from the limited number of wells that were available.

Lateral non-stationarity was also encountered by Doligez et al. (1994a, b) in their study of the Cajigar 2 succession. As the proportions changed from one area to another, they constructed a matrix of lithofacies proportions on a 300 m $\times$ 300 m grid. Each horizontal proportion curve was obtained by averaging the percentages of each lithofacies in the three levels above and below, as well as the level itself. This has the advantage of maintaining the vertical correlations between successive levels, as well as smoothing the results. Whereas in vertical proportion curves, the proportions are plotted horizontally as a function of the vertical height, the horizontal proportion curves show the proportions on the Y-axis with horizontal coordinate along the X-axis.

Non-stationarity can also arise when studying mining deposits. Compared with petroleum data, far more drill holes are usually available but there is often no chronostratigraphic marker to help interpret the geology. Betzhold and Roth (2000) studied the Mantos Blancos copper orebody which is located 45 km northeast of the city of Antofagasta in the north of Chile.

Their objective in simulating the orebody was to improve the ore homogenisation procedure by providing mine-planning engineers with more accurate images of the key mineralogical units. The rocks at Mantos Blancos come from a volcanic sequence consisting of andesitic flows and flow breccia at the top, then flow-breccia and flows of porphyritic dacite, and at the bottom, flows of augen (quartz-eye) dacite.

The sequence dips to the southwest at 10–20° in the mine area. The mineralisation forms an irregular blanket ranging from 100 to 200 m thick. As different mineralogical ensembles with similar average grades do not increase the grade variability, they can effectively be grouped together and treated as one new unit.

This led Betzhold and Roth to define three ore type classifications:

- High grade ore comprising chalcocite-bornite ore
- Low grade ore comprising chalcopyrite and chalcopyrite-pyrite ores
- Waste rocks that are not sent to the flotation plant. These include the oxide ore (atacamite-malachite), pyrite and the barren rock that is generally found beneath the orebody

Over the whole field, there is almost 10% of high-grade ore and about 45% of both low grade and non-copper sulphide material but these proportions are not constant in space. For example, there is much lower grade material near the top of the orebody. The proportions have to be modelled as a function of the point considered in the field.

Proportion curves were computed by a simple moving average procedure that was refined and complemented manually to incorporate knowledge about the geological characteristics of the site. Figure 4.9 shows two sets of proportion curves from a 2D test zone containing just over 900 samples. The horizontal axes are the east-west direction (on the left) and the vertical direction (on the right). In the top diagram, the horizontal proportion curve was calculated close to the surface, and the vertical proportion curve on the Western side of the field. In the lower diagram

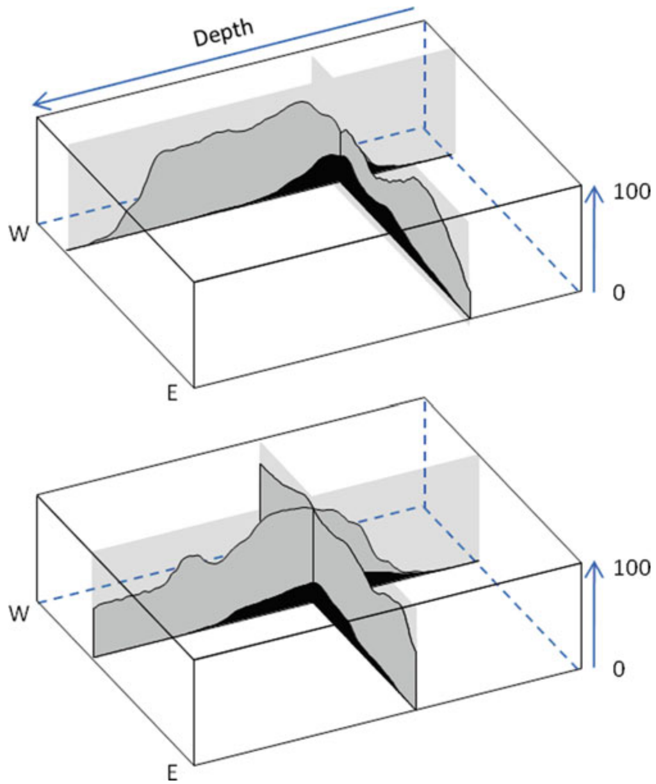

Fig. 4.9 Two sets of proportion curves, an HPC and a VPC, calculated using just over 900 samples. The horizontal axes are the east-west direction (on the *left*) and the vertical direction (on the *right*). In the top diagram, the HPC was calculated close to the surface, whereas the VPC came from the Western side of the field. In the lower diagram both proportion curves were computed closer to the centre of the field. The proportions are shown as a coloured surface on the vertical axis with high grade ore in black, low grade ore in *middle grey* and poor material in *light grey*

the proportion curves were computed closer to the centre of the field. The proportions of the three ore types are shown as a coloured surface on the vertical axis: high grade ore in black, low ore in middle grey and poor grade in light grey. The sum of the facies proportions is always equal to 100%. It is interesting to see how much the proportions of low and poor facies can vary over relatively small distances. In contrast the proportion of high grade ore seems more regular.

Choosing the Reference Level

The shape of the vertical proportion curves and the resulting simulations depend on the choice of the reference level. For oil reservoirs this is a specific geological marker which is used to restore the geometry of the reservoir at the time of deposition. This level must have been horizontal during sedimentation, and, should, if possible, correspond to a time line. That is, it should be a chronostratigraphic marker not an erosional unconformity. The reservoir is then flattened using this as the reference level.

Common choices for the reference level are the top of the unit or the bottom. Alternatively, a proportional grid could have been used to take account of differential subsidence. Figure 4.10 illustrates two of these three possibilities. Different reference levels can be used for each reservoir unit, or one serve for several of them.

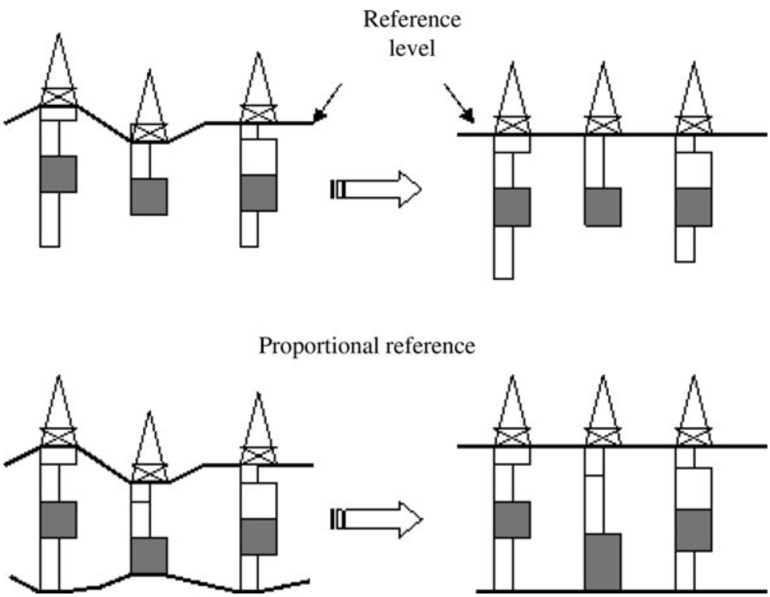

Fig. 4.10 Two ways of flattening a reservoir, (**a**) using the top as reference level, (**b**) proportionately to account for differential subsidence

To illustrate how important this choice is, Figure 4.11 illustrates the impact of three possible reference levels on the vertical proportion curve and then on the simulations. The Ellerbeck unit of the Ravenscar sequence which was described earlier in the chapter, was used as an example. The correct level is a flooding surface inside the Ellerbeck unit near the bottom. The two incorrect levels she considered were the base of the Ravenscar sequence and its top (denoted by R1 and R2 respectively). The base is a chronostratigraphic marker, the top is an unconformity.

Figure 4.12 shows the VPC for the three different choices. The best one (denoted by OK) shows a steady gradation in the depositional environment. Starting from the bottom these are:

- Mudstone coming from an open marine environment
- Argillaceous sandstone from the low to mid shore
- Transgressive sandstone
- Clean sandstone from the foreshore
- A mixture of argillaceous sandstone and mudstone from a lagoon environment
- Continental mudstone from a flood plain environment at the top

The second VPC correspond to reference level R1 which is not the correct one. But its irregularities could have been due to cycles within the depositional cycles. The third VPC corresponding to reference level R2 is obviously incorrect. Figure 4.13 shows one simulation of the unit obtained using the correct VPC which reproduces the type of continuity that would be expected at this scale, in this type of depositional environment. The lithotypes in the simulation corresponding to R1 are much less continuous

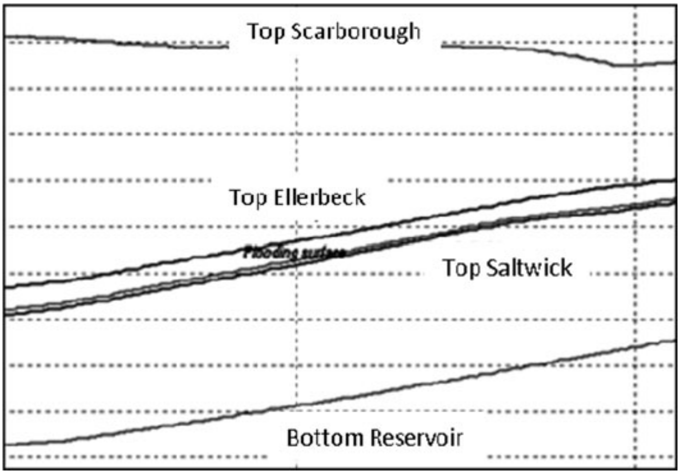

Fig. 4.11 Three possible choices for the reference level; the flooding surface inside the Ellerbeck formation (the correct choice), the bottom of the reservoir (R1) and the top (R2)

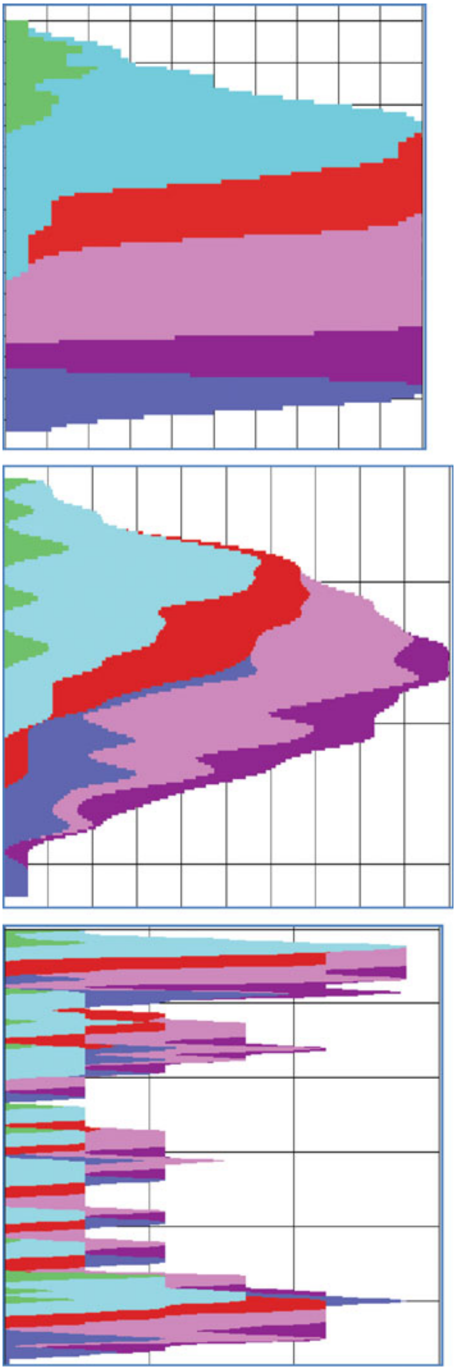

Fig. 4.12 Vertical proportion curve corresponding to the different reference levels: correct (top) – reference level R1 (middle) – reference level R2 (bottom)

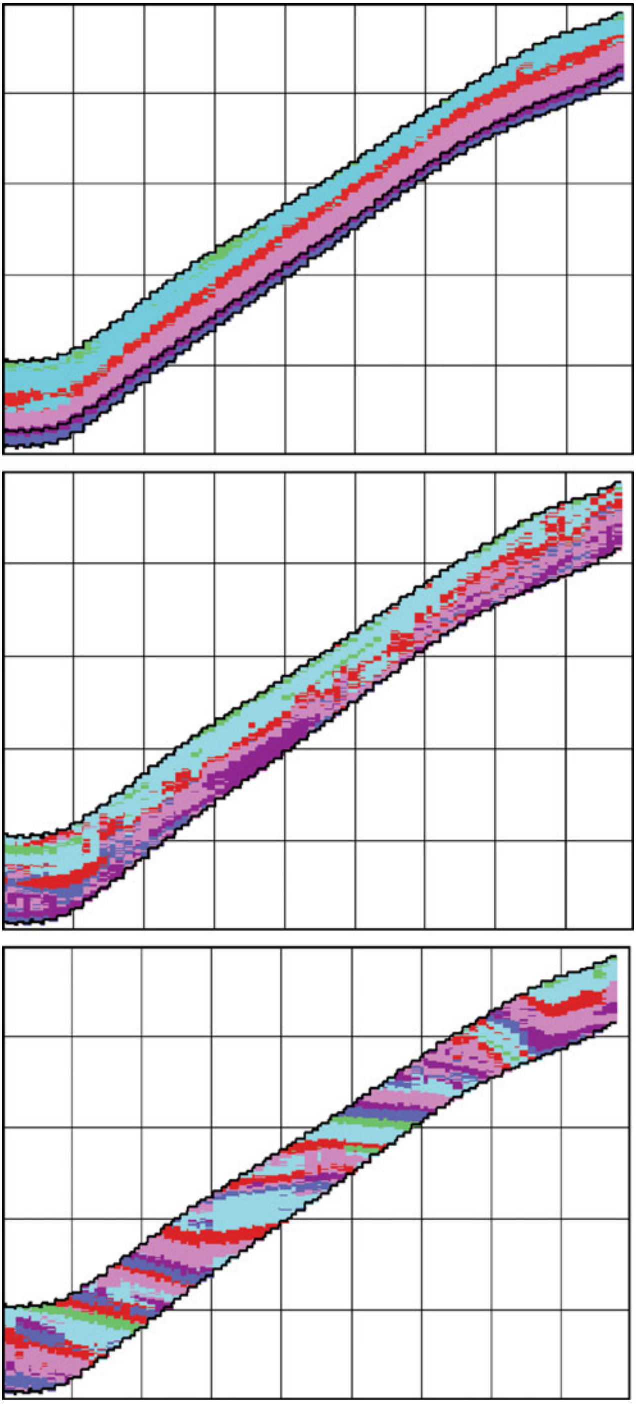

Fig. 4.13 One simulation of the unit obtained using different reference levels: correct (top) – reference level R1 (middle) – reference level R2 (bottom)

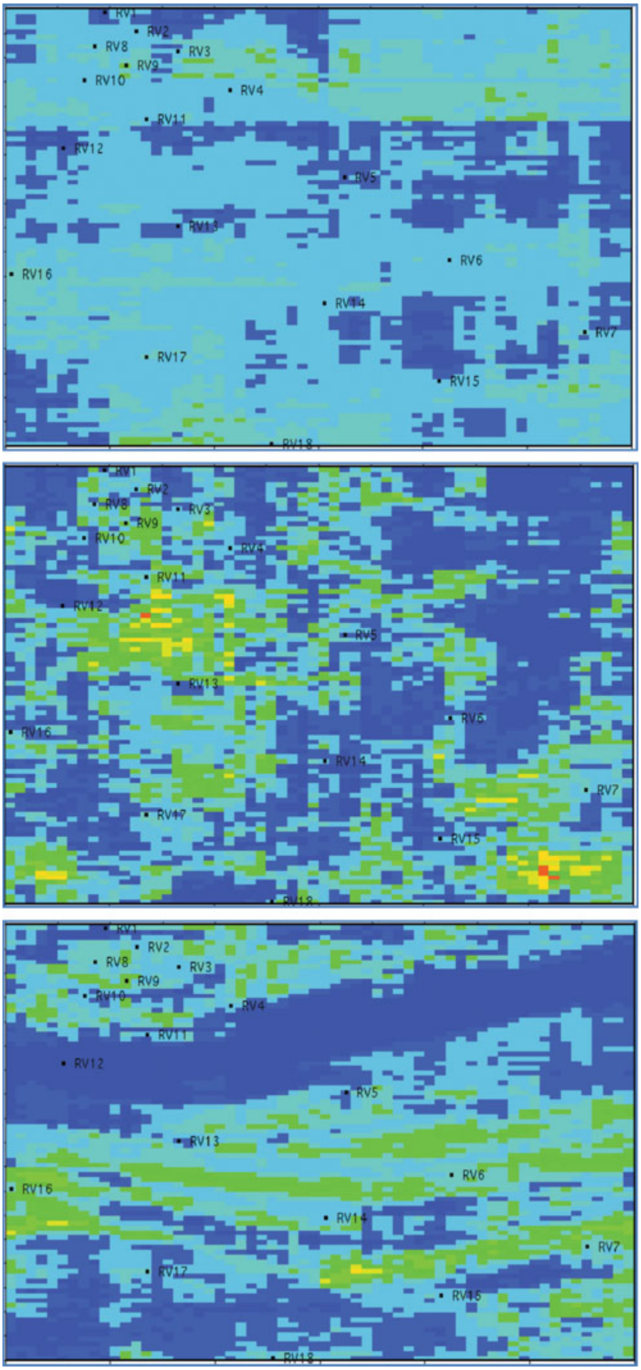

Fig. 4.14 Accumulated sand thickness obtained using the different reference levels: correct (top) – reference level R1 (middle) – reference level R2 (bottom)

and would lead to quite different fluid flows. The third simulation for R2 suggests no connectivity from one part of the unit to another.

It is not always obvious which level is the most appropriate for the reference. When in doubt, seismic data can sometimes provide guidance because it can give an indication of the cumulative thickness of sandstone. This could be compared to the cumulative thickness found in simulations. Figure 4.14 shows the cumulative sandstone thickness for the three simulations. The differences are quite marked. Volpi et al. (1997) and Ravenne et al. (2002) discuss vertical proportion curves in detail. The definition of a level has a marked impact on the result of the simulation. Onlap or toplap configurations can be produced depending on the choice of a reference level with regards to the unit geometry.

Non-stationarity

From a geological point of a view, a non-stationarity is characterised by a significant lateral change in the lithotype distribution in a reservoir unit, within the study area. It often shows up in the horizontal proportion curves as significant variations in the lithotype proportions in a given direction.

As geological phenomena always have some non-stationary aspects, depending on the scale of investigation, the choice between stationary or non-stationary models is subjective. It implies firstly a classification of the heterogeneity and then setting up a hierarchy. The distribution of the relevant heterogeneity will be then analysed to determine if it is stationary or not.

Non-stationarity concerns both object and sequence-based models. In object based models, it implies lateral variations of the object frequency. In sequence – based models, non stationarity can be handled by computing several proportion curves for lithotypes in different areas of the reservoir. This leads to the construction of a proportion matrix, in which vertical proportion curves are computed in each cell of a grid (Beucher et al. 1993; Doligez et al. 1999a, b, c). This approach is very flexible and gives realistic lateral variation of lithofacies. Furthermore, it is easy to integrate soft constraints such as an external drift into the computation of the proportion matrix. Geological information derived from seismic campaigns can then be integrated in the simulations (Moulière et al. 1997; Fournier and Derain 1997; Johann et al. 1996; Beucher et al. 1999).