Piezoelectric Effect

The piezoelectric effect is widely employed in microtechnique. Especially, piezos used as an actuator are appreciated because of the large force which can be generated in a small volume. But piezos can be employed also as sensors which provide a large output voltage.

If a piezoelectric material is loaded with a pressure, it is compressed as any other elastic material, and it is extended transversal to the direction of the load (cf. Fig. 99b). In addition, electrical charges are generated on electrodes on the surface of the piezoelectric material and it is possible to measure a voltage between the electrodes. The generation of charges is called the piezoelectric effect and the material is called a piezo. The origin of the word "piezo" is Greek and it means "I am pressing". The piezoelectric effect is employed to measure strains, forces, or pressures.

There is also an inverse piezoelectric effect: When a voltage is supplied to the electrodes, the piezo is straining both longitudinal and transversal to the electric field as shown in Fig. 99c. This inverse piezoelectric effect is employed to build actuators.

There are two kinds of piezoelectric materials: *monocrystalline* materials, such as quartz, zink oxide, and lithium niobate, and *ferroelectric* materials, such as the ceramics PZT (lead–zirconate–titanate) and barium titanate, and the polymer PVDF (poly vinyliden fluoride). The piezoelectric effect of monocrystalline materials is a function of crystal orientation. In general, it is comparatively small. Therefore, the piezoelectric effect of ferroelectric materials is employed in most cases.

Ferroelectric materials are not yet piezoelectric after they have been fabricated. They need to be polarized by a large electrical field before use; similar to ferromagnetic materials which need to be magnetized in a magnetic field before they become permanent magnets. This analogy is the reason for the name "ferro"electric, although these materials have nothing in common with iron.

Also in analogy to ferromagnetic materials, ferroelectric materials have a Curie temperature. If they are heated up over this temperature, the piezoelectric effect disappears again. Again in analogy to ferromagnetic materials, ferroelectric materials can be depolarized by an electric field opposite to their direction of polarization. As a consequence, ferroelectric materials should not be exposed to high temperatures and large electric fields opposite to their polarization.

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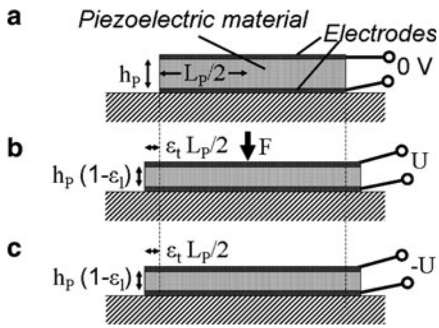


Fig. 99 A piezo pressed together generates electrical charges on its electrodes (**b**), and a voltage supplied to the electrodes is straining the piezo. In (**a**) the piezo is shown without mechanical load and discharged

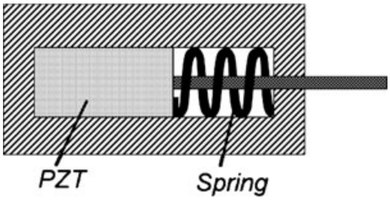


Fig. 100 Piezo mounted such that it is always under compressive stress

It needs to be noted also that the most commonly used piezoelectric material PZT, which shows the largest effect, is a ceramic. As a consequence, it is brittle and tends to break when loaded with a tensile force. Tensile stress in a piezo can be avoided by designing an actuator such that it is under compressive stress always. Figure 100 shows an example.

Usually, the directions of a piezo are denominated by the subscripts 1, 2, and 3. The direction of the polarization of the piezo is marked with the subscript 3. Typically, the direction of polarization is identical with the connecting line of the electrodes, because the piezo was polarized by applying a large voltage to these electrodes.

The strain ε_3 in polarization direction of a piezo is calculated with the following equation:

$$\varepsilon_3 = d_{33}\, E_{el} + \frac{\sigma_3}{E_P}, \tag{230}$$

where E_{el} is the electrical field and σ_3 is the stress acting in polarization direction. E_P is Young's modulus of the piezo when the electrodes are short-circuited. The quantity d_{33} is the piezoelectric modulus of the piezo. d_{33} is a material property, which denotes the strength of the piezoelectric effect.

Equation (230) consists of two terms. The second one is already known; it is the strain of an elastic material according to Hooke's law. The first term in (230) represents the strain generated by the inverse piezoelectric effect.

The deflection w_3 in polarization direction of a piezo is obtained form (230) by multiplying with the thickness h_P of the piezo which in this case is equal to the distance between its electrodes:

$$h_P\ \varepsilon_3 = w_3 = d_{33}\ h_P\ E_{el} + h_P \frac{\sigma_3}{E_P} = d_{33}\ U + \frac{h_P}{E_P\ A_P} F_3 = d_{33}\ U + F_3/k_3. \quad (231)$$

The last part of the equation was obtained with the definition of the homogeneous electrical field E_{el} in a capacitor as the quotient of the voltage and the distance of the electrodes, and the force F_3 acting in polarization direction was calculated as the product of the homogeneous stress σ_3 and the cross-sectional area A_P of the piezo perpendicular to the stress. The factor behind F_3 is the spring constant k of an elastic body.

The unit of the piezoelectric modulus obviously is (m/V). The first index denotes the direction in which the voltage is applied and the second stands for the direction in which the deflection is calculated. Thus, the strain in 1-direction (the direction of the length L_P of the piezo) under the action of forces in 1- and 3-direction is calculated with:

$$\varepsilon_1 = d_{31}\ E_{el} + \frac{\sigma_1}{E_P} - \nu_P \frac{\sigma_3}{E_P}, \quad (232)$$

where ν_P is Poisson's ratio of the piezo. The deflection in 1-direction is obtained by multiplying with the length L_P of the piezo:

$$L_P\ \varepsilon_1 = w_1 = d_{31} \frac{L_P}{h_P} U + \frac{L_P\ F_1}{E_P\ h_P\ b_P} - \nu_P \frac{F_3}{E_P\ b_P}, \quad (233)$$

where b_P is the width of the piezo. The absolute value of the piezoelectric modulus d_{31} is typically approximately half of the one of d_{33}. Therefore, it might be concluded that employing the effect in polarization direction would be preferable compared with the effect in transversal direction; but the opposite is true, because the length L_P of a piezo typically is much larger than its thickness h_P, and the factor L_P/h_P is much larger than two.

The electric field might also be applied in 1-direction transversal to the polarization and the deflection in 1-direction needs to be calculated. In this case, the piezoelectric modulus d_{11} needs to be employed:

$$\varepsilon_1 = d_{11}\ E_{el} + \frac{\sigma_1}{E_P} - \nu_P \frac{\sigma_3}{E_P} = d_{11} \frac{U}{L_P} + \frac{\sigma_1}{E_P} - \nu_P \frac{\sigma_3}{E_P} \quad (234)$$

$$\Rightarrow w_1 = L_P\ \varepsilon_1 = d_{11}\ U + L_P \left(\frac{\sigma_1}{E_P} - \nu_P \frac{\sigma_3}{E_P} \right). \quad (235)$$

However, applying a voltage perpendicular to the polarization is very difficult, because after fabrication and polarization of the piezo, electrodes need to be manufactured in 1-direction which are not short-cricuited with the electrodes employed for polarization. Therefore, d_{33} and d_{31} are sufficient to calculate the deflection of a piezo in most cases.

Equations (231) and (233) are employed for the calculation of the deflection of a piezo as an actuator. Figure 101 shows the deflection w_3 as a function of the force F_3, calculated with (231) for a piezo from PZT with a Young's modulus and piezoelectric modulus of 60 GPa and 250×10^{-12} m/V, respectively, an area of 10 mm^2, and a distance h_P between the electrodes of 0.5 mm. This graph is the characteristic curve of a piezo as an actuator. The maximum force F_0 generated by the piezo is achieved when it is hindered from deflecting, and no force can be generated at the maximum deflection w_E which can be generated. Between maximum force and maximum deflection, there is a linear curve which even extends over the points of maximum deflection and maximum force, because an external force may further deflect the piezo.

The maximum deflection w_E of a piezo, which is obtained when no external force is acting on it, is calculated by inserting no force in (231). The maximum force F_0 which can be generated by a piezo is obtained when it is hindered from deflecting. In polarization direction, this is calculated by setting the deflection in (231) to zero and to solve for the force F_3:

$$F_0 = -d_{33} \frac{E_P \, A_P}{h_P} U. \tag{236}$$

If the voltage supplied to the piezo is changed, the characteristic curve is shifted as shown in Fig. 101. Without any voltage supplied, the characteristic curves simply represents the deflection due to Hooke's law (cf. 231).

Equations (231) and (233) are used to calculate the deflection of a piezo employed as an actuator. If a piezo is to be used as a sensor, the voltage generated by the deformation of the piezo needs to be calculated. The charge density D_3 generated in 3-direction is obtained from the following equation:

$$D_3 = d_{33} \, \sigma_3 + \varepsilon_0 \, \varepsilon_r \, E_{el}. \tag{237}$$

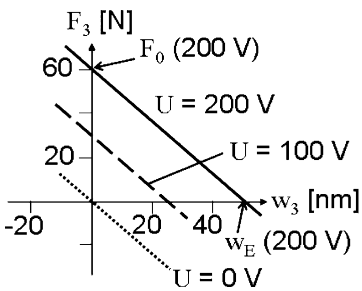


Fig. 101 Characteristic curves of a piezo as an actuator driven at various voltages

The charge Q_3 on the electrodes in polarization direction of the piezo is obtained from this equation by multiplying with the area A_P of the electrodes (A_P is equal to the area of the piezo in this case):

$$Q_3 = d_{33}\ F_3 + \frac{\varepsilon_0\ \varepsilon_r\ A_P}{h_P} U = d_{33}\ F_3 + C_{el}\ U. \tag{238}$$

The above equation consists of two terms. The second one represents the charge which is stored on a capacitor: $Q = C_{el}\ U$, where C_{el} is the electrical capacity of the piezo [cf. (221) on page 131]. The first term in (238) is the charge generated by the force acting on the piezo. The factor in front of the force F_3 is the piezoelectric modulus again. It is the same number as in (231), although the unit obviously is [C/N] instead of [m/V] in (231). However, both units are the same and it will be shown below that both numbers need to be equal because the piezoelectric effect is reversible.

If a piezo is strained perpendicular to its polarization direction, there is also a charge generated on the electrodes. Therefore, another term needs to be added in (237):

$$D_3 = d_{31}\ \sigma_1 + d_{32}\ \sigma_2 + d_{33}\ \sigma_3 + \varepsilon_0\ \varepsilon_r\ E_{el}. \tag{239}$$

$$\Rightarrow Q_3 = d_{31} \frac{L_P}{h_P} F_1 + d_{32} \frac{b_P}{h_P} F_2 + d_{33}\ F_3 + \frac{\varepsilon_0\ \varepsilon_r\ A_P}{h_P} U. \tag{240}$$

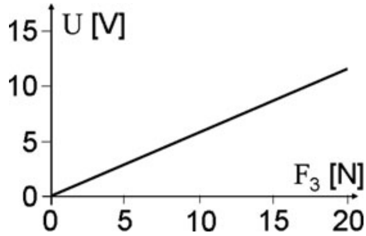
When a piezo is employed as a sensor, the above equation is used to calculate the charge generated by straining. Actually not the charge but the voltage is measured. The electrodes of the piezo are short-cricuited, the contact between the electrodes is opened, and the force is applied. As the contact between the electrodes is open, no charge can flow onto or leave the electrodes; i.e., Q_3 is zero in (240) yielding the voltage U which can be measured:

$$\begin{aligned} U &= \frac{d_{31}}{\varepsilon_0\ \varepsilon_r} \frac{L_P}{A_P} F_1 + \frac{d_{32}}{\varepsilon_0\ \varepsilon_r} \frac{b_P}{A_P} F_2 + \frac{d_{33}}{\varepsilon_0\ \varepsilon_r} \frac{h_P}{A_P} F_3 \\ &= g_{31} \frac{L_P}{A_P} F_1 + g_{32} \frac{b_P}{A_P} F_2 + g_{33} \frac{h_P}{A_P} F_3 \ \text{with}\ g_{ij} = \frac{d_{ij}}{\varepsilon_0\ \varepsilon_r}. \end{aligned} \tag{241}$$

The quantities g_{ij} are often also called piezoelectric moduli g_{31}, g_{32}, and g_{33}. In this book, it is avoided to use these quantities to prevent confusion of the different types of piezoelectric moduli.

Equation (241) describes the characteristic curve of a piezo employed as a sensor. Figure 102 shows the characteristic curve of a piezo with thickness h_P, area A_P, Young's modulus E_P, piezoelectric modulus d_{33}, and relative permittivity ε_r of 0.5 mm, 10 mm^2, 60 GPa, 250×10^{-12} m/V, and 2,400, respectively, calculated with (241). The force is acting in polarization direction and a few Newtons are enough

Fig. 102 Characteristic curve of a piezo as a sensor



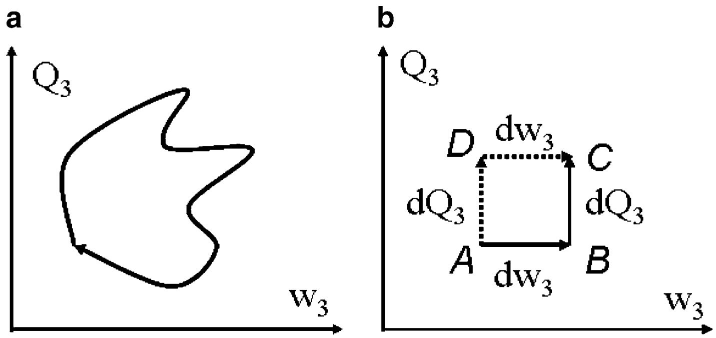


Fig. 103 Paths in the charge-deflection plane

to generate an output of several volts. This is an extraordinary result, because other sensors, e.g., strain gauges, produce only some millivolts without amplification. Therefore, electromagnetic disturbances usually are no problem for piezoelectric sensors.

On the other hand, it needs to be noted that the piezoelectric modulus is a function of temperature and in almost every case temperature compensation is necessary. Besides this, every piezoelectric material has a small electrical conductivity resulting in a slow discharging of the piezo. Therefore, some electronics is necessary if long-time measurements are to be performed. Short-time measurements of changes of force or pressure are realized easier.

The piezoelectric modulus d_{33} in (231) and (238) is the same. This is due to the fact that the potential energy of a piezo is reversible [48]. That is, we start from a certain state where the piezo has a deflection w_i and charge Q_i resulting in a potential energy V_{p0}. Then, deflection and charge are changed such that after a while the piezo arrives at the same state (deflection and charge) again (cf. Fig. 103a). The energy of a piezo is the same as before the changes, if the potential energy is reversible. In other words, the potential energy is a function of deflection and charge only, and not a function of the way how this state was approached. It is obvious that the potential energy of a piezo is reversible, because it is a function of deflection and charge only. It is the sum of the potential energy of a capacitor and a spring:

$$V_p = \frac{1}{2}\frac{Q_3^2}{C_{el}} + \frac{k_3}{2}w_3^2. \tag{242}$$

Starting from the above equation, the piezo is deflected by Δw_3 (path from *A* to *B* in Fig. 103b) resulting in the following energy:

$$V_p = V_{p0} + \frac{\partial V_p}{\partial w_3} \Delta w_3. \tag{243}$$

Then the charge of the piezo is changed by ΔQ_3 (path from *B* to *C* in Fig. 103b):

$$V_p = V_{p0} + \frac{\partial V_p}{\partial w_3} \Delta w_3 + \frac{\partial V_p}{\partial Q_3} \Delta Q_3 + \frac{\partial^2 V_p}{\partial w_3\ \partial Q_3} \Delta w_3 \Delta Q_3. \tag{244}$$

If the charge is changed first and then the deflection (path from *A* over *D* to *C* in Fig. 103b), the energy is calculated as:

$$V_p = V_{p0} + \frac{\partial V_p}{\partial Q_3} \Delta Q_3 + \frac{\partial V_p}{\partial w_3} \Delta w_3 + \frac{\partial^2 V_p}{\partial Q_3\ \partial w_3} \Delta Q_3\ \Delta w_3. \tag{245}$$

If the potential energy of the piezo is reversible, (244) and (245) need to be equal, because the same state is approached via different ways (first changing the deflection and then the charge or vice versa):

$$\frac{\partial^2 V_p}{\partial Q_3\ \partial w_3} = \frac{\partial^2 V_p}{\partial w_3\ \partial Q_3}. \tag{246}$$

Differentiation of (242) yields:

$$\frac{\partial V_p}{\partial Q_3} = \frac{Q_3}{C_{el}} = U \text{ and } \frac{\partial V_p}{\partial w_3} = k_3 w_3 = F_3. \tag{247}$$

Thus, (246) results in:

$$\frac{\partial U}{\partial w_3} = \frac{\partial F_3}{\partial Q_3}. \tag{248}$$

Solving (231) for U and calculating the differential with respect to w_3 on the one hand, and solving (238) for F_3 and calculating the differential with respect to Q_3 yields:

$$U = \frac{w_3 - F_3/k_3}{d_{33}} \Rightarrow \frac{\partial U}{\partial w_3} = \frac{1}{d_{33}}, \tag{249}$$

$$F_3 = \frac{Q_3 - C_{el} U}{d_{33}^*} \Rightarrow \frac{\partial F_3}{\partial Q_3} = \frac{1}{d_{33}^*}. \tag{250}$$

The asterisk in (250) denotes the piezoelectric modulus appearing in (238). According to (248), (249) and (250) are equal and thus d_{33} and d_{33}^* are equal.

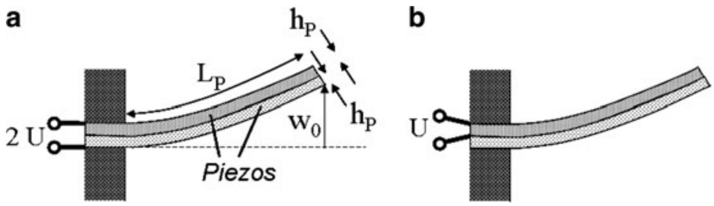


Fig. 104 Piezo bimorphs

The deflection of a piezo typically is on the order of several nanometers. This is not enough for most applications. On the other hand, the forces generated by piezos are comparatively large. Therefore, it is advantageous to partly sacrifice the force generated and to increase the deflection. This is typically achieved by an arrangement of two or more piezos bonded on each other. Figure 104a shows two piezos bonded onto each other. This is called a *piezo bimorph*. If a voltage is applied in polarization direction of a piezo, it shrinks laterally. Thus, the other piezo of the bimorph needs to be supplied with a voltage against its polarization direction resulting in a lateral extension. As a consequence, the bimorph is deflected transversally.

If voltage is applied to both piezos of a bimorph, the voltage is limited because always one of the piezos is charged against its polarization direction. Therefore, it is usual to apply power only to the piezo which shall shrink laterally (Fig. 104b). The deflection is reduced by a factor of 2, because only one piezo is active but the voltage may be raised more, e.g., a factor of 5. Thus, charging only one piezo yields larger deflections. Typically, the transversal deflection of the bimorph is a factor of ten larger than the lateral deflection of the two piezos.

The deflection $w_{0,B}$ of a bimorph composed of two piezos with length L_P, width b_P, thickness h_P, Young's modulus E_P, and piezoelectric modulus d_{33} is calculated with the following equation [49] (Only one piezo is active.):

$$w_{0,B} = w_{E,B} + w_{F,B} = \frac{3}{8} d_{31} \frac{L_P^2}{h_P^2} U + \frac{L_P^3}{2\, E_P\, b_P\, h_P^3} F. \tag{251}$$

The second term in (251) describes the deflection $w_{F,B}$ of a beam with a rectangular cross-section and thickness $2h_P$ which is clamped at one end and loaded transversally with a force F at the free end. This is already known from the theory of the deflection of beams [cf. Fig. 45 and (94) on page 65 and 66]. The first term in (251) describes the deflection $w_{E,B}$ of the beam generated by the voltage U.

The force F_P generated by a piezo bimorph is calculated by solving (251) for F and changing the sign, because the force of the bimorph is calculated instead of the force acting on it:

$$F_P = \frac{3}{4} d_{31} \frac{E_P\, b_P\, h_P}{L_P} U - 2 \frac{E_P\, b_P\, h_P^3}{L_P^3} w_0. \tag{252}$$

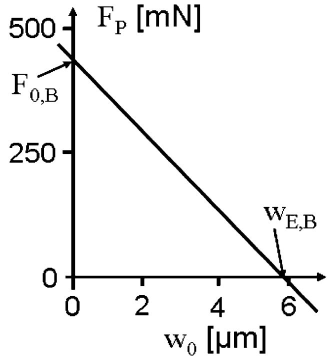


Fig. 105 Characteristic curve of a piezo bimorph as an actuator

Equation (252) describes the characteristic curve of a piezo bimorph. Figure 105 shows this curve for a bimorph with piezos with length, width, thickness, Young's modulus, and piezoelectric modulus of 20 mm, 5 mm, 1 mm, 60 GPa, and 190×10^{-12} m/V, respectively, and driven by 200 V. The comparison with Fig. 101 on page 140 shows that a larger deflection and a smaller force are achieved. The characteristic curve is linear and subtends ordinate and abscissa at the maximum force $F_{0,B}$ obtained when the deflection of the bimorph is prevented by outer forces and the maximum deflection $w_{E,B}$ which is obtained when no outer forces are acting on the bimorph. The characteristic curve extends over the intersections, because a larger force than $F_{0,B}$ acting against the bimorph bends it backward and an additional force in forward direction results in a larger deflection than $w_{E,B}$ and a negative counter force generated by the elastic forces of the beam.

The maximum force $F_{0,B}$ is the first term in (252), which calculates the force of the bimorph when it is not deflected. The maximum deflection is found as the first term of (251), which calculates the deflection when no force is acting on the bimorph:

$$F_{0,B} = \frac{3}{4} d_{31} \frac{E_P \, b_P \, h_P}{L_P} U, \tag{253}$$

$$w_{E,B} = \frac{3}{8} d_{31} \frac{L_P^2}{h_P^2} U. \tag{254}$$

The two above equations show that the characteristic curve of a piezo can be adapted in certain limits to the needs of a given application. A larger ratio of the thickness to the length increases the maximum force and reduces the maximum deflection. With a larger width, the maximum force can be enhanced without changing the maximum deflection.

If the bimorph is deflected in only one direction, the inactive piezo can be exchanged by a non-piezoelectric material. This opens up the door for optimizing deflection and force by varying Young's modulus and thickness of the inactive layer.

The deflection w_0 of a *piezo bonded onto an inactive carrier* consists of two terms describing the deflection due to the electric field w_E and due to external forces w_F similar as in the case of a bimorph (251) [49]:

$$w_0 = f_{w_E}\ w_{E,B} + f_{w_F}\ w_{F,B} =: w_E + w_F. \tag{255}$$

Here, $w_{E,B}$ and $w_{F,B}$ are the deflections of a bimorph due to the piezoelectric effect and outer forces as in (251), and f_{w_E} and f_{w_F} are correction factors which describe the influence of the carrier layer:

$$f_{w_E} = \frac{2}{(1+h_V) + \left((1-E_V\ h_V)^2/(4\ E_V(1+h_V))\right)}, \tag{256}$$

$$f_{w_F} = \frac{8(1+E_V)}{4\ E_V(1+h_V)^2 + (1-E_V\,h_V)^2}. \tag{257}$$

In the two equations above, the quantities h_V and E_V appear which are defined as ratios of the thickness of carrier h_T and piezo h_P and their Young's moduli E_T and E_P, respectively:

$$h_V = \frac{h_T}{h_P} \text{ and } E_V = \frac{h_T\ E_T}{h_P\ E_P}. \tag{258}$$

The maximum force F_0 generated by the piezo on an inactive carrier is obtained by solving (255) for the force (contained in $w_{F,B}$) and assuming zero deflection w_0:

$$F_0 = \frac{E_V(1+h_V)}{(1+E_V)}\frac{3}{4}d_{31}\frac{E_P\ b_P\ h_P}{L_P}U = f_{F_0}\ F_{0,B} \text{ with } f_{F_0} := \frac{E_V(1+h_V)}{(1+E_V)}. \tag{259}$$

The first fraction in the above equation is defined as the correction factor f_{F_o} which describes the effect of bonding the piezo to a carrier beam, while the rest of the equation is the maximum force $F_{0,B}$ of a bimorph as shown in (253).

The characteristic curve of a piezo on an inactive carrier is obtained by calculating the force F_P from (255) for the force and not assuming zero deflection:

$$F_P = f_{F_0}\ F_{0,B} - \frac{2\ E_P\ b_P\ h_P^3}{f_{w_F}\ L_P^3} w_0. \tag{260}$$

Now the question rises what is the optimum thickness and Young's modulus to obtain maximum deflection w_E and maximum force F_0, respectively. According to (255), the maximum deflection w_E is calculated as the product of the maximum deflection $w_{E,B}$ of the bimorph and the correction factor f_{w_E} which describes the effect of the carrier layer. Therefore, the maximum deflection is achieved with the parameters for which the correction factor is largest. Figure 106a shows the

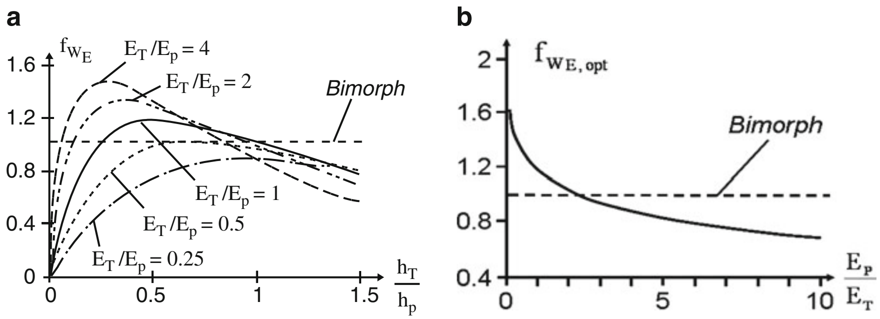


Fig. 106 Correction factor f_{w_E} of a piezo bonded to an inactive carrier calculated with (256) as a function of the ratio of the thicknesses of carrier and piezo (**a**), and value obtained with the thickness ratio optimum for large deflection (**b**)

correction factor f_{w_E} calculated with (256) as a function of the thickness ratio of carrier E_T and piezo E_P and the ratio of the Young's moduli. The dashed straight line corresponds to a bimorph. This shows that the bimorph is not optimum with respect to a large deflection. 20% more deflection can be achieved, if the thickness of the carrier is only half of the thickness of the piezo. The larger the ratio of the Young's moduli of carrier and piezo is, the larger the deflections can be obtained, if the thickness ratio of carrier and piezo is optimum.

The optimum thickness ratio is found by calculating the maximum of f_{w_E} as a function of this ratio. Thus, the derivative of f_{w_E} with respect to the thickness ratio is calculated, set to be zero, and solved for the thickness ratio. This calculation is hard to be done by hand but easily performed with the help of suitable computer codes:

$$\frac{\partial f_{w_E}}{\partial (h_T/h_P)} = 0 \Rightarrow \left(\frac{h_T}{h_P}\right)_{opt} = \frac{1}{2}\sqrt{\frac{E_P}{E_T}}. \tag{261}$$

If the above equation is inserted into (256), the maximum correction factor $f_{w_E,opt}$ can be calculated which is obtained with the optimum thickness ratio:

$$f_{w_E,opt} = 32\frac{2+\sqrt{E_P/E_T}}{32+41\sqrt{E_P/E_T}+8\ E_P/E_T}. \tag{262}$$

The maximum correction factor $f_{wE,opt}$ calculated with the above equation is shown in Fig. 106b and clearly shows that larger deflections can be obtained when the carrier is made of a material with a larger Young's modulus. However, it needs to be considered that a different material for the carrier results in a thermal expansion different from the one of the piezo. Therefore, the deflection will change with temperature. Employing a carrier with a larger Young's modulus is only an advantage, if the cross sensitivity to temperature changes does not perturb the desired performance.

If not the deflection but the force generated by a piezo on a carrier shall be maximum, the correction factor f_{F_0} needs to be maximized. Figure 107 shows f_{F_o} as a function of the ratios of thicknesses and Young's moduli of carrier and piezo. The larger the thickness and the Young's modulus of the carrier are, the larger the forces are generated. There are no optimum values. However, the possible deflection will vary also as a function of these parameters, and a piezo on a carrier is normally not only chosen to obtain the maximum force, but the maximum deflection.

A piezo *bimorph* may be equipped *with* an inactive *carrier between the piezos*. This arrangement is shown in Fig. 108. This also corresponds to two piezos glued onto each other to form a bimorph. The glue behaves as an inactive layer, and the following equations show what is the effect of the glue on deflection and force of such a bimorph with an inactive carrier between the piezos.

In the following, the calculations assume that only one piezo is powered. In most applications, however, it will be advantageous to move the beam up by driving the upper piezo and down by the other one. That way, larger deflections are achieved.

The deflection of a bimorph with an inactive carrier between the piezos is calculated with (255) with modified correction factors [49]:

$$f_{w_E} = \frac{(1 + h_V)}{1 + (3/2)h_V + (3/4 + (1/8)E_V)h_V^2} \tag{263}$$

$$f_{w_F} = \frac{1}{1 + (3/2)h_V + (3/4 + (1/8)E_V)h_V^2}. \tag{264}$$

The maximum force and force as a function of the deflection (characteristic curve) are calculated with (259) and (260) but with a different correction factor:

$$f_{F_0} = (1 + h_V). \tag{265}$$

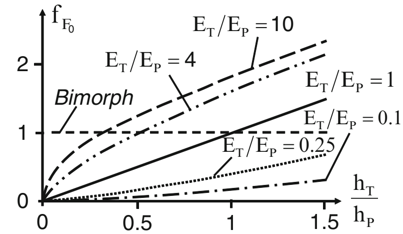


Fig. 107 Correction factor f_{F_o} for a piezo bonded on an inactive carrier calculated with (259)

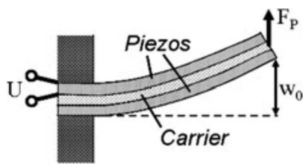


Fig. 108 Bimorph with an inactive carrier between two piezos

If the thickness and Young's modulus of the carrier shall be optimized to obtain either maximum deflection or maximum force, the correction factors according to (263) and (265) need to become maximum. These correction factors are drawn in Fig. 109 as a function of the ratios of the thicknesses and Young's moduli of carrier and piezos, respectively.

The thicker the carrier and the larger its Young's modulus are the smaller is the deflection. If the deflection shall be large, a carrier between the piezos is no advantage and the glue between the piezos of a bimorph should be as thin as possible and its Young's modulus should be as small as possible.

On the other hand, the maximum force F_0 generated by a bimorph with a carrier between the piezos is rising as a linear function of the thickness of the carrier but is not a function of its Young's modulus.

Piezos and carriers do not need to be beams with a rectangular cross-section. Deflections and forces of *piezos and carriers as circular plates* simply supported at their rim (cf. Fig. 110) are calculated also with (255) and (259), if maximum deflection $w_{E,B}$, deflection due to outer forces $w_{F,B}$, and maximum force $F_{0,B}$ of the bimorph are calculated with the following equations [49]:

$$w_{E,B} = \frac{3}{8} d_{31} \frac{R_P^2}{h_P^2} U, \tag{266}$$

$$w_{F,B} = 0.069 \frac{R_P^2}{E_P\ h_P^3} F, \tag{267}$$

$$F_{0,B} = 5.45\ d_{31}\ E_P\ h_P\ U. \tag{268}$$

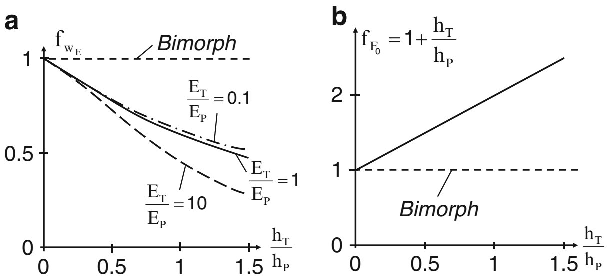


Fig. 109 Correction factors f_{w_E} and f_{F_0} for bimorphs with an inactive carrier in between as a function of the ratios of the thicknesses and Young's moduli of carrier and piezo

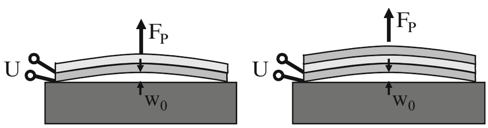


Fig. 110 Circular piezo plates simply supported at their rim

In the above equations, it is assumed that Poisson's ratio both of piezo and carrier layer are 0.3. If arrangements of piezos and carriers are used, the same correction factors f_{w_E}, f_{w_F}, and f_{F_o} are to be used as in (256), (257), and (259) for a piezo on a carrier or (263), (264), and (265) for two piezos with a carrier in between, respectively. This means also that the optimum ratios of thicknesses and Young's moduli of piezo and carrier are the same as described above for rectangular beams.

The characteristic curves of all piezos and arrangements of piezos with inactive carriers employed as actuators are strait lines defined by their maximum force F_0 and maximum deflection w_E as shown in Fig. 111a. These parameters are listed in Table 13 and the required correction factors are found in Table 14. Thus, the characteristic curve of any arrangement of piezos discussed here is described by the following equation and can be calculated with the help of the two tables:

$$F_P = F_0\left(1 - \frac{w_0}{w_E}\right). \qquad (269)$$

If the force which has to be overcome is not a function of the deflection of the piezo arrangement, e.g., a weight is to be lifted; the mechanical energy W_A generated by the piezo is the product of force and deflection:

$$W_A = F_P\ w_0 = F_0\ w_0\left(1 - \frac{w_0}{w_E}\right). \qquad (270)$$

This means that for the maximum force no work is done, because the deflection is zero, and for no force acting on the piezo also no work is done, although the deflection is maximum. Figure 111b shows the work W_A (= energy output) as a function of the deflection w_0. The maximum work $W_{A,opt}$ can be calculated from the maximum of the parabola described by (270):

$$W_{A,opt} = \frac{1}{4} F_0\ w_E. \qquad (271)$$

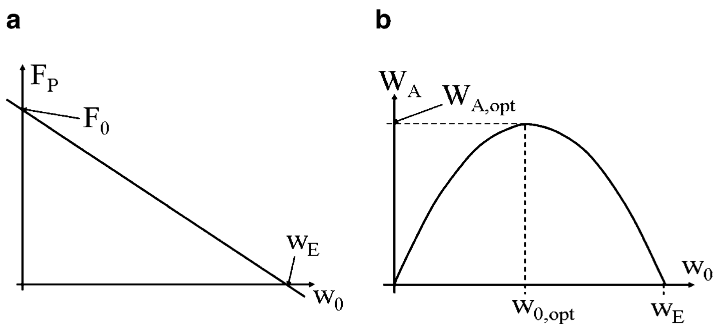


Fig. 111 (**a**) Characteristic curve of a piezo or an arrangement of piezos with a carrier and (**b**) work done by the piezo

The optimum deflection $w_{0,opt}$, where the work is maximum, is found at half of the maximum deflection w_E. Half of the maximum deflection corresponds also to half of the maximum force, because the characteristic curve in Fig. 111a is a strait line. The energy input is approximately the same independent of the deflection; it is the electrical energy W_C necessary to charge the capacity of the piezo [cf. (221) on page 133]. To obtain the exact energy input, the work done by the actuator would need to be added, but the energy required for charging is at least ten times larger.

So, if a piezo is designed for a certain application and the energy consumption is an issue, e.g., in battery powered devices, the piezo should be dimensioned such that it is able to achieve the double of the desired deflection and force. Both a smaller and a larger piezo will result in a small efficiency.

The efficiency η_A is calculated from the ratio of the energy output $W_{A,opt}$ and the energy input W_C. It is shown for every arrangement of piezos in Table 13. The maximum efficiency is on the order of 10%. Sometimes people say that the efficiency of piezos would be approximately 95%, but this is only true when the electrical energy stored during deflection of the piezo is recovered when the deflection is reduced again. Normally, this is not done and the 95% efficiency is only a theoretical value, while in practical applications less than 10% are achieved.

Thickness and Young's modulus of carriers can also be optimized for maximum efficiency. As seen in Table 13, the product of the correction factors f_{w_E} and f_{F_o} needs to be optimized to achieve this. This product is shown in Fig. 112 both for a piezo on a carrier and a bimorph with a carrier between the piezos as a function of the ratios of the thicknesses and the Young's moduli of piezo and carrier.

For a piezo on a carrier, the product is:

$$f_{w_E}\, f_{F_0} = \frac{2\, E_V}{\left(1 + (1 - E_V\, h_V)^2 \big/ (4\, E_V (1 + h_V)^2)\right)(1 + E_V)}. \tag{272}$$

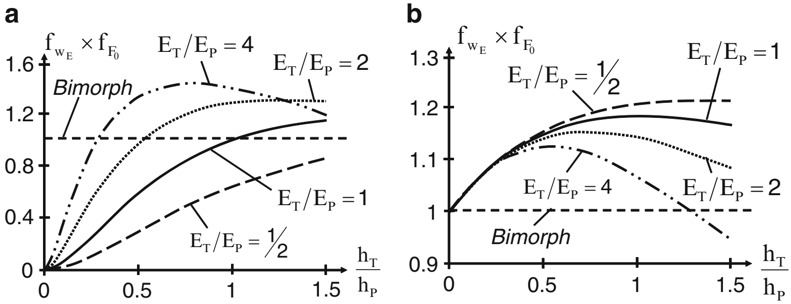


Fig. 112 Product of the correction factors f_{w_E} and f_{F_0} which need to be optimized to obtain maximum efficiency. (**a**) Piezo with a carrier, (**b**) bimorph with a carrier between the piezos

For a bimorph with a carrier between the piezos, the product is:

$$f_{w_E} f_{F_0} = \frac{(1 + h_V)^2}{1 + (3/2)h_V + \left(3/4 + (1/8)\,E_v\right)h_V^2.} \tag{273}$$

Figure 112a shows that for a piezo on a carrier, the energy output and efficiency can be largest when Young's modulus of the carrier is maximum. A bimorph is not optimum with respect to efficiency; 20% more energy output could be achieved with the same input, if the inactive piezo would be two times thicker.

A bimorph with a carrier between the piezos achieves a higher efficiency when the carrier is made of a material with a small Young's modulus and adapted thickness. The optimum thickness ratios is found by calculating the derivative of (273), setting it equal to zero, and solving for the thickness ratio. This calculation is very lengthy and yields a very complex result, but it can easily be obtained with suitable computer codes:

$$\left(\frac{h_T}{h_P}\right)_{opt} = \sqrt[3]{\frac{2 - E_T/E_P + 2\sqrt[2]{1 - E_T/E_P}}{E_T/E_P}} + \sqrt[3]{\frac{E_T/E_P}{2 - E_T/E_P + 2\sqrt[2]{1 - E_T/E_P}}} - 1. \tag{274}$$

The above equation is not easy to understand but when drawn in a graph shows its simple nature (cf. Fig. 113).

If the largest possible deflection is to be achieved, a piezo on a carrier is the best possible arrangement. The deflection can be enhanced by choosing a carrier with a large Young's modulus. However, it needs to be considered that the deflection will be a function of temperature also, if the carrier is not a piezo (see "Thermal Actuators"). An alternative for obtaining a larger deflection with a suitable carrier layer is often to employ a larger bimorph. The thickness of the carrier should be close to the ratio given in (261) on page 147.

If the largest possible force is to be achieved, the best option is a piezo without any carrier pushing in polarization direction.

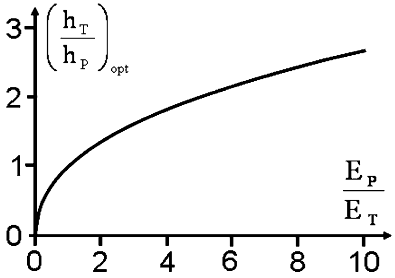


Fig. 113 Optimum thickness ratio of carrier and piezo for maximum efficiency of a bimorph with a carrier between the piezos

Energy output and efficiency are largest for a piezo without any carrier layer. However, if at a considerably large deflection energy output and efficiency are to be maximized, a bimorph with an inactive carrier between the piezos often will be the best solution. Young's modulus of the carrier should be as small as possible and its thickness should be designed according to (274). Besides the fact that the piezoelectric moduli are a function of temperature (see below), deflection and force of this arrangement will not be sensitive to temperature changes, because it is symmetrical with respect to the neutral fiber.

The maximum energy output density of a piezo actuator is calculated from (271) together with (236) (page 140) and (231) (page 139):

$$\frac{W_{A,opt}}{V} = \frac{1}{4} d_{33}^2 \ E_P \frac{U^2}{h_P^2}. \tag{275}$$

If for maximum voltage, thickness, Young's modulus, and piezoelectric modulus 400 V, 0.5 mm, 60 GPa, and 250×10^{-12} V/m are assumed, respectively, 0.6 μJ/μL is obtained for the maximum output energy density.

The piezoelectric moduli, e.g., d_{33} and d_{31}, are a function of temperature (cf. Fig. 114). As a consequence, both sensors and actuators are cross sensitive to temperature changes, and this fact needs to be taken into account, when devices based of the piezoelectric effect are designed.

Besides the fact that the piezoelectric moduli are a function of temperature, there is the *pyroelectric effect*: When the temperature changes, electrical charges are generated on the electrodes of a piezo, and these charges cause deflection of the piezo. The charge change ΔQ_3 generated on the electrodes in polarization direction by a temperature change ΔT is calculated with the following equation:

$$\Delta Q_3 = A_P \ p_3 \Delta T. \tag{276}$$

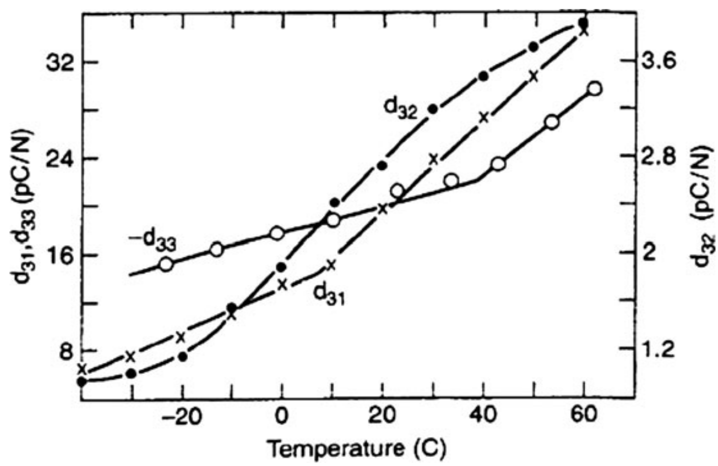


Fig. 114 Piezoelectric moduli of PVDF as a function of temperature [50]

In the above equation, p_3 is the pyroelectric coefficient in the direction of polarization which is a material property of the piezo. The charge change ΔQ_3 corresponds to a voltage change ΔU on the capacitor according to the definition of the electric capacity C_{el}:

$$\Delta Q_3 = C_{el}\ \Delta U = \varepsilon_0\ \varepsilon_r \frac{A_P}{h_P} \Delta U = A_P\ p_3 \Delta T \Rightarrow \Delta U = \frac{h_P\ p_3}{\varepsilon_0\ \varepsilon_r} \Delta T. \tag{277}$$

In principle, the pyroelectric effect could be employed to measure temperature changes or to actuate a piezo. However, the slow discharging of piezos is a disadvantage for sensor applications and there are a lot of alternatives for temperature measurement. The actuation of a piezo by a voltage is much quicker than by the pyroelectric effect. Therefore, the pyroelectric effect mostly causes cross-sensitivity and is not employed in any way.

For every case in Table 13, the deflection is calculated as $w_0 = w_E + w_F$. The correction factors describing the effect of the arrangement of the piezos are given in Table 14.

Exercises

Problem 21

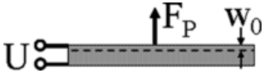
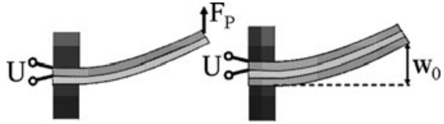
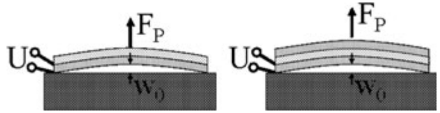
Your chief tells you to design a microvalve which employs piezo actuators. It shall be a two-way valve with an orifice which is closed by a piezo beam from two combined layers from PZT (cf. Fig. E20). This way, temperature changes do not affect switching.

Only the upper piezo layer shall be driven, while the lower one is short-circuited. This way, it is possible to apply the whole available voltage of up to 200 V and no piezo gets depolarized. The diameter of the orifice is 200 µm. The piezo material is commercially available with a width of 500 µm. The thickness of each of the two layers is 500 µm.

The distance between piezo beam and valve seat of an open valve needs to be a fourth of the diameter of the inlet and outlet to avoid that the flow resistance is a function of the gap between the beam and the valve seat.

(a) What length do you choose for the piezo beam to allow a sufficient opening of the valve?
(b) Up to what pressure difference can the valve be opened, if the piezo beam is designed with a length of 30 mm?
(c) What would be the maximum pressure difference against which the valve could work, if the active piezo would not be combined with an inactive one, but would press against the valve seat as shown in Fig. E21?
(d) Do you think that the design of Fig. E21 is feasible?

Table 13 Equations for the calculation of deflections, forces, work outputs, and efficiencies of piezos

	Parallel to polarization	Perpendicular to polarization	Rectangular piezos with carrier	Circular plate with carrier
				
w_E	$d_{33}\ U$	$d_{31}\dfrac{L_P}{h_P}U$	$f_{w_E}\dfrac{3}{8}d_{31}\dfrac{L_P^2}{h_P^2}U$	$f_{w_E}\dfrac{3}{8}d_{31}\dfrac{R_P^2}{h_P^2}U$
w_F	$\dfrac{h_P}{E_P\ A_P}F$	$\dfrac{L_P}{E_P\ b_P\ h_P}F$	$f_{w_F}\dfrac{L_P^3}{2\ E_P\ b_P\ h_P^3}F$	$f_{w_F}\ 0.069\dfrac{R_P^2}{E_P\ h_P^3}F$
F_0	$d_{33}\ \dfrac{E_P\,A_P}{h_P}U$	$d_{31}\ E_P\ b_P\ U$	$f_{F_0}\dfrac{3}{4}d_{31}\dfrac{E_P\ b_P\ h_P}{L_P}U$	$f_{F_0}\ 5.45\ d_{31}\ E_P\ h_P\,U$
$W_{A,opt}$	$\dfrac{A_P\ E_P}{4\ h_P}U^2d_{33}^2$	$\dfrac{A_P\ E_P}{4\ h_P}U^2\ d_{31}^2$	$f_{w_E}\ f_{F_0}\dfrac{9}{32}\dfrac{A_P\ E_P}{4\ h_P}U^2\ d_{31}^2$	$f_{w_E}\ f_{F_0}\ 2.04\dfrac{R_P^2\ E_P}{4\ h_P}d_{31}^2$
η_A	$\dfrac{E_P\ d_{33}^2}{2\ \varepsilon_0\ \varepsilon_r}$	$\dfrac{E_P\ d_{31}^2}{2\ \varepsilon_0\ \varepsilon_r}$	$f_{w_E}\ f_{F_0}\dfrac{9}{64}\dfrac{E_P\ d_{31}^2}{\varepsilon_0\ \varepsilon_r}$	$f_{w_E}\ f_{F_0}\ 1.02\dfrac{E_P\ d_{31}^2}{\varepsilon_0\ \varepsilon_r}$

For the circular plates, a Poisson's ratio of 0.3 is assumed both for piezo and carrier

Here $h_V=\dfrac{h_T}{h_P}$ and $E_V=\dfrac{h_T\ E_T}{h_P\ E_P}$.

Table 14 Correction factors describing the effect of the arrangement of the piezos in Table 13

	Bimorph	Piezo with carrier	Two piezos on a carrier
f_{w_E}	1	$\frac{2}{(1+h_V)+(1-E_V\ h_V)^2/(4\ E_V(1+h_V))}$	$\frac{(1+h_V)}{1+(3/2)h_V+(3/4+(1/8)E_V)h_V^2}$
f_{w_F}	1	$\frac{8(1+E_V)}{4\ E_V(1+h_V)^2+(1-E_V\ h_V)^2}$	$\frac{1}{1+(3/2)h_V+(3/4+(1/8)E_V)h_V^2}$
f_{F_0}	1	$\frac{E_V(1+h_V)}{(1+E_V)}$	$(1+h_V)$

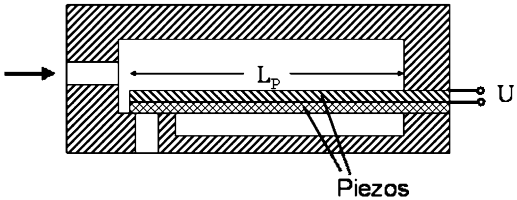


Fig. E20 Schematic view of the two-way valve

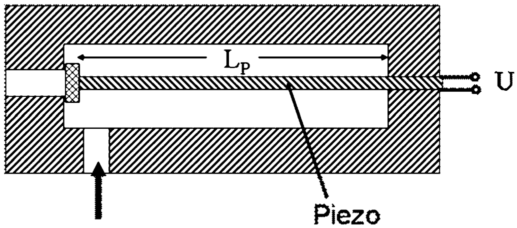


Fig. E21 Cross-section of the alternative two-way valve

Piezoelectric modulus d_{31} of PZT	-170×10^{-12} C/N
Piezoelectric modulus d_{33} of PZT	250×10^{-12} C/N
Young's modulus of PZT	60 GPa
Relative permittivity of PZT	2,400

Problem 22

The 30-mm long beam of Fig. E21 shall now be used as a sensor.

(a) What voltage can be measured when the beam is compressed with a force of 13 mN?

(b) How much is the beam compressed by the load of 13 mN if the electrodes are short-circuited?

(c) How much is the beam compressed by the load of 13 mN if the electrodes are not short-circuited but insulated to the environment? Take into account that a charge is generated by the force which diminishes the compression.

(d) How much are the combined piezos of Fig. E20 deflected if the force of 13 mN is acting at their free end perpendicular to the direction of the beam and only one of the piezos is short-circuited and the other one has electrodes insulated to the environment?

Problem 23

A piezo layer from PZT is bonded onto a circular silicon plate. The thickness of the piezo layer and the silicon plate are 0.5 and 1 mm, respectively, and the radius of both is 10 mm. The applied voltage is 200 V.

(a) Calculate the maximum force and the maximum deflection of this arrangement. What is the maximum energy output?
(b) Calculate the electrical energy which is needed to deflect this actuator. Calculate the ratio of energy output to energy input and, thus, calculate the efficiency. Hint: The piezo actuator forms a capacitor together with the electrodes. The energy stored in the capacitor is calculated with (221) on page 131.
(c) Calculate the efficiency also for the case that the piezo is not combined with the silicon carrier and is working only by extension in the direction of polarization.
(d) Calculate the efficiency for the case that the lower side of the silicon is bonded to another piezo layer of the same size as on the upper side. There is always voltage applied to only one piezo layer.
(e) What is your conclusion, when you compare the results of (b)–(d)?

Young's modulus of PZT	60 GPa	Young's modulus of silicon	190 GPa
d_{31} of the PZT layer	-170×10^{-12} C/N	Relative permittivity of PZT	2,400
d_{33} of the PZT layer	250×10^{-12} C/N		