Design and Analysis of Hybrid Systems, with Applications to Robotic Aerial Vehicles

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Abstract. Decomposing complex, highly nonlinear systems into aggregates of simpler hybrid modes has proven to be a very successful way of designing and controlling autonomous vehicles. Examples include the use of motion primitives for robotic motion planning and equivalently the use of discrete maneuvers for aggressive aircraft trajectory planning. In all of these approaches, it is extremely important to verify that transitions between modes are safe. In this paper, we present the use of a Hamilton-Jacobi differential game formulation for finding continuous reachable sets as a method of generating provably safe transitions through a sequence of modes for a quadrotor performing a backflip maneuver.

1 Introduction

As robotic and automated systems become more complex, it has become increasingly difficult to design and analyze these systems, and it is especially hard to provide provable guarantees on safety and performance. One successful approach is to break down complex nonlinear systems into a hybrid collection of discrete modes, with different continuous dynamics for each mode. This decomposition can greatly

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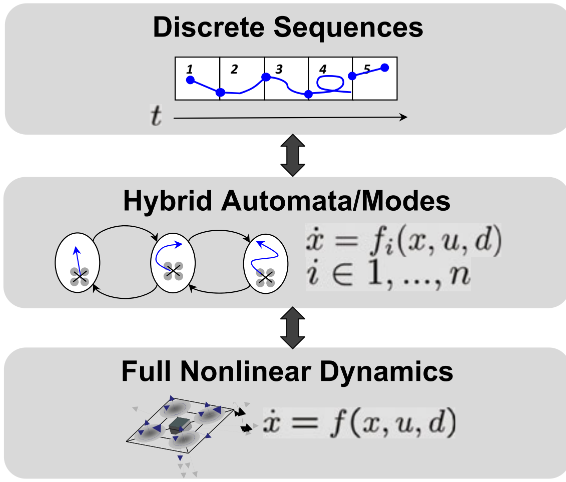

Fig. 1 Hybrid hierarchical representation of maneuvers for a quadrotor helicopter

simplify the analysis of the behavior of the overall system, and planning and control is also simplified by the ability to generate plans at the level of the discrete modes (see Figure 1). This hybrid, hierarchical approach to the design and control of autonomous systems has proven to be very powerful. Successful examples of this approach include aerobatic maneuver design (Frazzoli et al, 2005), linear-temporal logic specifications for generating robot behaviors (Kress-Gazit et al, 2008), and the use of motion primitives for robotic manipulator motion planning in complex dynamical tasks (Burridge et al, 1999).

A key consideration in the use of hybrid modes is the question of verifying that in transitioning between modes safety or performance criteria are met. For example, if constructing a sequence of maneuvers for an aircraft, it is necessary to know if one maneuver can be safely followed by another maneuver. In other words, an allowable grammar over the discrete modes must be constructed. Previous work has addressed this problem in a variety of ways. In the maneuver sequencing for helicopters, steady state ”trim states” were designated, with all maneuvers starting and ending in a trim state (e.g. level flight). Thus a maneuver that had a given end state could be followed by another maneuver with the same trim state as its initial condition (Frazzoli et al, 2005). The work on motion primitives proceeded similarly, with Lyapunov functions designated for each mode that guaranteed that the output of one action would be within the stable capture region of the subsequent action (Burridge et al, 1999).

In much of the existing literature, the particular methods to ensure continuity between modes have been specific to the application at hand. Moreover, in many cases, while ensuring feasibility of transitions with respect to the dynamics, the methodologies may require separate external mechanisms to meet safety criteria, such as avoiding obstacles. Ding, et. al. demonstrated the use of reachable sets in UAV refueling as a method for ensuring both safety relative to another aircraft and guaranteed arrival at a target state (Ding et al, 2008). In this work, we propose the use of reachable sets as a mechanism for combining both dynamic feasibility of switching and simultaneously the imposition of verifiable safety constraints on system trajectories for a quadrotor helicopter performing an aerobatic maneuver. We demonstrate the use of the Hamilton-Jacobi differential game formulation of reachable sets (Mitchell et al, 2005) to construct maneuvers that safely transition through a sequence of modes for a backflip maneuver, arriving at a target state while avoiding unsafe states en route.

The organization of this paper is as follows. Section 2 provides background on the theory of the Hamilton-Jacobi game formulation for generating reachable sets. Section 3 describes the dynamics of the quadrotor helicopter considered, and the details of the analysis of the backflip maneuver are described in Section 4. Finally, simulation results for the flip maneuver are presented in Section 5, with conclusions and future work in Section 6.

2 Backwards Reachable Sets

The backwards reachable sets in this work are generated according to the Hamilton-Jacobi game formulation as described in Mitchell et al (Mitchell et al, 2005). Two types of reachable sets are used: avoid sets, which are reachable sets generated to avoid undesired states, and capture sets, which are defined in order to reach certain desired states. The formulation will be summarized in the discussion of avoid sets, with the description of capture sets highlighting the differences.

2.1 Avoid Sets

The backwards reachable set $G(t)$ is defined to be the set of all states x such that, for any inputs u, the disturbance d can drive the system into a set G_0 in time t. The system dynamics are defined by

$$\dot{x} = f(x, u, d) \tag{1}$$

where x is the system state, u is the control input, and d is a disturbance input, where u and d are assumed to be constrained in some sets U and D, respectively. As detailed in (Mitchell et al, 2005), the boundary of the reachable set is defined by the solution to the modified Hamilton-Jacobi-Isaacs equation

$$-\frac{\partial J(x,t)}{\partial t} = \min\{0, \max_u \min_d \frac{\partial J(x,t)}{\partial x} f(x,u,d)\} \quad (2)$$

where the level set $J(x,0) = 0$ defines the initial undesired set G_0.

2.2 Capture Sets

The same principle behind the avoid set can also be used to reverse the role of the control and disturbance to generate capture sets. Given a desired target state region, the backwards reachable set can be calculated with the control input attempting to drive the state into the desired state region and the disturbance attempting to keep the state out. The capture set so generated is the set of all states such that for any possible action that the disturbance might take, the input will drive the state to the desired region in some time t. The formulation for the capture set is identical to that for the avoid set, except for reversing the roles of the input and disturbance. The conditions so derived are then

$$-\frac{\partial J(x,t)}{\partial t} = \min\{0, \min_u \max_d \frac{\partial J(x,t)}{\partial x} f(x,u,d)\} \quad (3)$$

It should be noted that this problem can be simplified even further if one has a desired control law $u(x)$; in this case the capture set is given by

$$-\frac{\partial J(x,t)}{\partial t} = \min\{0, \max_d \frac{\partial J(x,t)}{\partial x} f(x,u(x),d)\} \quad (4)$$

and is simply the set of all states such that for any possible disturbance, the given control law will drive the state to the desired region in some time t.

2.3 Maneuver Sequencing with Reachable Sets

Capture and avoid sets can be used to construct safe sequences of maneuvers. Starting with the final (target) set, the dynamics for the final (n^{th}) maneuver can be run backwards to generate a capture set for that maneuver. Then a target region can be selected within the capture set of the final maneuver as the target set for the previous ($n-1^{st}$) maneuver. Thus an initial condition within the capture set of the $n-1^{st}$ maneuver is guaranteed to arrive within the capture set of the n^{th} maneuver, allowing a safe switch into the n^{th} maneuver and eventual safe arrival at the final target set (see Figure 2). This process can be repeated for any number of desired maneuvers to identify a start region for the entire sequence. Safety considerations such as avoiding a particular unsafe set can be encoded either by choosing capture sets that avoid the unsafe regions of particular reach sets, or by generating reach-avoid sets that reach target sets while avoiding the unsafe sets.

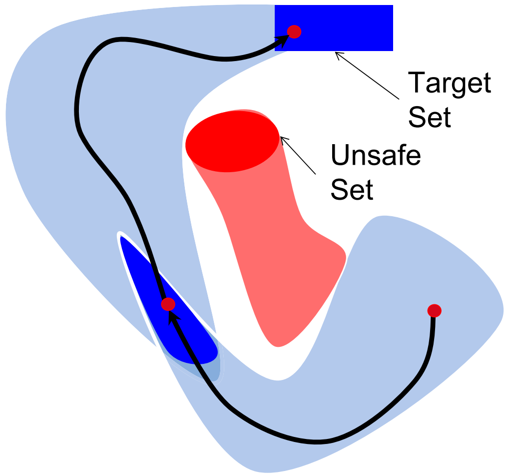

Fig. 2 Capture and avoid sets for sequencing two modes/maneuvers

3 Planar Quadrotor Dynamics

To simplify the problem the quadrotor's dynamics were modeled in a plane (as opposed to $\mathbb{R}^3$). It is assumed that the vehicle's out of plane dynamics can be stabilized, which we believe is a valid assumption. The resulting dynamics (based on the original dynamics in (Hoffmann et al, 2007)) are:

$$\frac{\partial}{\partial t}\begin{bmatrix} x \\ \dot{x} \\ y \\ \dot{y} \\ \theta \\ \dot{\theta} \end{bmatrix} = \begin{bmatrix} \dot{x} \\ -\frac{1}{m}C_D^v\dot{x} \\ \dot{y} \\ -\frac{1}{m}\left(mg + C_D^v\dot{y}\right) \\ \dot{\theta} \\ -\frac{1}{Iyy}C_D^\theta\dot{\theta} \end{bmatrix} + \begin{bmatrix} 0 \\ D_x \\ 0 \\ D_y \\ 0 \\ D_\theta \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ -\frac{1}{m}\sin\theta & -\frac{1}{m}\sin\theta \\ 0 & 0 \\ \frac{1}{m}\cos\theta & \frac{1}{m}\cos\theta \\ 0 & 0 \\ -\frac{l}{Iyy} & \frac{l}{Iyy} \end{bmatrix}\begin{bmatrix} T_1 \\ T_2 \end{bmatrix} \tag{5}$$

where m is the vehicle's mass, g is gravity, C_D^v is the linear drag constant, C_D^θ is the rotational drag constant, D_x, D_y and D_θ are disturbances, I_{yy} is the moment of inertia, and all other variables are as depicted in figure 3. Several assumptions went into this formulation of the dynamics, including that the vehicle undergoes linear

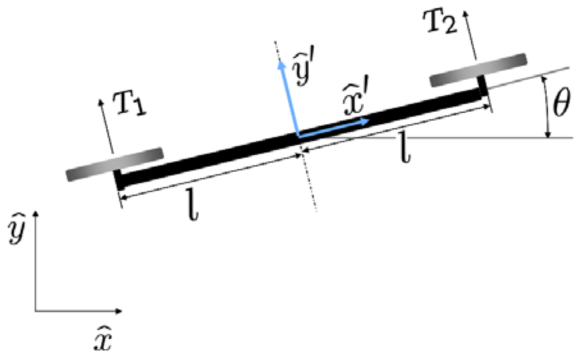

Fig. 3 The quadrotor's two-dimensional dynamics.

drag (as opposed to drag proportional to velocity), and that the thrust from each motor saturates at some value T_{max}.

4 Backflip

A diagram of the backflip maneuver is pictured in figure 4. The maneuver was broken down into three modes: impulse, drift, and recovery. (This break down was due to the fact that for at least part of the flip, the vehicle's motors must be turned off in order to prevent the vehicle from propelling itself into the ground.) In the impulse mode, a moment is applied to the vehicle that initiates the backflip. In the drift mode, the vehicle's motors are turned off and the vehicle completes the flip. Finally, in the recovery mode, the motors are turned back on and the vehicle is returned to a stable state (from which other maneuvers could potentially be initiated).

Of course, sequencing the maneuver in a manner that is guaranteed to be safe is a difficult problem: doing so requires hitting some target sets (e.g. a target set that ensures the vehicle is upside down) while avoiding some unsafe sets (e.g. the unsafe set consisting of states below $y = 0$). This problem is compounded by the fact that the quadrotor system dynamics are six-dimensional, and existing computational methods for computing reachable sets are only tractable for systems with

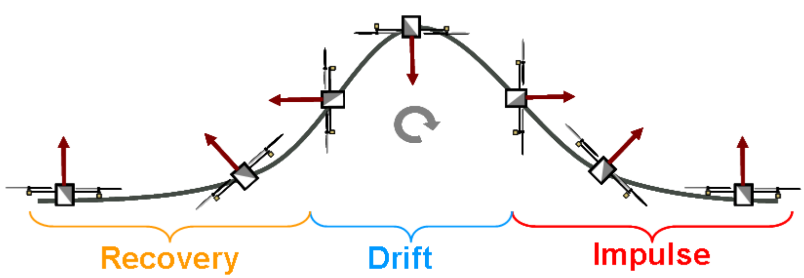

Fig. 4 The backflip maneuver, broken down into three modes.

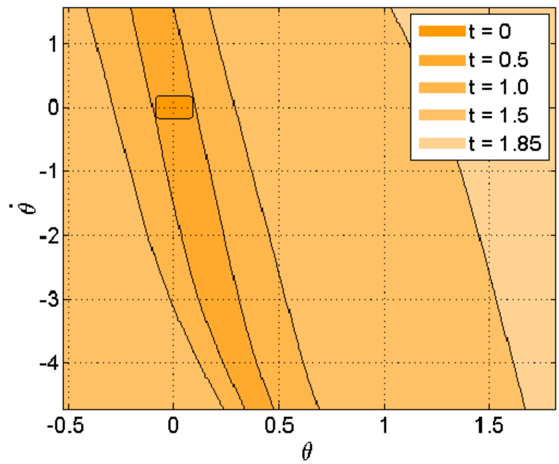

Fig. 5 The recovery mode target set and capture basin.

dimensions of four or less. To get past this problem, the system's states were broken apart apart into three sets and analyzed separately. The rotational dynamics (θ and $\dot{\theta}$) were analyzed to ensure that the vehicle achieved a flip; the vertical dynamics (y and $\dot{y}$) were analyzed to ensure the vehicle remained above some minimum altitude; and the horizontal dynamics (x and $\dot{x}$) were ignored to keep the problem as simple as possible.

4.1 Attainability

The general method for calculating the maneuver was as described in section 2. In particular, the target for the final state of the recovery mode was chosen to be $\theta = 0 \pm 5°$, $\dot{\theta} = 0 \pm 20°$, so as to have the vehicle end in a fairly level configuration with very little rotational velocity. Additionally, as mentioned in section 2, a fixed control law was chosen to drive the vehicle to this target set; in this case, a standard PD controller of the form $u = k_p\theta + k_d\dot{\theta}$ was used.[1] This target set was then propagated backwards using the reachable set toolbox in MATLAB, taking into account the worst-case disturbances (due to motor noise and wind). The resulting level sets represented the capture set for this maneuver, as pictured in figure 5.

For the drift mode, a similar procedure was followed. The target set was chosen as $\theta = 90 \pm 5°$, $\dot{\theta} = -138 \pm 20°$, and was propagated back (this time with no control input, and thus reduced worst-case disturbances due to the lack of motor noise) to produce the capture set for the drift mode (figure 6).

[1] It should be noted that the actual commanded thrust was of the form $T_1 = T_{nom} - u$, $T_2 = T_{nom} + u$, where T_{nom} was the nominal total thrust necessary to counteract gravity, and u was as given above.

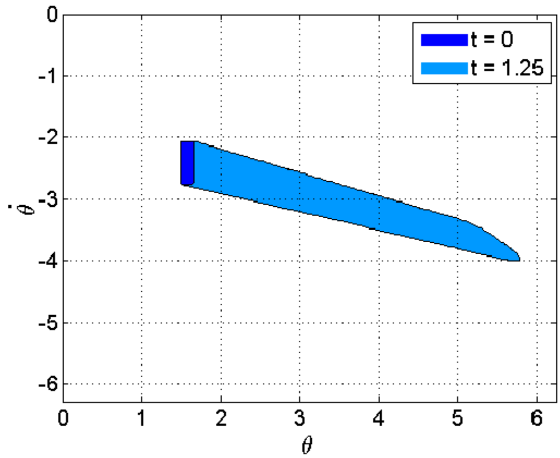

Fig. 6 The drift mode target set and capture basin.

Finally, for the impulse mode, the target set was $\theta = 300 \pm 5°$, $\dot{\theta} = -210 \pm 10°$. Once again, a fixed controller (of the form $u = k_p\theta + k_d\dot{\theta} + k_c$) was used, and the worst-case disturbances were chosen so as to account for motor noise and wind. The resulting capture set is pictured in figure 7.

4.2 Safety

To ensure safety, an initial unsafe set of $y < 0$ was chosen. Because the vehicle's vertical dynamics are coupled to its rotational dynamics when thrust is applied (see

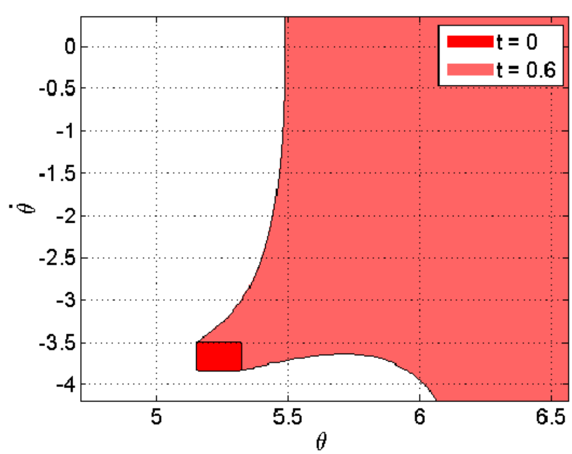

Fig. 7 The impulse mode target set and capture basin.

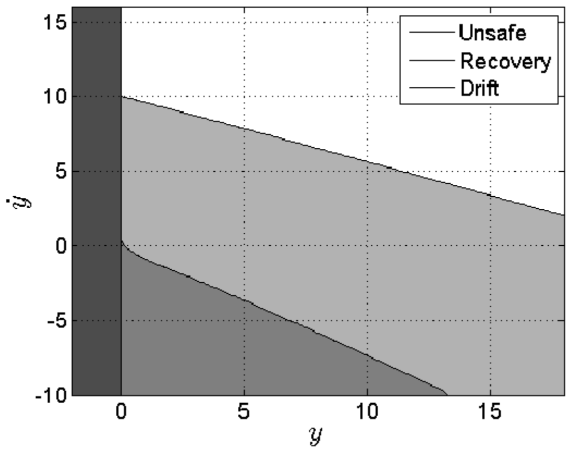

Fig. 8 The unsafe reachable sets in the vertical dynamics.

equation 5), the interaction between the two systems could not be ignored when trying to calculate the unsafe reachable set for this mode. However, because the recovery mode was designed with a fixed control law, a nominal trajectory in the θ, $\dot{\theta}$ space could be created as a function of y and $\dot{y}$. The unsafe set could then be calculated by plugging this nominal trajectory into the system dynamics, and proceeding as usual. The set was propagated backward for a fixed time T, based on the maximum time that the rotational part of the recovery mode could take.

In the drift mode, things are less complicated; the vehicle's dynamics decompose into three separate two-dimensional systems, given below.

$$\frac{\partial}{\partial t}\begin{bmatrix} x \\ \dot{x} \\ y \\ \dot{y} \\ \theta \\ \dot{\theta} \end{bmatrix} = \begin{bmatrix} \dot{x} \\ -\frac{1}{m}C_D^v\dot{x} \\ \dot{y} \\ -\frac{1}{m}\left(mg + C_D^v\dot{y}\right) \\ \dot{\theta} \\ -\frac{1}{I_{yy}}C_D^\theta\dot{\theta} \end{bmatrix} + \begin{bmatrix} 0 \\ D_x \\ 0 \\ D_y \\ 0 \\ D_\theta \end{bmatrix} \tag{6}$$

Thus, it was easy to simply propagate the unsafe set from the recovery mode backwards using the y, $\dot{y}$ dynamics. Again, this was done for a fixed time based on the maximum length of the maneuver as calculated from the rotational dynamics. Finally, it was assumed that there would be no loss in altitude during the impulse mode because of the way the modes and their switching criteria were designed (i.e. the vehicle would never have a negative vertical velocity during the impulse mode). The resulting unsafe sets are pictured in figure 8; as long as the vehicle began each mode outside the unsafe set for that mode, the overall safety of the system was guaranteed.

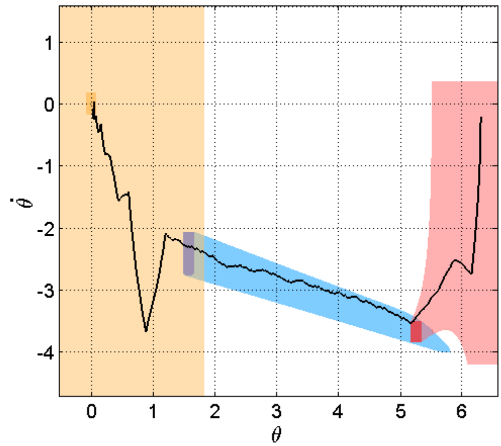

Fig. 9 A trajectory of the vehicle in the rotational state space. The yellow region is the capture basin and target for the recovery mode, blue is for the drift mode, and red is for the impulse mode.

5 Results

The combined results of the reachable set computations for the rotational state space are pictured in figure 9. Additionally, a sample trajectory in the rotational state space is overlaid, indicating that the vehicle does indeed remain inside each of the given capture basins as it completes the maneuver.

Figure 10 shows a time-lapse image of a resulting simulated trajectory. In this simulation, the transition between the different modes was triggered whenever both of the rotational states satisfied the conditions of the target set for the given mode.

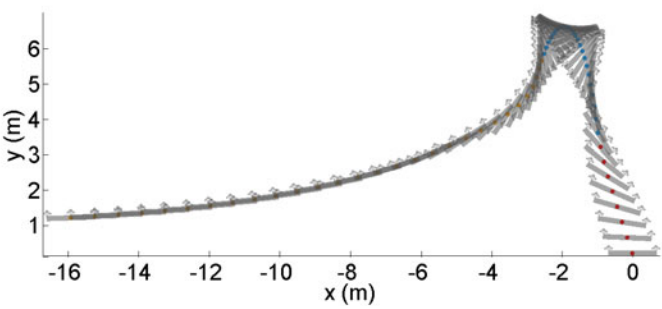

Fig. 10 A time-lapse image of a simulated trajectory.

6 Conclusions and Future Work

While the method we have proposed for using reachable sets to generate provably safe transitions between different modes shows great promise, several open questions remain. First, in future work we hope to explore how to parametrically describe reachable sets. In particular, it is apparent that in the current framework the resulting reachable set (whether it be for safety or attainability) depends a great deal on the parameters used when generating it. For example, the reachable set for a flip that ends with a slow rotational velocity would look very different from one that ends with a high rotational velocity. As a result, the reachable sets for each version of the same maneuver must be calculated offline using predefined parameters. Instead, it would be preferable if a reach set could be calculated and represented in such a way that the effect of a simple change in parameters could be quickly computed, resulting in the ability to choose between different versions of the same maneuver in an online manner.

Of course, the most immediate goal which we intend to accomplish is the implementation of the work described in this paper on an actual quadrotor vehicle. While this goal will likely entail a sizable amount of engineering work, we believe that due to the robustness of the theory we have developed, doing so is well within the realm of possibility.

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