Thermal Actuators

If the linear dimensions of a device are reduced, its mass decreases with the third power, while its surface decreases only proportional to the square (cf. page 3). As a consequence, the ratio of surface to volume or mass is very large for microdevices. This means, that a microdevice is heated up much more quickly and with less energy consumption, because of its small mass, and is cooling down more quickly due to its large surface to mass ratio. Therefore, heating up is a suitable actuation principle for a lot of microdevices, while it is not a good solution for macroscopic devices. The ink-jet printer is a good example (cf. page 251).

If an isotropic rigid body with a coefficient of thermal extension α_{th} is heated up by the temperature change ΔT, it extends to all directions by the strain ε_{th} (cf. Fig. 115):

$$\varepsilon_{th} = \alpha_{th}\ \Delta T. \tag{278}$$

The coefficient of thermal extension α_{th} is a material constant which needs to be measured or found in a suitable book.

If a pressure load p is acting on the heated body against the direction of thermal extension, the strain generated by the pressure according to Hooke's law needs to be added:

$$\varepsilon_{th} = \alpha_{th}\ \Delta T - \frac{p}{E_S}. \tag{279}$$

In the above equation, E_S is Young's modulus of the heated body. The deflections w_x and w_z of the rigid body in direction of its length L_S and thickness h_S, respectively, under the action of compressive forces F_x and F_z are derived from the above equation (b_S is the width of the body):

$$w_z = \alpha_{th}\ h_S\ \Delta T - \frac{h_S}{L_S\ b_S}\frac{F_z}{E_S} + \frac{\nu_S}{b_S}\frac{F_x}{E_S}, \tag{280}$$

$$w_x = \alpha_{th}\ L_S\ \Delta T - \frac{L_S}{h_S\ b_S}\frac{F_x}{E_S} + \frac{\nu_S}{b_S}\frac{F_z}{E_S}. \tag{281}$$

W.K. Schomburg, *Introduction to Microsystem Design*, RWTHedition,
DOI 10.1007/978-3-642-19489-4_13, © Springer-Verlag Berlin Heidelberg 2011

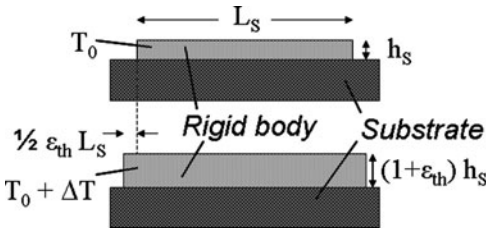


Fig. 115 Extension of a heated rigid body

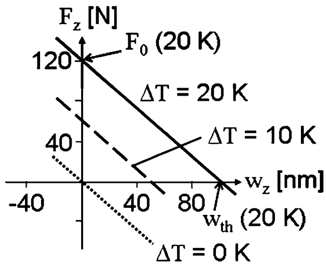


Fig. 116 Characteristic curves of a thermal actuator as calculated with (282)

From the two above equations, the characteristic curves of this thermal actuator can be calculated by solving for the force. For exampel, if deflection and force are acting in z-direction only, it is obtained:

$$F_z = L_S\ b_S\ E_S \left(\alpha_{th}\ \Delta T - \frac{w_z}{h_S} \right). \tag{282}$$

Figure 116 shows the characteristic curve of a thermal actuator with length, width, and thickness of 5 mm, 2 mm, 0.5 mm, respectively. A Young's modulus of 60 GPa is assumed, so that this actuator corresponds to the piezo described by the characteristic curve in Fig. 101 on page 140. The comparison of Figs. 116 and 101 shows that a thermal actuator is able to produce similar forces and deflections as a piezo when it is heated up by just 10 K, and it can be heated up much more. Therefore, it needs to be noted that thermomechanical actuators are capable to produce large forces and small deflections.

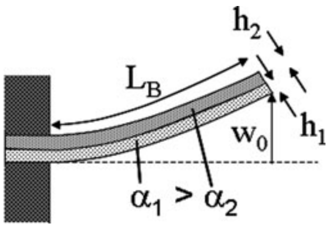
Similar as for piezos, the maximum force F_0 and maximum deflection w_{th} without any force acting on the actuator are calculated from (282) and (280), respectively:

$$F_0 = L_S\ b_S\ E_S\ \alpha_{th}\ \Delta T. \tag{283}$$

$$w_{th} = h_S\ \alpha_{th}\ \Delta T. \tag{284}$$

As a consequence of small deflections and large forces generated by a thermo-mechanical actuator, as in the case of piezo actuators, it is advantageous to employ

Fig. 117 Bimaterial actuator



beams with different deflection which are bonded to each other. Such beams are called *bimaterial actuators*. They consist of two stripes of materials with different thermal extension. Figure 117 shows such a bimaterial actuator. The thermal extension of beam 1 is larger than the one of beam 2. That is why the beams are bending upward. Obviously, the coefficient of thermal expansion of beam 1 should be as large as possible and the one of beam 2 as small as possible or even negative. There are some ceramic materials which show a negative coefficient of thermal extension and, therefore, shrink when heated.

When heated, both beams of a bimaterial actuator undergo a change in their length according to (281) (with no forces acting). The difference in their straining takes the role which was played by the straining of a piezo when a voltage is applied. Thus, the same equations can be employed as for bimorphs when the piezoelectric strain is replaced by the difference in the thermal strain of the two beams:

$$\varepsilon_1 = d_{31} \frac{U}{h_P} \rightarrow \varepsilon_{th} = \Delta\alpha_{th}\ \Delta T. \tag{285}$$

This way, the deflections and characteristic curves of a bimaterial actuator are obtained from (255) (on page 146) and (259)/(260):

$$w_0 = w_{th} + w_F = f_{w_E} \frac{3}{4} \frac{L_B^2}{h_2} \Delta\alpha\ \Delta T + f_{w_F} \frac{1}{2} \frac{L_B^3}{E_2\ b_B\ h_2^3} F, \tag{286}$$

$$F_{th} = f_{F_0} \frac{3}{2} \frac{E_2\ b_B\ h_2^2}{L_B} \Delta\alpha\ \Delta T - \frac{2\ E_2\ b_B\ h_2^3}{f_{w_F}\ L_B^3} w_0. \tag{287}$$

In the above two equations, the correction factors f_{w_E}, f_{w_F}, and f_{F_0} are the same as in (256), (257) and (259) (see page 146), respectively, h_1 and E_1 are thickness and Young's modulus of the beam with the larger thermal expansion, and b_B and L_B are width and length of the beams, respectively.

The characteristic curve of a bimaterial actuator consisting of two beams with the same dimensions and Young's modulus as in Fig. 116 and with a difference in the coefficients of thermal expansion $\Delta\alpha = 5 \times 10^{-6}$ is shown in Fig. 118 as calculated with (287). The characteristic curve is a strait line as in Figs. 116 and 105 (page 145). As a consequence, the optimum deflection for maximum energy

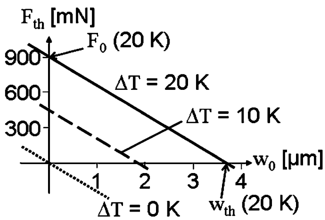


Fig. 118 Characteristic curves of a bimaterial actuator as calculated with (287)

output and efficiency is half of the maximum deflection w_{th} and the maximum energy is a fourth of the product of maximum deflection w_{th} and maximum force F_0 [cf. (271) on page 150].

If the dimensions of a bimaterial actuator shall be optimized for maximum deflection, force, or efficiency, the correction factors f_{w_E}, f_{F_0}, and their product need to be maximized as in the case of the piezo actuators. Therefore, the same ratios of thickness and Young's modulus are optimum as described on pages 146f and 152f, respectively.

The energy input W_E necessary to deflect a thermomechanical actuator approximately is the heat necessary to enhance its temperature, because this is much larger than the energy output which can be generated. An actuator with mass m_K and heat capacity C_{th} which is to be heated by the temperature difference ΔT consumes the following power:

$$W_E = m_K\, C_{th}\, \Delta T. \tag{288}$$

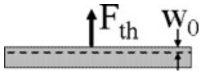
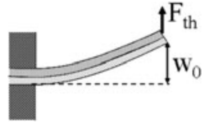
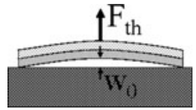
The efficiency of a bimaterial actuator is calculated as the ratio of energy output to input. The maximum efficiency typically is on the order of 0.1%. In Table 15, the efficiency is shown together with deflection, force, and maximum energy output of thermomechanical actuators. The correction factors are the same as for piezos with a carrier which are listed in Table 14 on page 156. There is no thermomechanical equivalence to two piezos with a carrier in between, because it is virtually impossible to heat only one side of a stack of beams bonded to each other.

Maximum force is achieved with a plate in normal direction. Maximum deflection is obtained with a bimaterial beam.

Table 15 shows also that a certain deflection or force can be achieved (in certain limits) either by enlarging the actuator or by raising the temperature at which it is driven. When the efficiency shall not become too small, it is better to drive a small actuator at large temperatures. A larger temperature will also help to extenuate two other principles of thermal actuators: cross sensitivity to changes of ambient temperature and long cooling time.

The maximum energy output density of the actuator is calculated from the optimum output energy in Table 15 divided by the volume $h_S\, A_S$. If for Young's modulus, thermal expansion, and temperature difference 120×10^9, 15×10^{-6}, and 200°C are assumed, respectively, a maximum energy output density of 270 μJ/μL is obtained.

Table 15 Equations for the calculation of deflections, forces, work outputs, and efficiencies of thermomechanical actuators

	In normal direction, one plate	In lateral direction, one plate	Two rectangular beams	Circular bimaterial plates
				
w_{th}	$h_S\ \alpha_{th}\ \Delta T$	$L_B\ \alpha_{th}\ \Delta T$	$f_{w_E}\frac{3}{8}\frac{L_B^2}{h_2}\Delta\alpha_{th}\ \Delta T$	$f_{w_E}\frac{3}{8}\frac{R_P^2}{h_2}\Delta\alpha_{th}\ \Delta T$
w_F	$\frac{h_P}{E_S\,A_S}F$	$\frac{L_S}{E_S\,b_S\,h_S}F$	$f_{w_F}\frac{L_B^3}{2\ E_2\ b_B\ h_2^3}F$	$f_{w_F}\ 0.069\frac{R_P^2}{E_2\ h_2^3}F$
F_0	$A_S\ E_S\ \alpha_{th}\ \Delta T$	$h_S\ b_S\ E_S\ \alpha_{th}\ \Delta T$	$f_{F_0}\frac{3}{4}\frac{E_2\ b_B\ h_2^2}{L_B}\alpha_{th}\ \Delta T$	$f_{F_0}\ 5.45\ E_2\ h_2^2\ \Delta\alpha_{th}\ \Delta T$
$W_{A,opt}$	$\frac{h_S\ A_S}{4}E_S\ \alpha_{th}^2\ \Delta T^2$	$\frac{h_S\ L_S\ b_S}{4}\ E_S\ \alpha_{th}^2\ \Delta T^2$	$f_{w_E}\ f_{F_0}\frac{9}{64}h_2\ L_B\ b_B\ E_2\ \Delta\alpha^2\ \Delta T^2$	$f_{w_E}\ f_{F_0}\ 0.51\,h_2\ R_P^2\ E_2\ \Delta\alpha^2\ \Delta T^2$
η_A	$\frac{h_S\ A_S\ E_S}{4\ m_K\ C_{th}}\alpha_{th}^2\ \Delta T$	$\frac{h_S\ L_S\ b_S\ E_S}{4\ m_K\ C_{th}}\alpha_{th}^2\ \Delta T$	$f_{w_E}\ f_{F_0}\frac{9}{64}\frac{h_2\ L_B\ b_B\ E_2\ \Delta\alpha^2\ \Delta T}{m_1\ C_{th,1}+m_2\ C_{th,2}}$	$f_{w_E}\ f_{F_0}\ 0.51\frac{h_2\ R_P^2\ E_2\ \Delta\alpha^2\ \Delta T}{m_1\ C_{th,1}+m_2\ C_{th,2}}$

For the circular plates, a Poisson's ratio of 0.3 is assumed for both layers of the bimaterial actuator

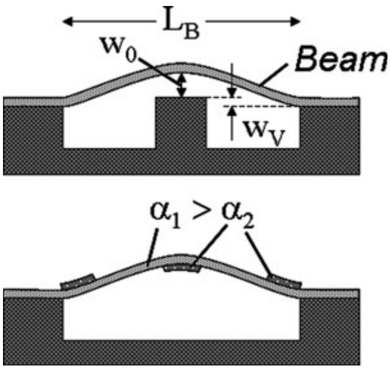


Fig. 119 Beam clamped at both ends deflected by heating only the beam

Another kind of thermomechanical actuator may be built of a *heated beam clamped at both ends*. If only the beam and not its bearing is heated, the beam generates compressive stress and buckles when its critical stress is exceeded. Typically, the beam is heated by an electrical current through a part of the beam or a conductor path on it. Some predeflection w_V of the beam is necessary to ensure buckling to the desired side (cf. Fig. 119 top). Another possibility is a material with a different heat expansion at least on a part of the beam forming a bimaterial. Figure 119 bottom shows some possibilities. Even a layer of the same material as the membrane on one of its sides will result in a buckling towards this side, because the neutral fiber is shifted towards that side. It needs also to be taken into account that a thin film may deflect a beam due to initial stress as shown in Figs. 7 or 9 on page 12 and 13, repectively.

When heated, a beam not clamped and without any outer forces acting on it, expands according to (278) (page 159). When the beam is clamped at both ends its stress becomes more compressive by the thermal stress σ_{th} which is the strain without clamping ($\alpha_{th}\ \Delta T$) times Young's modulus E_B of the beam. When the buckling stress is overcome, the beam buckles. The deflection as a function of the stress of the beam is calculated with (116) (page 75). Inserting σ_{th} into (116) yields:

$$w_0(F=0) = 3\sqrt{2}\sqrt{\frac{I}{A_B}}\sqrt{\frac{\sigma_0 - E_B\ \alpha_{th}\ \Delta T}{\sigma_k} - 1}. \tag{289}$$

Figure 120 shows the deflection w_0 of a rectangular beam with Young's modulus $E_B = 100$ GPa, thermal expansion $\alpha_{th} = 5 \times 10^{-6}$, thickness 20 μm, width 60 μm, length 1 mm calculated with the above equation as a function of temperature. For temperatures where the stress of the beam is less than the critical stress σ_k, there is no deflection. When the stress exceeds the critical stress, the deflection quickly rises as calculated with (289).

The interrelationship between the force F acting perpendicular to the center of the beam and its deflection w_0 is calculated by inserting the thermal stress σ_{th} into

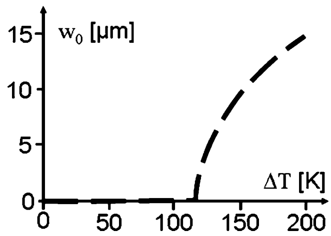


Fig. 120 Deflection of a heated beam clamped at both ends as calculated with (289)

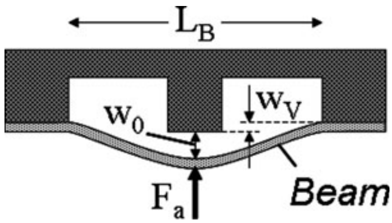


Fig. 121 Heated beam working against an external force F_a

(114) (on page 74). The force F_B generated by the beam is obtained from this by changing the sign:

$$F_B = -4\frac{A_B}{L_B} w_0 \left(48 \frac{E_B\ I}{A_B\ L_B^2} + \sigma_0 - E_B\ \alpha_{th}\ \Delta T + \frac{8}{3} E_B \frac{w_0^2}{L_B^2} \right). \quad (290)$$

The force F_B generated by the heated beam is shown in Fig. 122 as a function of the deflection w_0 for several temperature changes ΔT. The curves have been calculated with (290) and the parameters used for Fig. 120. The temperature changes correspond to different stresses in the beam, and, therefore, Fig. 122 shows a similar graph as Fig. 54 on page 75.

If a beam as shown in Fig. 121 is pressed by an external force F_a onto the structure ensuring the predeflection w_V, heating of the beam will cause no deflects until the curve shown in Fig. 122a reaches F_a. In Fig. 122, this is achieved at 100 K. The black filled circles denote the force and deflection of the beam at certain temperature changes ΔT. When the temperature exceeds 100 K, the beam deflects as indicated in the figure.

If the external force F_a is larger, the beam achieves this force at a larger temperature (150 K in Fig. 122b). At this temperature, the beam suddenly snaps up from 5 μm to approximately 11 μm as indicated by the arrow. This is an effect which might be employed in a microsystem to achieve a certain effect. For example, a valve or an electrical switch could be opened quickly by a certain amount. If this is not desired, the predeflection w_V needs to be designed larger that the snapping over deflection w_U which can be calculated with (117) (on page 75).

It is also seen in Fig. 122b that a predeflection $w_{V,2}$ which is designed too small or no predeflection results in no deflection of the beam even at very high temperatures.

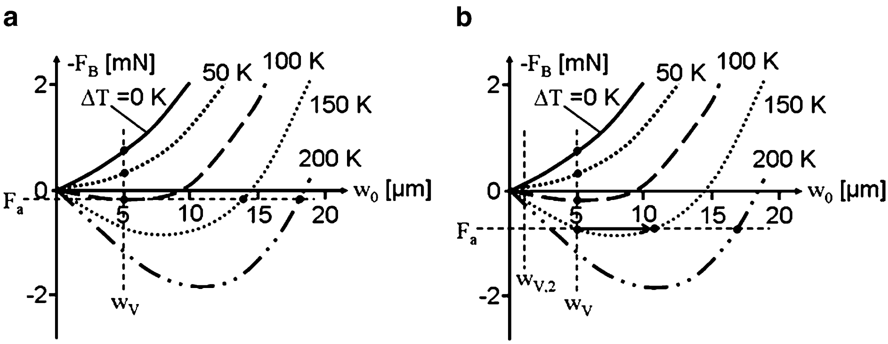


Fig. 122 Heated beam working against an external force F_a

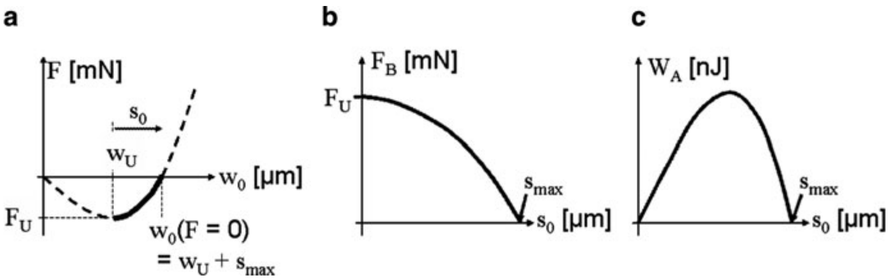


Fig. 123 Redefinition of the deflection s of the heated beam (**a**) and characteristic curve (**b**). (**c**) The energy output

The reason why the beam does not deflect without any predeflection is, that it is at its unstable equilibrium where only small or theoretically no force is necessary to hold it in position.

The deflection of the heated beam starts at its predeflection w_V. Therefore, it appears to be appropriate to define a deflection s_0 starting at w_V and to redraw the characteristic curve of the heated beam clamped at both ends as an actuator (cf. Fig. 123a, b). Maximum force can be achieved, if for a given temperature the predeflection is designed equal to the deflection w_U of the snapping over of the beam resulting in the characteristic curve shown in Fig. 123b.

The characteristic curve is calculated by substituting w_0 in (290) by $(w_U + s_0)$:

$$F_B = 4\frac{A_B}{L_B}[w_U + s_0]\left(48\frac{E_B\ I}{A_B\ L_B^2} + \sigma_0 - E_B\ \alpha_{th}\ \Delta T + \frac{8}{3}E_B\frac{[w_U + s_0]^2}{L_B^2}\right). \quad (291)$$

The maximum force F_U of the actuator is either found from (118) (on page 76) when the thermal stress is added to the residual stress or from (291) by inserting (117) (page 75) for w_U and setting s_0 to zero:

$$F_U = 8\sqrt{\frac{2}{3}}\frac{\sqrt{A_B\ I}}{L_B}\sigma_k\left(\frac{\sigma_0 - \alpha_{th}\ E_B\ \Delta T}{\sigma_k} - 1\right)^{\frac{3}{2}} \tag{292}$$

The maximum deflection s_{max} is the difference between the predeflection w_U and the deflection without any external force acting (cf. Fig. 123a). It is found by subtracting (117) from (116) and adding the thermal stress to the residual one:

$$s_{max} = 3\sqrt{2}\left(1 - \frac{1}{\sqrt{3}}\right)\sqrt{\frac{I}{A_B}}\sqrt{\frac{\sigma_0 - \alpha_{th}\ E_B\ \Delta T}{\sigma_k} - 1}. \tag{293}$$

The energy output W_A of the actuator when loaded with a force which is independent of the deflection is calculated as the product of the force F_B (291) and the deflection s_0:

$$W_A = F_B\ s_0. \tag{294}$$

The energy output is drawn in Fig. 123c as a function of the deflection. The comparison of the characteristic curve and the energy output of a heated beam clamped at both ends (Fig. 123), and a bimaterial actuator (Fig. 118 on page 162) shows only small differences. However, the bimaterial actuator is not sensitive to changes of the stress due to outer forces. The tolerance required to achieve the suitable predeflection of a heated beam clamped at both ends may also be very small.

Another kind of thermal actuator is the *thermo-pneumatic actuator*. A thermo-pneumatic actuator consists of a chamber closed by a membrane and some means to heat the gas (mostly air) inside of the chamber. An example is shown in Fig. 124 where a heater coil on the membrane is heated up. As a consequence, the heated air is expending and generates a pressure drop Δp over the membrane. The pressure difference bulges up the membrane to a deflection w_0. The behavior of a closed gas volume is described quite well by the ideal gas law:

$$p\ V = n_{mol}\ R_G\ T = \frac{V_0}{V_{mol}} R_G\, T. \tag{295}$$

In the above equation, p, V, and T represent pressure, volume, and temperature of the gas, respectively, while n_{mol} and $R_G = 8.31$ J / (mol K) denote the number of

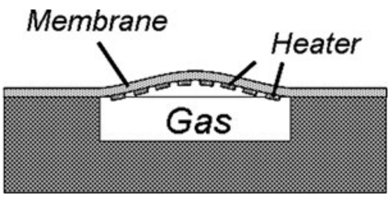


Fig. 124 Thermo-pneumatic actuator

moles of the gas contained in the volume and the gas constant which is a natural constant. V_0 is the volume of the gas when the membrane is not deflected and V_{mol} is the volume of a mole of the gas at normal temperature and pressure (22.4 L at 101.3 kPa and 295 K). As an approximation, it is assumed here that the entire gas in the actuator chamber is heated to the same temperature. In general, this is not true and the temperature of the heater needs to be much larger than the mean temperature of the gas. Therefore, the temperature discussed here is a kind of effective temperature which generates the calculated effects.

When the gas in the actuator is heated and an outer force F_0 prevents the deflection of the membrane, the volume inside the actuator chamber equals V_0. The absolute values of the outer force and the force generated by the actuator are equal to the ratio of the pressure drop Δp over the membrane and its area A_M. From (295) it is obtained:

$$\Delta p = \frac{F_0}{A_M} = \frac{R_G}{V_{mol}} \Delta T \Rightarrow F_0 = \frac{A_M\ R_G}{V_{mol}} \Delta T. \tag{296}$$

The above equation shows that the force generated at no deflection of the membrane is not a function of the volume V_0 of the actuator.

The deflection of the membrane itself requires a certain pressure difference (cf. page 33ff). If an actuator is to be designed, the radius of the membrane will be chosen so large and its thickness so small that the pressure drop required to deflect the membrane itself is negligible compared with the effect of outer forces. Therefore, it appears to be reasonable to neglect the effect of the membrane on force and deflection in the following calculations. However, it needs to be checked in every case that this assumption is really true, and, if necessary the following equations need to be adapted.

If the deflection (i.e., volume change) and pressure difference of a thermo-pneumatic actuator is to be calculated under the action of outer forces in general, it helps to assume that the deflection was achieved in two steps. In the first step, the volume is extended by ΔV by enhancing the temperature by ΔT_1, while the pressure is kept constant at the value p_0 when the membrane is not deflected. From (295):

$$\Delta V = \frac{V_0\ R_G}{V_{mol}\ p_0} \Delta T_1 \Rightarrow \Delta T_1 = \frac{V_{mol}\ p_0}{V_0\ R_G} \Delta V. \tag{297}$$

In the second step, the pressure is enhanced by a temperature change $(\Delta T - \Delta T_1)$ at a constant volume. Equation (295) now results in:

$$\begin{aligned} \Delta p &= \frac{V_0\ R_G}{V_{mol}(V_0 + \Delta V)} (\Delta T - \Delta T_1) = \frac{V_0\ R_G\ \Delta T - V_{mol}\ p_0\ \Delta V}{V_{mol}(V_0 + \Delta V)} \\ &= \frac{R_G\ \Delta T - V_{mol}\ p_0(\Delta V/V_0)}{V_{mol}(1 + (\Delta V/V_0))}. \end{aligned} \tag{298}$$

Equation (297) was inserted in (298) for ΔT_1. Equation (298) is the characteristic curve of the thermo-pneumatic actuator. Figure 125a shows this characteristic curve for an actuator with a volume V_0 of 50 and 100 nL, respectively. The pressure p_0 at no deflection and temperature enhancement ΔT are 101 kPa and 100 K, respectively. It is seen that the maximum pressure Δp_{max} at no deflection of the membrane is not a function of the volume V_0 of the actuator. The characteristic curves extend over the coordinate axes, i.e., the maximum pressure Δp_{max} and maximum volume change ΔV_{max} can be exceeded when an outer force or pressure pushes the membrane more down to negative values or pulls it more up. Maximum pressure and maximum volume change can easily be calculated by inserting $\Delta V = 0$ and $\Delta p = 0$ into (298), respectively:

$$\Delta p_{max} = \frac{R_G \, \Delta T}{V_{mol}} \text{ and } \Delta V_{max} = \frac{V_0 \, R_G \, \Delta T}{V_{mol} p_0}. \tag{299}$$

The energy output W_A of a thermo-pneumatic actuator is the product of the generated pressure difference Δp and volume change ΔV. Thus from (298) it is obtained:

$$W_A = \frac{R_G \, \Delta T - V_{mol} \, p_0 (\Delta V / V_0)}{V_{mol} \, (1 + (\Delta V / V_0))} \Delta V. \tag{300}$$

The energy output of the actuator shown in Fig.125a, when working against a constant load is displayed in Fig.125b for an actuator volume $V_0 = 100$ nL. As for a piezo, a thermal bimorph or a heated beam clamped at both ends, there is an optimum volume change ΔV_{opt} (optimum deflection for the other actuators) at which the energy output is maximum. The efficiency at the maximum energy output is only on the order of 1% instead of approximately 10% for a piezo. This optimum volume change ΔV_{opt} is found by calculating the zero of the derivative of (300):

$$\frac{\partial W_A}{\partial \Delta V} = 0 \Rightarrow \Delta V_{opt} = V_0 \left(\sqrt{1 + \frac{\Delta V_{max}}{V_0}} - 1 \right) = V_0 \left(\sqrt{1 + \frac{R_G \, \Delta T}{V_{mol} \, p_0}} - 1 \right). \tag{301}$$

The volume change ΔV_{opt} as calculated with the above equation is approximately half of the maximum volume change ΔV_{max}, if the ΔV_{max} is small compared

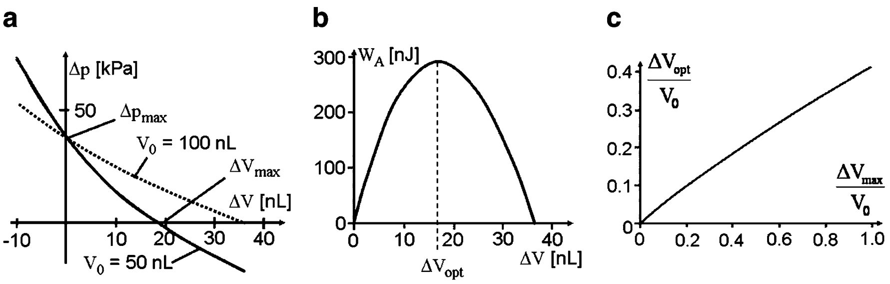


Fig. 125 Characteristic curves of thermo-pneumatic actuators with a volume of 50 and 100 nL, respectively (**a**), energy output W_A (**b**), and optimum volume change for maximum energy output (**c**)

with the actuator volume without membrane deflection. Figure 125c shows the optimum volume change as a function of the maximum one calculated with (301).

The maximum energy output which can be generated is obtained now by inserting (301) for the volume change into (300):

$$W_{A,opt} = \frac{R_G\,\Delta T - V_{mol}\,p_0\left(\sqrt{1+(R_G\,\Delta T/(V_{mol}\,p_0))}-1\right)}{V_{mol}\,\sqrt{1+(R_G\,\Delta T/(V_{mol}\,p_0))}}V_0\left(\sqrt{1+\frac{R_G\,\Delta T}{V_{mol}\,p_0}}-1\right). \tag{302}$$

The maximum energy output density is obtained from the above equation by dividing by V_0. If 200°C and 100 kPa are assumed for the maximum possible temperature change and the environmental pressure, respectively, the maximum energy output density is 10 μJ/μL.

For most applications not only the volume change but also the deflection of the thermo-pneumatic actuator is important. The deflection of an actuator with a circular membrane can be calculated with sufficient accuracy, if a spherical cap is assumed for the form of the deflected membrane. The volume V_W of a spherical cap with height w_0 (corresponding to the membrane deflection) and cap diameter $2R_M$ (corresponding to the membrane diameter) is given by:

$$V_W = \frac{1}{6}\pi\,w_0\left(3\,R_M^2 + w_0^2\right) \approx \frac{1}{2}\pi\,w_0\,R_M^2, \tag{303}$$

V_W is the volume change generated by a temperature change. Therefore, from (297) the maximum membrane deflection $w_{0,max}$ is obtained now:

$$V_W = \frac{V_0\,R_G}{V_{mol}\,p_0}\Delta T \approx \frac{1}{2}\pi\,w_{0,max}\,R_M^2 \Rightarrow w_{0,max} = \frac{2}{\pi\,R_M^2}\frac{V_0\,R_G}{V_{mol}\,p_0}\Delta T. \tag{304}$$

If a cylindrical actuator chamber with height h_A and radius R_M is assumed under the membrane, its volume is $V_0 = h_A\,\pi\,R_M^2$, and it is obvious that the maximum deflection of the membrane is not a function of the lateral dimensions but only of the height h_A of the actuator chamber:

$$w_{0,max} = \frac{2\,h_A\,R_G}{V_{mol}\,p_0}\Delta T. \tag{305}$$

Similarly, the volume change V_W is calculated for square membranes. The volume change is found by integrating the deflection w_0 as described by (65) on page 45 over the entire membrane:

$$V_W = \int_{-a_M/2}^{a_M/2}\int_{-a_M/2}^{a_M/2} w_0\left(1-4\frac{x^2}{a_M^2}\right)^2\left(1-4\frac{y^2}{a_M^2}\right)^2 dx\,dy = \left(\frac{8}{15}\right)^2 a_M^2\,w_0. \tag{306}$$

In the same way as above, it is found now:

$$V_W = \frac{V_0\ R_G}{V_{mol}\ p_0}\Delta T \approx \left(\frac{8}{15}\right)^2 w_{0,max}\ a_M^2 \Rightarrow w_{0,max} = \left(\frac{15}{8}\right)^2 \frac{V_0\ R_G}{a_M^2\ V_{mol} p_0}\Delta T. \quad (307)$$

If an actuator chamber in the form of a hollow cuboid with height h_A is assumed, the maximum deflection is:

$$w_{0,max} = \left(\frac{15}{8}\right)^2 \frac{h_A\ R_G}{V_{mol}\ p_0}\Delta T. \quad (308)$$

Very large pressure changes can be generated by the *phase transition* of some material inside of an actuator chamber (see Fig.126 top). A simple example is a micro-steam engine: A small quantity of water is enclosed in an actuator chamber and the water is heated by some means until it is partly evaporated and the vapor pressure deflects the membrane. The vapor pressure p_V is only a function of temperature T and vaporization heat Λ of 1 mol:

$$p_V = e^{-\frac{\Lambda}{R_G T}}, \quad (309)$$

That is, the pressure generated is not a function of the deflection of the membrane as long as there are both liquid and gaseous water in the actuator. If the deflection of the membrane is changed by an outer force acting on the membrane, vapor is turned into liquid or vice versa and the pressure remains to be the same (cf. Fig.126 bottom).

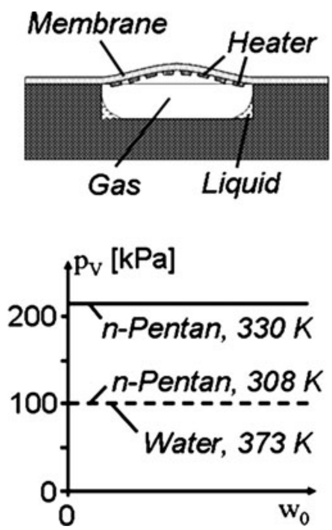


Fig. 126 Thermo-pneumatic actuator with phase transition (*top*) and characteristic curve (*bottom*)

A thermal actuator with phase transition can achieve comparatively large deflections and large forces even if the volume of the actuator chamber is small. If the actuator chamber is shallow and the diameter of the membrane is comparatively large, the deflection of the membrane is not a limiting factor for the volume expansion. Thus, a volume expansion of a factor of 10 appears to be possible. If from Fig. 126 a pressure of only 100 kPa is taken and multiplied by a volume change of 10, an energy output density of 1 MPa = 1,000 μJ/μL is obtained.

However, it is difficult to fill a small amount of liquid into the actuator chamber and to make sure that the liquid, and especially the vapor, does not leave it again by diffusion through the membrane.

The thermal energy W_{th} needed to generate deflection of the membrane consists of two terms. The first one is the energy necessary to heat up the liquid and the second one is the energy required for evaporation:

$$W_{th} = c_p\ \rho_F\ V_{fl}\ \Delta T + \lambda_V\ \rho_F\ \Delta V_{fl}. \tag{310}$$

Here c_p, ρ_F, λ_V, V_{fl}, and ΔT are the heat capacity, density, steam heat, volume, and temperature change of the liquid, respectively. The first term in the above equation is needed to generate the actuator pressure, while the second one generates the deflection of the membrane. In general, the first term is much larger than the second one. Therefore, in the light of minimizing the energy consumption the volume of the liquid inside the actuator needs to be chosen not larger than enough to allow for the desired maximum deflection.

The mass Δm of the liquid which needs to be evaporated to obtain a volume change ΔV can be calculated from the mass m_{mol} and the volume V_{mol} of a mole of a gas:

$$\Delta m = \frac{\Delta V}{V_{mol}} m_{mol}. \tag{311}$$

The volume of a mole as a function of temperature and pressure is calculated from (295) (page 167) and the volume V_{fl} of a liquid is the ratio of its mass Δm and density ρ_F. Taking all this together with the above equation yields the minimum liquid volume $V_{fl,min}$ required to generate a certain volume change ΔV:

$$V_{fl,min} = \frac{\Delta V}{\rho_F\ R_G} \frac{p}{T} m_{mol}. \tag{312}$$

In general, the efficiency of all thermal actuators cannot be larger than the efficiency η_C of the Carnot process:

$$\eta_C = \frac{\Delta T}{T_0}. \tag{313}$$

In the above equation, T_0 and ΔT are the ambient temperature and the temperature rise generated for the process. As a consequence, it is advantageous to drive a thermal actuator high above the ambient temperature. Besides a higher efficiency, a higher driving temperature also reduces the effect of changes of the ambient temperature on the performance of the actuator. In addition, this is necessary to cope with another typical disadvantage of thermal actuators: In general, it is easy to heat them up very quickly by an electrical current, but it is difficult to cool them down in a short time again. Cooling is achieved by heat diffusion in most cases and that is a comparatively slow process. Therefore, the heat capacity of thermal actuators needs to be designed as small as possible allowing quick heating and especially cooling.

In Table 16, the order of magnitude of typical properties of microactuators are compared with each other. The values are only an orientation for choosing the right actuator for a certain application and more exact results need to be calculated with the equations in the corresponding chapters.

Exercises

Problem 24

Figure E22 shows a microvalve which was introduced at the conference Actuator 96 by the company Bosch [51]. On a nearly circular silicon membrane, there are arranged two annular aluminum stripes. The valve is opened against pressure acting at the inlet by heating the inner aluminum ring. The geometry of the valve is designed such that it is closed when the inner ring is not heated.

(a) What is the purpose of the outer aluminum ring? It is not heated to switch the valve.

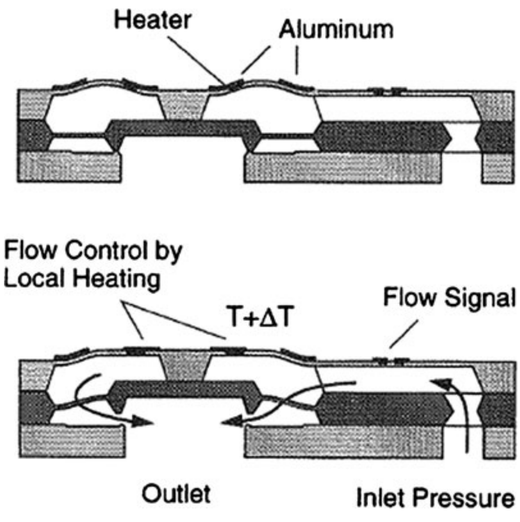


Fig. E22 Cross-section of the microvalve [51]

Table 16 Typical orders of magnitude of properties of microactuators. For details see the corresponding chapters

	Electrostatic		Electroosmotic	Piezo		Thermal					
	normal	lateral		general	bimorph	general	bimaterial	beam	Thermo pneumatic	Phase transition	Pneumatic/ hydraulic
Stroke length	5 μm	100 μm	>10 mm	10 nm	5 μm	500 nm	50 μm	10 μm	100 μm	1 mm	1 mm
Force	100 μN	5 μN	50 μN	10 N	500 mN	500 N	1 N	1 mN	50 mN	1 N	10 N
Max. energy output density	0.7 μJ/μL	0.7 μJ/μL		0.6 μJ/μL	0.4 μJ/μL	270 μJ/μL	200 μJ/μL	50 μJ/μL	10 μJ/μL	1000 μJ/μL	
Efficiency	<100%			<10%		<1%			<1%	<1%	
Required voltage	100 V		100 V	100 V		10 V			10 V	10 V	
Activation time	ns			ns		1–10 ms			1–10 ms	1–10 ms	ms
Deactivation time	ns			ns		10–500 ms			10–500 ms	10–500 ms	ms
Sensitive to temperature changes	No		Yes	Yes		Yes			Yes	Yes	No

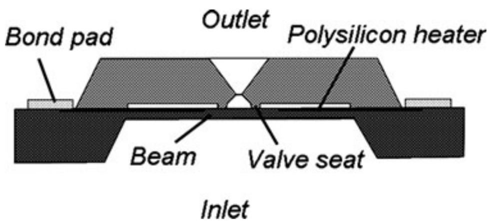


Fig. E23 Cross-section of the microvalve [52]

(b) It is difficult to calculate the behavior of an annular bimaterial. Therefore, rectangular bimaterial stripes shall be calculated here which are as wide as the circumference of the real aluminum and as long as their width.
Calculate maximum force and maximum deflection of the bimaterial stripe. What is the maximum energy output?

(c) Calculate the thermal energy needed to deflect the bimaterial stripe. Calculate the ratio to the energy output, and, thus find out the maximum efficiency. Hint: The energy required to deflect the stripe is approximately the same as the energy needed to heat it up.

Young's modulus of Al	70 GPa	Young's modulus of Si	190 GPa
Circumference of inner aluminum stripe	1 mm	Width of inner aluminum stripe	3.5 mm
Thickness of aluminum	6.7 μm	Thickness of membrane	12 μm
Thermal expansion of Al	23.8×10^{-6}/K	Thermal expansion of Si	2.7×10^{-6}/K
Heat capacity of Al	896 J/(kg K)	Heat capacity of Si	703 J/(kg K)
Density of aluminum	2,730 kg/m^3	Density of silicon	2,330 kg/m^3
Temperature increase of inner aluminum ring	200 K		

Problem 25

On the conference Actuator 94, a microvalve with a silicon beam as a switching element was presented [52]. On the beam, there was a conductor path employed as a heater. The beam was deflected downward when heated and opened the valve (cf. Fig. E23).

When the beam is not heated, it lies horizontally on the valve seat and closes the valve (residual stress $\sigma_0 = 0$). In the drawing, it is not shown that the beam is deflected a little bit downward also when the valve is closed, because the valve seat is a bit protruding.

(a) The beam needs to move down for at least a quarter of the diameter of the outlet to avoid a limitation of the flow through the open valve. How much needs the beam to be heated to achieve the necessary deflection? (Assume that there is no pressure drop over the beam when the valve is open and that, therefore, no force is acting on the beam).

(b) What temperature change is necessary to open the valve against a pressure difference of 100 kPa?
(c) What predeflection is necessary to allow opening of the valve with the temperature difference calculated at (b).
(d) What is the deflection of a beam over the valve seat with the predeflection from (c) and the temperature difference from (b)?

Length of the beam	2.6 mm	Young's modulus of Si	190 GPa
Width of the beam	600 μm	Thermal expansion of Si	2.3×10^{-6}/K
Thickness of the beam	20 μm	Residual stress of beam	0 MPa
Diameter of valve outlet	360 μm		

Problem 26

A thermo-pneumatic actuator with a cylindrical chamber which is built up as shown in Fig. 124 (page 167) shall close the inlet of a microvalve against an outer pressure. With (301) (page 169), the optimum stroke of the actuator can be calculated with which the energy is used most efficiently.

(a) Please derive from the characteristic curve [(298) on page 168] an equation with which the counter pressure can be calculated at which the energy of the actuator is used optimally.
(b) Which counter pressure is optimal with respect to energy consumption, if the actuator is heated by 100 and 200 K, respectively?
(c) What is the necessary height of the actuator chamber if the stroke is 20 μm at a temperature enhancement of 200 K and if the energy of the actuator is to be used optimally?
(d) How large should the radius of the actuator be chosen to achieve the largest possible efficiency for the actuator?
(e) How many energy is necessary at least to heat up the actuator by 200 K if the chamber is 30 μm high and has a radius of 500 μm? Assume (unrealistically) that no heat is lost and only the air in the actuator is to be heated.
(f) How large is the efficiency in this idealized case?

Heat capacity of air at constant pressure	1.005 kJ/(kg K)	Density of air	1.29 kg/m^3
Pressure in the actuator when deflected without counter pressure			101.3 kPa