Elastic Deformations

Before the mechanics of basic elements of microtechnique is described, it is necessary to introduce the main parameters which govern the elastic deformation of rigid bodies under the action of forces. There are mainly three parameters: *Stress due to straining*, *residual stress*, and *bending*.

Stress due to straining is generated when a force is pulling or pushing at a rigid body. For example, a force F pulls in longitudinal direction at the free end of a beam which is fixed at its other end (cf. Fig. 2a). This force results in a strain ε_B of the beam and a stress σ_B. The stress is the force acting at the beam per cross-section area A_B. Stress and strain are proportional to each other according to Hooke's law:

$$\frac{F}{A_B} = \sigma_B = E_B \varepsilon_B. \tag{1}$$

The proportionality constant E_B is a function of the material of the beam and is called Young's modulus.

While the length of the beam is increased according to Hooke's law, its diameter is decreased by a factor which is smaller than the longitudinal strain ε_B by Poisson's ratio ν_B. Thus, the change of the width Δb_B and the thickness Δd_B of the beam are:

$$\Delta b_B = -\nu_B \ \varepsilon_B \ b_B \quad \text{and} \quad \Delta d_B = -\nu_B \ \varepsilon_B \ d_B. \tag{2}$$

If the force acting at the beam is released again and if the force was not too large, the beam comes back to its original position and there is neither stress nor strain any more. Besides the stress due to straining, a rigid body may show some residual stress also. This occurs only if the body is fixed at more than one part, e.g., a beam fixed at both ends (cf. Fig. 2b). The residual stress in microtechnique often is generated during the fabrication process. For the example of a beam fixed at both ends, it may be assumed that the beam was stretched or compressed to fit between the fixation points. If the residual stress does not exceed certain values and no outer forces are acting, the beam looks the same way as a beam without residual stress, but a significant change is observed when an outer force is acting on the beam. For example, if a force is pulling transversally at the center of the beam, it may be deflected much more with a compressive residual stress and much less with a tensile one.

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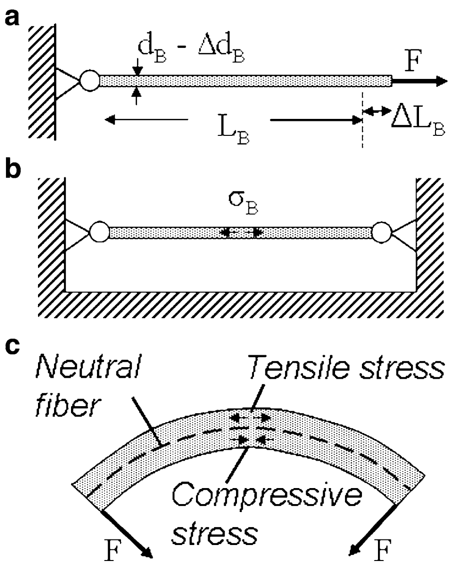

Fig. 2 Beams with (**a**) stress due to straining, (**b**) residual stress, and (**c**) bending

Again, if the fixation points are moving or released, the residual stress is changing and the beam shows some movement, e.g., becomes thicker and shorter.

Bending generates tensile stress and strain on one side of a body and compressive ones on the opposite side (cf. Fig. 2c). The gradient of the stress over the thickness of the body results in a bending moment, which tends to bring the body back into its original position when outer forces are taken away again. The integral of the stress over the cross-section is zero when only bending is involved. So, bending does not alter the length of, e.g., a beam. There is an infinitesimally thin layer in every bent body at which the stress is zero. This is called the neutral fiber. The neutral fiber is strained, however, when a force or a force component is acting longitudinal to the beam. Beams clamped at one end and loaded only in transversal direction are elastically deformed only by bending, because the neutral fiber cannot be strained that way.

The strain ε of a beam in axial direction generated by bending is the change of an infinitesimal length dL_B, cf. Fig. 3a:

$$\varepsilon = \frac{ds - dL_B}{dL_B} = \frac{(R_c + z)d\alpha - R_c d\alpha}{R_c d\alpha} = \frac{z}{R_c}. \tag{3}$$

In this equation, R_c denotes the radius of curvature of the beam. The radius of curvature of a function w(x) can be calculated, in general, by [21]:

$$R_c = \pm \frac{\left(1 + (\partial w/\partial x)^2\right)^{3/2}}{\partial^2 w/\partial x^2}. \tag{4}$$

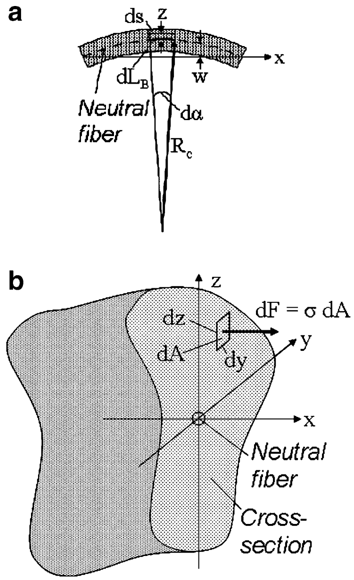

Fig. 3 Calculation of strain and bending moment of a beam

For small slopes of the curve (if the x-axis is parallel to the beam axis), the first derivative can be neglected compared with one, and (3) becomes:

$$\varepsilon = -z\frac{\partial^2 w}{\partial x^2}. \tag{5}$$

The bending moment M acting at the beam is the integral of the product of the force dF acting on the infinitesimal surface element dA of the cross-section of the beam and the distance z to the axis around which the beam is bending (the y-axis in Fig. 3b):

$$M = \int_A z\ dF = \int_A z\sigma\ dA. \tag{6}$$

In this equation, the stress according to Hooke's law (1), and (5) is inserted:

$$M = \int_A z\ \sigma\ dA = -\int_A z^2\ E_B\ \frac{\partial^2 w}{\partial x^2} dA = -E_B\ \frac{\partial^2 w}{\partial x^2} \int_A z^2\ dA = -E_B\ I\ \frac{\partial^2 w}{\partial x^2}. \tag{7}$$

In this equation, I denotes the area momentum of inertia which is defined by:

$$I = \int_A z^2\ dA. \tag{8}$$

The area momentum of inertia of cross-sections important in microtechnique are listed in Table 3 on page 67.