Microoptics

Miniaturized optical components are very popular nowadays. For example, lenses in cellular phones need to be small and light weight and shall be equipped with an optical zoom. Micro-optical components are employed also in sensors, for data transmission, and chemical analysis.

Miniaturization is a disadvantage for optical imaging because every optical component is an aperture in the optical path and every aperture results in diffraction. As shown on the left of Fig. 127, light due to its wave nature does not propagate as straight lines but shows diffraction. The light intensity far behind a narrow aperture illuminated by parallel light does not show a rectangular profile. For the same reason, the focus of a lens with diameter D_L and focal length f_B cannot be arbitrarily small but the intensity distribution I_L is given by the following equation:

$$I_L = I_0 \left[\frac{\sin(\pi\ x(D_L/\lambda\ f_B))}{(\pi\ x(D_L/\lambda\ f_B))} \right]^2. \quad (314)$$

In the above equation, λ, x, and I_0 are the wavelength of the light used, distance from the optical axis, and intensity of the light illuminating the aperture, respectively. The intensity distribution calculated with (314) is shown on the right of Fig. 127. Obviously, a smaller diameter D_L of the lens broadens the intensity distribution. As a consequence, an image projected with a lens becomes sharper when the lens diameter is enlarged and miniaturization of the lens results in a diffuse projection. Therefore, there are always other reasons for miniaturization of optical components which are more important for a certain application than the loss in sharpness, e.g., small mass, small size, and low cost.

If an optical spectrum is to be analyzed, usually a *diffraction grating* is employed. A diffraction grating is a large number of narrow optical apertures arranged next to each other. Typically, transparent or reflecting parallel slits are made on a surface which is opaque besides the slits.

To describe how an optical grating works, first only two transparent slits are considered – a so-called double slit. A plane wave arriving at the two slits can be considered behind the double slit as two point sources of light which are emitting

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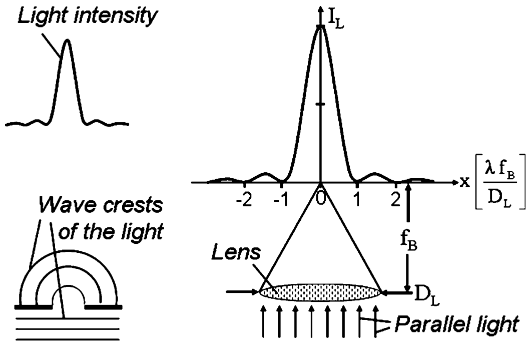


Fig. 127 Optical diffraction of light waves at an aperture (*left*) and a lens (*right*)

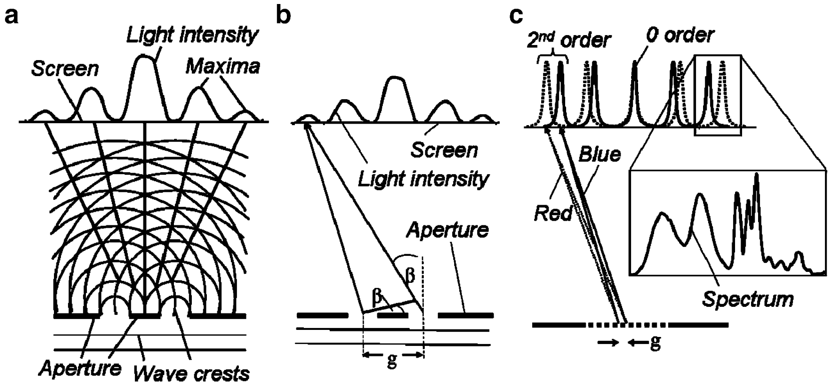


Fig. 128 Light interference behind a double slit (**a**, **b**) and a grating (**c**)

light in phase as spherical waves (cf. Fig. 128a). At certain angles, the wave crests and wave troughs of these waves coincide. On a screen far behind the double slit, the light intensity shows maxima at these angles. The formation of maxima is called constructive interference, the formation of minima where the crest of one wave compensates the trough of another wave is called destructive interference.

The angles β of maximum intensity are found where the optical path difference between the two waves is an integer multiple of the wavelength λ. Thus, as seen in Fig. 128b, intensity maxima are found where the following equation is fulfilled:

$$\mathrm{m}\,\lambda = \mathrm{g}\sin(\beta) \Rightarrow \sin(\beta) \approx \beta = \frac{\mathrm{m}\,\lambda}{\mathrm{g}}. \tag{315}$$

In the above equation, g is the distance of the centers of the two slits. It is called the grid constant. m is an integer which is called the diffraction order.

If not only two but also several or many slits with a constant distance g are employed, the maxima of all the spherical waves propagating from these slits interfere with each other and generate a grid of narrow maxima of equal intensity on the screen (cf. Fig. 128c). Except the zero order maximum (m = 0), the angle of constructive interference β is a function of the wavelength. The higher the order of a maximum is, the more is the spectrum spread and the larger is the resolution for analyzing it. As a consequence, a grid can be used to separate the light into its colors and to display and analyze its spectrum.

If the light is not illuminating the grid perpendicular, the angle of incidence needs also to be taken into account when the position of the maxima are calculated. Figure 129 shows that the following equation needs to be satisfied for constructive interference:

$$m\,\lambda = g(\sin(\alpha) + \sin(\beta)) \Rightarrow \sin(\alpha) + \sin(\beta) = \frac{m\,\lambda}{g} \approx \alpha + \beta. \tag{316}$$

Note that α and β as shown in Fig. 129 are defined to be positive. One of them can be negative when it is on the opposite side of the dashed line normal to the grid. By choosing α the designer can control the position of, e.g., the first-order maximum of a certain wavelength.

Besides transmission grids, there are also reflection grids. As shown in Fig. 130a, at a reflection grid, the reflected light is subject to interference. The angle β in (316) is negative in most cases of a reflection gird.

If a grid is to be used with low-intensity light, it is a disadvantage that roughly half of the light to be analyzed is absorbed. This problem is avoided when a *step grid* is employed as shown in Fig. 130b. The light reflected at a step is interfering with those reflected at the neighboring steps. As a result of the step, there is a phase difference between light reflected at different steps.

According to the reflection law, the angle α_r of the reflected light is the same as the angle of incidence α (cf. Fig. 131):

$$\alpha = \alpha_r. \tag{317}$$

Due to the slope angle Φ_{St} of the steps, there is a wavelength for which the angles of reflection and constructive interference coincide. For this wavelength, the so-called *blaze* wavelength λ_B, and, to some smaller extent, the neighboring wavelengths

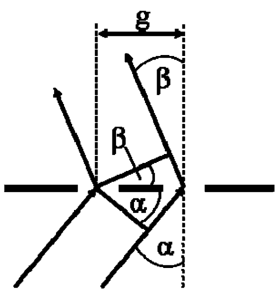


Fig. 129 Interference at a not perpendicularly illuminated grid

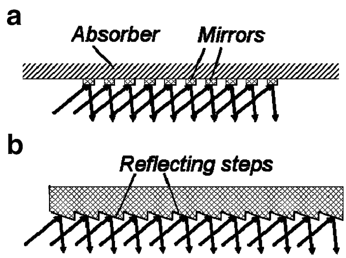


Fig. 130 Reflection grid (**a**) and step gird (**b**)

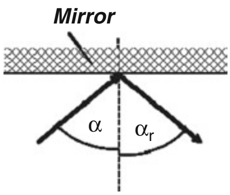


Fig. 131 Reflection at a mirror

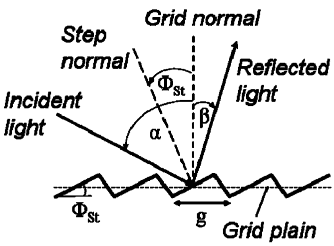


Fig. 132 Blaze angle at a step grid

the intensity of the interference maximum is enhanced. This effect can be employed for the design of spectrometers. If the detector is less sensitive in a certain range of wavelengths, e.g., extreme blue light, this can be compensated partly with a blaze in this range. Another possibility is that a certain range of the spectrum is of high importance for analytical purposes.

In Fig. 132, it is seen that the reflection law results in:

$$\alpha - \Phi_{St} = \Phi_{St} - \beta \Rightarrow \Phi_{St} = \frac{\alpha + \beta}{2}. \tag{318}$$

Note that β in Fig. 132 is negative according to the definition of (316). If β is the blaze angle for the blaze wavelength λ_B, both (318) and (316) need to be fulfilled, and the step angle Φ_{St} is derived as:

$$\Phi_{St} = \frac{m\,\lambda_B}{2\,g}. \tag{319}$$

The blaze angle β at which constructive interference and reflection coincide is a function of the angle of incidence. It is found from the two above equations:

$$\beta = \frac{m\ \lambda_B}{g} - \alpha. \tag{320}$$

If the screen shown in Fig. 128b is far behind the double slit, the angles β of light emitted from the neighboring slits and interfering at a certain position on the screen are nearly the same. Therefore, (315) and (316) have been derived correctly. In microtechnique, however, the distance of the screen is never far from the gird. Therefore, the plain of the grid needs to be bent to achieve that light emitted at the same difference between incident angle α and diffraction angle β, i.e., the same phase difference, arrives at the same point. This problem is solved when the light source (or entrance slit of a spectrometer), the gird, and the screen (or detector of a spectrometer) are arranged on a circle (cf. Fig. 133a). Such a circle is called a *Rowland circle*. It is a geometrical fact that every triangle inside of a given circumcircle and with one side identical to a certain secant shows the same angle opposite to the secant (cf. Fig. 133b).

The surfaces of optical components may show only small roughness, because even a small roughness will result in interference effects. Typically, the components need to be even within a margin which is a 20th of the wavelength.

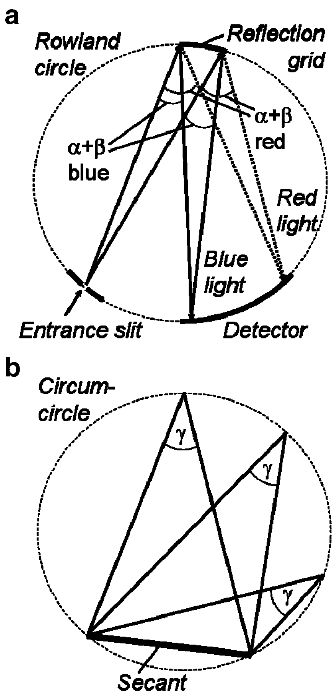


Fig. 133 Rowland circle

If light is arriving at a mirror at an angle of incidence of 0°, the incident wave is interfering with the reflected one. As a consequence, the so-called *standing wave* is generated and the intensity of the light in front of the mirror shows maxima and minima (cf. Fig. 134). At the surface of the mirror, there is a minimum and the distance from one minimum to the next one is half of the wavelength of the light. This phenomenon is observed when a partly transparent screen is placed very near to the surface of the mirror and moved back and forth to the mirror. Standing light waves are also the reason for interference patterns in photo resists on a reflecting substrate which cause uneven side walls of resist patterns after development.

An optical component suitable for miniaturization which is often used for analytical purposes is the *Fabry-Perot interferometer*. It consists of two mirrors which are arranged in parallel to each other (cf. Fig. 135). At least one of these mirrors is partly transparent. Therefore, light can enter into the space between the mirrors and a small part of this light is escaping out of the interferometer again. The integer multiples of the wavelengths λ of the light leaving the interferometer equal the double distance d between the mirrors:

$$m\ \lambda = 2\ d. \tag{321}$$

The reason for this effect is that the standing waves developing on the surfaces of the two mirrors can only interfere constructively if their minima and maxima coincide. That is, the standing wave between the mirrors needs to have a minimum on the surfaces of both mirrors.

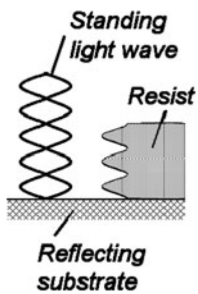


Fig. 134 Standing light wave on a reflecting substrate (*left*) and resist edge after developing (*right*)

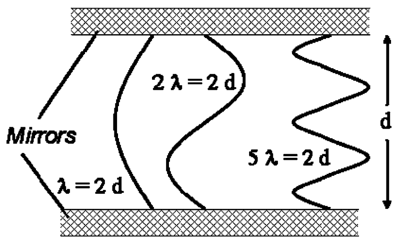


Fig. 135 Fabry-Perot interferometer and standing light waves developing inside

The path of light may also change its direction by *refraction*. Refraction occurs when the medium in which the light is propagating shows a change of its refractive index. The refractive index is the ratio of the velocity of light in vacuum and in the medium. So, the refractive index of the vacuum equals 1. In a medium, the velocity of light is a little bit smaller than in vacuum, and, therefore, the refractive index of media is a little bit larger than 1.

When light is propagating towards a down step in refraction index at an angel of incidence α larger than 0°, it is partly reflected at an angle α_r according to (317), and partly transmitted at an angle α_t into the medium with the smaller refraction index (see Fig. 136a). One side of each wave crest will arrive earlier at the interface than the other (cf. left side of wave crest in Fig. 136b). The transmitted part of the side arriving earlier will continue propagating with a larger velocity c_1, while the other side for a small time t retains its former velocity c_2. The distances c_1t and c_2t the light is traveling within the time t are not equal. Therefore, the direction of propagation will change to a larger angle α_t.

From the geometry in Fig. 136b, it is seen that:

$$c_1 t = z\cos(90^\circ - \alpha_t) = z\sin(\alpha_t) \text{ and } c_2\, t = z\sin(\alpha)$$

$$\Rightarrow \frac{c_1\, t}{\sin(\alpha_t)} = \frac{c_2\, t}{\sin(\alpha)} \Rightarrow \frac{c_1}{c_2} = \frac{n_2}{n_1} = \frac{\sin(\alpha_t)}{\sin(\alpha)}. \qquad (322)$$

Equation (322) is *Snell's law* with which the refraction angle can be calculated. Since n_2 is larger than n_1, at a certain angle of incidence α_{Tot}, α_t needs to become larger than 90°. This would mean that the light goes back into the medium where it comes from. As a consequence, for angels of incidence larger than α_{Tot}, there is no refraction but only reflection according to the reflection law (317). α_{Tot} is called the *angle of total reflection*. It can be calculated from (322) by choosing 90° for the angle α_t of the transmitted light, i.e., the sine equals 1:

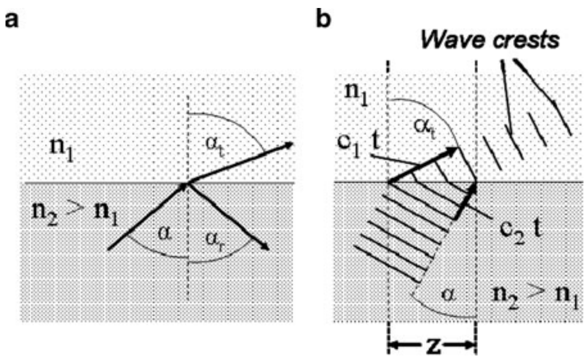


Fig. 136 Reflection and refraction of light

$$\sin(\alpha_{Tot}) = \frac{n_1}{n_2} \Rightarrow \alpha_{Tot} = \arcsin\left(\frac{n_1}{n_2}\right). \quad (323)$$

If a layer with a larger refraction index n_2 is surrounded by layers with a smaller one n_1, it can be employed as an *optical waveguide*. Light propagating at an angle larger than the angle of total reflection is confined in the middle layer. *Optical fiber*s are based on this principal, but light is also guided in a layer which is extending laterally and surrounded by layers with a smaller refractive index.

The angle of total reflection limits the maximum angle θ_{max} at which light may enter or exit an optical waveguide (see Fig. 137). The refraction index n_0 outside of the waveguide is smaller than inside, and the refraction angel α_t is calculated by Snell's law (322). From Fig. 137, it is seen that $\sin(\alpha_t) = \cos(\alpha_{Tot})$. Snell's law is now:

$$n_0 \sin(\theta_{max}) = n_2 \sin(\alpha_t) = n_2 \cos(\alpha_{Tot}). \quad (324)$$

The sum of the squares of sine and cosine of every angle α equals 1: $\sin^2(\alpha) + \cos^2(\alpha) = 1$. If the angle of total reflection is inserted into this equation and (323) is used for $\sin(\alpha_{Tot})$, (324) yields:

$$n_0 \sin(\theta_{max}) = n_2 \cos(\alpha_{Tot}) = n_2 \sqrt{1 - \frac{n_1^2}{n_2^2}} = \sqrt{n_2^2 - n_1^2} := NA. \quad (325)$$

The quantity NA defined in the above equation is the *numerical aperture* of the optical waveguide which is a measure of the illumination angle which can be achieved. The numerical aperture is only a function of the refraction indices of the materials it is made of, while the maximum angel θ_{max} at which light may enter into or exit from the waveguide is also a function of the refraction index of the medium outside the fiber.

In optical waveguides, the component of the light perpendicular to the mirrors needs to fulfill the condition of standing waves as in a Fabry-Perot interferometer

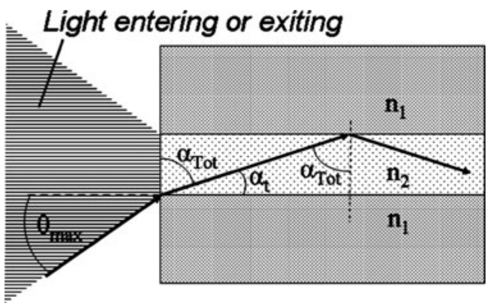
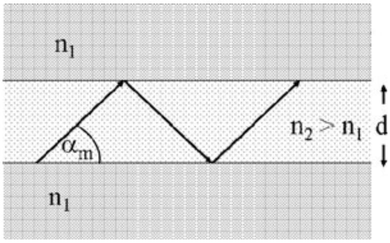


Fig. 137 Reflection and refraction of light

Fig. 138 Propagation of light in a waveguide at mode m



(cf. 321) [78]. As a consequence, the following equation needs to be fulfilled by the light propagating in the waveguide [53]:

$$m\ \lambda = 2\ d \cos(\alpha_m) \Rightarrow \alpha_m = \arccos\left(\frac{m\ \lambda}{2\ d}\right). \tag{326}$$

In the above equation, α_m is the angle of incidence inside of the optical fiber (Fig. 138). Obviously, light can propagate inside an optical fiber only at distinct angles α_m. The light propagating at a certain angle is called a mode. The number of modes is limited because the angle of incidence cannot become larger than the angle of total reflection:

$$m\ \leq \frac{2\ d}{\lambda} \cos(\alpha_{Tot}). \tag{327}$$

If the height d of the waveguide is small, only a small number or even only one mode can propagate inside. The kernel of one mode waveguides typically is approximately 1 µm high, while multimode waveguides have a kernel, 100 µm or more in height. An advantage of one mode waveguides is that all the light is traveling the same distance when propagating through the guide. Light which is reflected at larger angles α_m of incidence has a longer way to travel, and, therefore, arrives a bit later than the light of other modes. As a consequence, a data bit transmitted through several modes becomes wider, while propagating through an optical fiber. This effect limits the data rate which can be sent through the fiber. Therefore, one mode fibers allow larger data rates.

Equation (327) can be interpreted also in another way: There is a lower limit for the wavelength λ of light which can propagate through a thin waveguide. Thus, a waveguide works as a filter:

$$\lambda\ \leq \frac{2\ d}{m} \cos(\alpha_{Tot}). \tag{328}$$

The light confined in an optical waveguide is not able to leave the middle layer according to classical theory. However, quantum mechanics is needed to describe all aspects of light propagation in waveguides. According to quantum mechanics, the wave which is employed to describe the light is a measure of the probability that a light quantum, a so-called photon, is found at a certain position. The wave function is not zero in the layer with the larger refraction index but extends into this layer with decreasing amplitude. The part of the wave function which is in the "forbidden" region is called the *evanescent field* (cf. Fig. 139). Thus, there is a little probability that photons exist in the layers with the larger refraction index and even outside of the entire waveguide. As a consequence, the light inside of the waveguide can be affected by media outside. Some sensors employ this effect.

Due to the evanescent field, there is a way for the light in a waveguide to "tunnel" through the barrier of the lower refractive index into a neighboring region with higher refraction index. After some time and propagation length, the entire wave function has shifted to the other waveguide (cf. Fig. 139a), and after the same distance of propagation it has shifted back into the initial waveguide. A part of the light is absorbed during tunneling, and, therefore, the amplitude of the wave function is reducing.

This effect can be employed to distribute light from a single into two waveguides. The power as a function of the position x in waveguides 1 and 2, respectively, can be calculated with the following equations:

$$P_1(x) = \cos^2(\kappa x)e^{-\alpha_{op}x}, \tag{329}$$

$$P_2(x) = \sin^2(\kappa x)e^{-\alpha_{op}x}. \tag{330}$$

The above equations are plotted in Fig. 139b. The coupling constant κ and the damping constant α_{op} are a function of the geometry of the waveguides and the modes of the light propagating in them.

When light is to be distributed from one waveguide into two, they need to be designed in parallel for the distance $\pi/4\kappa$, as seen from Fig. 139b and (329) and (330). Figure 140 shows such a design and as an alternative a so-called Y-coupler. The Y-coupler is easier to design, because nothing needs to be known about the wave function and modes in the waveguide, but the angle at the junction may not be

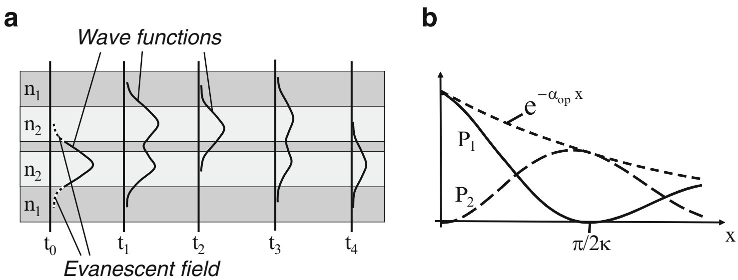


Fig. 139 Light tunneling from a waveguide to a neighboring one (**a**) and power in the two waveguides (**b**)

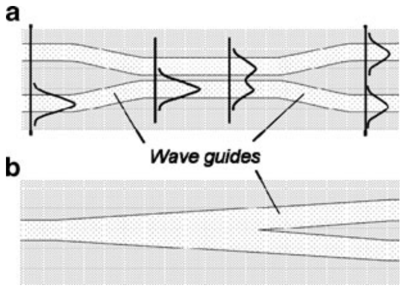


Fig. 140 Distribution of light from one into two waveguides by (**a**) tunneling into a neighboring waveguide and (**b**) a Y-coupler

larger than about 0.5° to avoid too much loss of the light where the required angle of incidence at the change in refraction index cannot be matched.

In some materials, the refractive index n_{op} is a function of the electric field E_{el} applied to them. This is called the *electro-optical effect*. By employing this effect, it is possible to modulate the amplitude of light and to switch it on and off. In general, the refractive index can be described by:

$$n_{op} = n_2 - \frac{1}{2} r_P n_2^3 E_{el} - \frac{1}{2} r_K n_2^3 E_{el}^2. \tag{331}$$

In the above equation, n_2 is the refractive index without any electric field and r_P and r_K are material constants: the Pockels constant and the Kerr constant, respectively. Two kinds of the electro-optical effect are distinguished: the *Pockels effect* and the *Kerr effect*. The Pockels effect is linear with the electrical field. It is represented by the term in (331), which is linear with the electric field. The other electro-optical effect is the Kerr effect. With the Kerr effect, the refractive index of a material is changed with the square of the electric field [the last term in (331)]. The part of an electro-optical material in which the Pockels or the Kerr effect is employed is called a Pockels cell or a Kerr cell, respectively.

Tables 17 and 18 show some materials with comparatively large Pockels and Kerr constants, respectively.

As described above, the refractive index is the ratio of the velocity of light in a certain material and in vacuum. Therefore, decreasing the refractive index results in a smaller delay of the propagating light.

This is employed in a device called *Mach-Zehnder interferometer* (cf. Fig. 141). Light coming from the left is distributed into two waveguides and changed in one of them. After reunification of the waveguides, the two waves interfere resulting in a changed amplitude.

If a Pockels cell is installed in one of the arms of a Mach-Zehnder interferometer, the delay of the wave in that arm can be manipulated by an electric signal and, this way, the amplitude of the outgoing light is controlled. The phase shift $\Delta\varphi$ in a Pockels cell with length L_{PZ} of a wave which shows a wavelength of λ_0 in vacuum is according to (331):

Table 17 Pockels constants

Material with Pockels effect	Pockels constant r_P
Potassium dihydrogen phosphate (KH_2PO_4)	3.6×10^{-11} m/V
Deuterated potassium dihydrogen phosphate (KD_2PO_4)	8×10^{-11} m/V
Lithium niobate ($LiNbO_3$)	37×10^{-11} m/V

Table 18 Kerr constants

Material with Kerr effect	Kerr constant r_K
Nitrobenzene	2.4×10^{-12} m/V^2
Glasses	3×10^{-16} to 2×10^{-25} m/V^2
Water	4.4×10^{-14} m/V^2

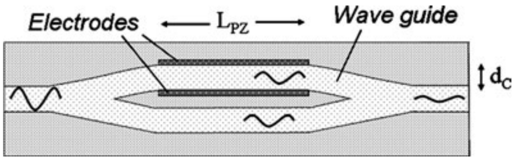


Fig. 141 Mach-Zehnder interferometer with a Pockels cell (region between the electrodes) in one of its arms

$$\Delta\varphi = \frac{\pi\, L_{PZ}}{\lambda_0} r_P\, n_2^3\, E_{el}. \tag{332}$$

The light is switched off when the phase shift of the wave is $\Delta\varphi = \pi$. Therefore, the above equation yields for switching off the light after a Mach-Zehnder interferometer with a Pockels cell:

$$E_{el} = \frac{\lambda_0}{L_{PZ}\, r_P\, n_2^3} \Rightarrow U = \frac{d_C\, \lambda_0}{L_{PZ}\, r_P\, n_2^3}. \tag{333}$$

In the above equation, U and d_C are the voltage applied to the electrodes and their distance, respectively. Light switching with a Pockels cell is very quick. Thus, light pulses, only a few nanoseconds in length, can be generated and sent as data bits into an optical fiber.

It is also possible to modulate the intensity of the light by changing the phase shift in a Pockels cell. If I_e and I_a denote the intensity of the light at the input and output of the cell, respectively, the modulation is described by:

$$I_a = I_e \cos^2\left(\frac{\pi\, L_{PZ}}{\lambda_0\, b_{PZ}} r_P\, n_2^3\, U\right). \tag{334}$$

The above equation is shown in Fig. 142. When light is to be modulated with a Pockels cell, the nearly linear part of the $\cos^2$-function around $\pi/4$ is used.

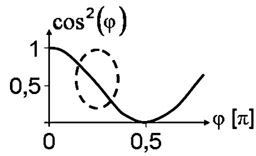


Fig. 142 Linear part of the $\cos^2$-function marked by a *dashed ellipse*

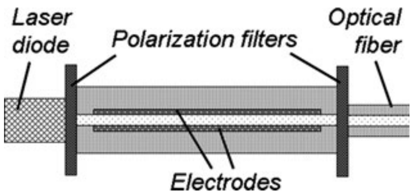


Fig. 143 Device employing turning of polarization direction of light for sending data bits into an optical fiber

There are also devices employing the effect that some materials turn the polarization angle of light (cf. Fig. 143). Before entering into the device, the light passes a polarization filter. At the end of the device, a second polarization filter is arranged adjusted for light polarized perpendicular to the first filter. Thus, only light with a polarization direction turned by 90° in the cell can pass it. This is also a way to generate data bits which are sent through an optical fiber.

Exercises

Problem 27

The resolution of an optical instrument is defined as the distance between neighboring points of the image which can be distinguished from each other. According to Lord Rayleigh, two points can be distinguished from each other, if the intensity maximum of one point lies in the minimum of the neighboring one.

(a) Find a way to calculate the resolution of an ideal lens on the basis of (314) (page 177). Derive an equation for the calculation of the resolution limit.
(b) It does not help to make the distance of the pixels of a CCD chip narrower than the resolution of the lens. The lens of the digital camera of a mobile phone has a diameter of 2 mm and a focus length of 28 mm. Calculate for the wavelength of visible light with the maximum intensity in the sun light (550 nm) the minimum distance of pixels which makes sense.
(c) For comparison, calculate the resolution of the lens of a 35 mm camera at the same wavelength. Assume that the lens of that camera has a diameter of 30 mm and a focal length of 50 mm.