Chapter 4
Complexity Theories of Cities (CTC)

4.1 Introduction

4.1.1 Bénard Cells

Henri Bénard, a French physicist working at the beginning of the 20th century, found the following about a liquid in a round vessel heated from below: At the beginning of the process, when the temperature difference between the heated bottom and the cool top is low, the heat is being transferred by conduction and no macro-motion can be observed in the liquid. However, as the temperature difference increases and a certain threshold is reached, the movement in the liquid becomes instable, chaotic and then a strikingly ordered pattern appears: The molecules of the liquid which at the beginning were moving in random, suddenly exhibit a coherent macro-movement in roles which are millions of times larger than the molecules. As can be seen in Fig. 4.1, the motion of the roles forms a hexagonal pattern on the surface of the liquid. This pattern is in fact an outcome of the movement of the hot liquid, which rises through the center of the honeycomb cells, and of the cooler liquid, which falls along their walls. All this happens as if by an external force. Yet no such force exists – the spatial order appears spontaneously, by means of self-organization.

In the early 1900s this experiment was registered as just another interesting example of *convection* – a process in fluid dynamics referring to the flow of heat, e.g., molecules, from the hot to the cold region of the liquid. In the 1960s it became one of the canonical experiments of several new theories about systems that are *open* in the sense that they exchange matter and information with their environment and *complex* in the sense that their parts are numerous and form a complex network with feed-forward and feedback loops. Such system are never in rest – they are far from equilibrium and exhibit phenomena of chaos, bifurcation, phase transition, fractal structure and the like. In the 1960s it was common to refer to these theories as theories of *self-organization*. In the last decade, it has become more common to refer to this body of studies as *complexity theory* or more precisely *complexity theories*.

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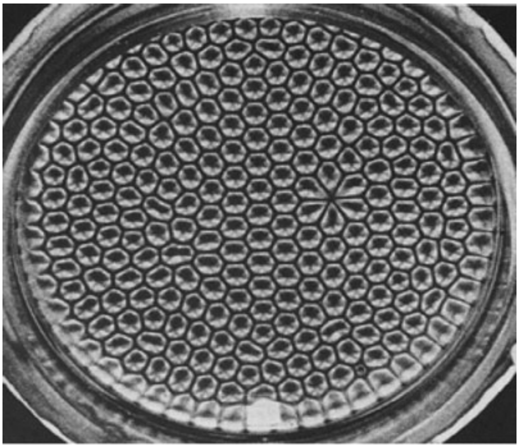

Fig. 4.1 Top view of a liquid in a circular vessel (from Haken 1996). When the liquid is heated from below and the temperature gradient exceeds a critical value, hexagonal cells are formed. In the middle of each cell the liquid rises, sinking back down at the edges of the hexagon

4.1.2 A Concise Introduction to Self-Organization and Complexity

The notion of *self-organization* appeared already in the early years of cybernetics, implicitly in the writings of McCulloch and Pitts (1943), and explicitly in studies such as Ashby's (1947) psychological discussion of the nervous system, and Yovits and Cameron's (1959) and Forester and Zopf's (1962) studies in the domain of system theory. Its modern form, however, is related to several theories developed since the mid-sixties, in particular to Haken's (1977, 1987, 1990, 1996) theory of *synergetics*, to Prigogine's *dissipative structures* (Nicolis and Prigogine 1977; Prigogine 1980; Prigogine and Stengers 1985), to Eigen's (1971) *catalytic networks*, as well as to Lovelock (1979), Maturana and Varela (1980), and Margulis (1995). Of the latter, Haken's synergetics and Prigogine's dissipative structures were probably the first to be applied to cities and urbanism. The authors who first coined the notion of 'self-organization' were fascinated mainly by the property of noncausality of the systems they have confronted. That is to say, by the finding that in certain situations external forces acting on the system do not determine or cause its behavior, but instead trigger an internal and independent process by which the system spontaneously self-organizes itself. The authors of the second wave of studies, led, as noted, by physicists such as Haken and Prigogine, were attracted by an even more complex process; the latter can be nicely illustrated by the above Bénard experiment that has since become a classical way to convey the notion of self-organization.

Most theories of complexity have been applied to cities with the implication that we now have a whole domain of theory and research that I have called *complexity theories of cities* (CTC). The domain of CTC already includes a rich body of research on *fractal cities* (Batty and Longley 1994), *self-organization and the city* (Portugali 2000), *cities and complexity* (Batty 2005), cellular automata and agent base urban simulation models (Benenson, and Torrens 2004), studies on cities from the perspective of Bak's *self-organized criticality* (Batty, ibid), studies on cities as networks (ibid) and much more.

The Bénard experiment, which has been repeated and elaborated by others, exhibits the main features of self-organization. First, that a system that is open and is thus part of its environment can attain a spatio-temporal structure and maintain it in far from equilibrium conditions; not in spite of, but as a consequence of, a sufficient flow of energy and matter. This contradicts the traditionally held view in physics, that systems must be looked at as essentially isolated from their environment. According to the second law of thermodynamics, such systems tend to move toward molecular disorder, that is to say, toward an increase of entropy.

Second, that this flow of energy and matter through its boundaries allows the system not only to spontaneously self-organize, attain a certain structure, and maintain it in far from equilibrium conditions, but also to "create" or "invent" novel structures and new and novel modes of behavior. Self-organized system are thus said to be "creative".

Third, self-organized systems are complex in two respects; one, in the sense that their parts (e.g., the molecules in the liquid) are so numerous that there is no technical way to establish causal relations among them; two, in that their parts and components are interconnected in a nonlinear fashion by a complex network of feedback and feedforward loops. Mathematically this form of behavior can be described by a set of nonlinear equations.

While the various complexity/self-organization theories and approaches that have been suggested since the emergence of this paradigm in the 1960s, and by implication also the various CTC, share the above-noted properties, they differ in two interrelated respects: Firstly, they differ with respect to their time scale and as a consequence in their mathematics, and also in the properties and processes they emphasize. More specifically, some refer to the longer time-scale evolution of complex systems – in our case cities and urbanism, while others to their shorter-term evolution and/or behavior. Based on this time-scale distinction, we can identify two major types of CTC (Fig. 4.2): long-term (or *comprehensive*), CTC versus short-term complexity theories, and complexity theories of cities.

Long-term comprehensive complexity theories of cities are the urban theories derived from the founding theories; first and foremost from Haken's theory of *synergetics* (Haken 1983, 1987) and also from Prigogine's theory of *dissipative structures* (Nicolis and Prigogine 1977; Prigogine and Stengers 1984); to some extent urban interpretations derived from Bak's *self-organized criticality* (Bak and Chen 1991) can also be added to this group. They are long-term and 'comprehensive' in the sense that they refer to all stages of the evolution of such systems: the bottom-up process of *emergence* that brings complex systems into a global ordered state and the process of *steady state* that keeps them is a structurally stable state. Of the latter, Haken's *synergetics* is the most comprehensive one due to its *slaving principle* (see below) and its emphasis on *circular causality*, that is to say, the feedback process by which the system "enslaves" the parts that brought it into being.

Short-term complexity theories limit their focus of interest to one part of the process. Most theories in this domain are short-term *emergence* complexity theories

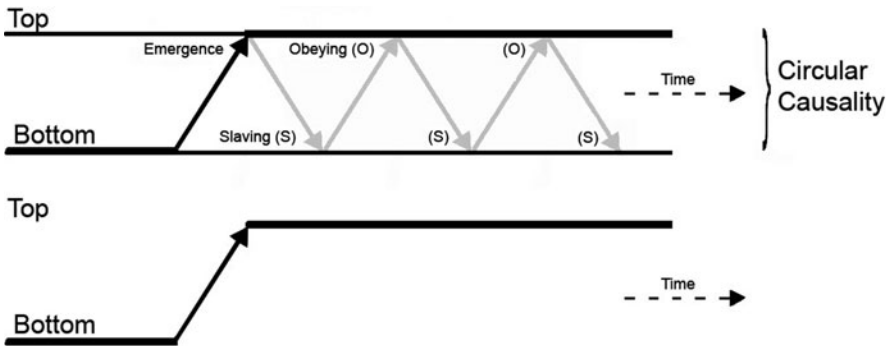

Fig. 4.2 Long-term (*upper drawing*) vs. short-term (*lower drawing*) complexity theories and CTC. A typical comprehensive complexity theory (*upper drawing*) is Haken's synergetics according to which local interactions/synergy between the parts (*bottom*) gives rise to an order parameter (*top*) that then enslaves the behavior of the parts (*bottom*). By "obeying", the parts strengthen and reproduce the order parameter and so on in circular causality. Compare to Fig. 4.4. Short-term CTC (*lower drawing*) focus exclusively on the bottom-up process of emergence by which the local interaction between the parts gives rise to a global systemic property, behavior, or phenomenon

that concentrate on the process of emergence, that is, the bottom-up process by which local interaction between the parts gives rise to a global structure. These theories do not theorize on the conditions and dynamics of the steady-state stage that keeps the system in a structurally stable state for long periods. Mandelbrot's theory of fractals is of this nature. On the other hand, chaos theory (or theories) is (are) a short-term complexity theory that looks at the reverse process of the "emergence" of chaos out of order.

The second distinction concerns *complexity theories* vs. *complexity models*. That is, theories that theorize about the dynamics and properties of complex systems versus models by which one can study the various phenomena and properties that typify complex systems. To be sure, all complexity theories and CTC come with their specific formalism and models. However, only CA (cellular automata), AB (agent base), and graph theoretic approaches can be described as "pure" models; firstly, in the sense that they do not make strong statements about the very dynamics of cities as complex systems. Secondly, and as a consequence of the above, they can be employed as tools to simulate various phenomena theorized by the "genuine" theories. One might object to this distinction by saying that these complexity models emphasize bottom-up emergence processes of complex systems. This is indeed so. However, the use of these models is not exclusive to the study of complex systems – they can and have been used, as means to simulate mechanistic simple systems, and, can and have been employed as means to simulate top-down processes for instance. A case in point is the FACS models introduced in Sect. 4.4.2 below, and employed in Part IV, Chaps. 17, 18.

Looking at the short history of CTC studies, one can observe a movement of interest from CTC to complexity models of cities. As we shall see below, CA, AB and graph-theoretic network models have become the most dominant approaches in the study of cities as complex, self-organizing systems. Chap. 5 below discusses the advantages but also the disadvantages of this situation.

The aim of this chapter is twofold: To introduce and discuss the various CTC, and, by so doing also to further elaborate on the various complexity theories from which they were derived. The discussion proceeds under the title of eight categories of "cities" which are related to general theories or specific methodologies. These eight cities are grouped into long-term CTC, short-term CTC and complexity models of cities. Under the long-term CTC come 'dissipative cities', 'synergetic cities' and self-organized criticality 'sandpile cities' (Sect. 4.3), under short-term CTC come 'chaotic cities' and 'fractal cities', while under complexity models of cities come 'cellular automata' and 'agent-based cities', 'FACS cities' and network 'small world cities' (Sect. 4.4). The discussion of each category of cities starts with a short introduction to the general principles of the approach and then elaborates its complementary self-organizing city.

4.2 Long-Term Complexity Theories of Cities

4.2.1 Dissipative Cities

Dissipative cities are the product of Prigogine's theory of *dissipative structures* and its application, by Allen and coworkers, to the study of cities and systems of cities (Allen 1981; Allen and Sanglier 1981; Allen et al. 1985). As the name indicates, Prigogine's theory of self-organization puts specific emphasis on the process of dissipation. A good starting point for his argument might be the conjunction between Boltzmann's order principle and the Bénard experiment as described above. The latter, as noted, exhibits a coherent motion, which "means that many molecules travel with nearly the same speed". According to Boltzmann's principle (which relates entropy to probability), there is almost no chance for this coherent self-organized motion to occur; "yet it occurs" explain Prigogine and Stengers. The reason is that in

> *far-from-equilibrium condition, the concept of probability that underlies Boltzmann's order principle is no longer valid .... Classical thermodynamics, leads to the concept of 'equilibrium structures' such as crystals. Bénard cells are structures too, but of a quite different nature. That is why we have introduced the notion of "dissipative structures", to emphasize the close association, at first paradoxial, in such situations between structure and order on the one side, and dissipation or waste, on the other. We have seen ... that heat transfer was considered a source of waste in classical thermodynamics. In the Bénard cell it becomes a source of order. .... The interaction of the system with the outside world, its embedding in nonequilibrium conditions, may become in this way the starting point for the formation of new dynamic states of matter – dissipative structures. .... Bénard cells, like all dissipative structures, are essentially a reflection of the global situation of nonequilibrium producing them* (Prigogine and Stengers 1985, p 142–144).

For anyone familiar with location theory the close resemblance between the hexagonal structure of the Bénard cells and the Christaller's (1966/1933) and

Lösch's (1954) hexagonal landscapes of central places (above, Chap. 2 Figs. 2.6–2.9) not just invites, but almost demands, a comparison. And indeed, this challenging similarity was taken up by Allen and coworkers who in a series of studies have reformulated the static central place theory of Christaller and Lösch in the dynamic terms of Prigogine's dissipative structures.

Allen and coworkers have developed a sequence of several models, which elaborated their theoretical treatment of hierarchical landscapes of central places, first with respect to systems of cities in a given region and later at the intra-urban scale in connection with a single city. At a later stage they have also applied their models to real case studies of Brussels and the Belgian provinces (Sanglier and Allen 1989; see also: Pumain, Saint-Julien, and Sanders 1987).

A typical model of Allan's starts with an infrastructure of localities in a region, each with its residents and jobs. The actors are individuals who migrate in order to get employment, and employers who offer or take away jobs depending on the market's situation. The migration (or interaction) between localities and the introduction and extraction of economic activities (i.e. employment opportunities), create for each locality a kind of local "carrying capacity" and for the system as a whole nonlinearities and feedback loops which link population growth and manufacturing activities. For example, in Allen and Sanglier's study from 1981, the positive feedback is due to common infrastructure, economies of scale, etc. and the negative feedback is due to crowding, pollution and other factors. An example for a simulated scenario produced by the model is Fig. 4.3. This specific scenario starts with a hypothetical region (not represented in Fig. 4.3) characterized by a rectangular lattice of homogeneous localities (similar to the region shown in Fig. 2.7 *bottom, right*). Then, the mere play of chance factors, such as the place and time where different enterprises and migrations start, produce symmetry breakings, which entail an uneven distribution of population and employment (Figs. 4.3a–d). The result is an evolutionary process by which new urban centers emerge, grow, and form the whole of the regional system of central places; as the system evolves, some old localities grow, others decline or even disappear, thus constructing the specific history of this region.

The application, by Allen and coworkers, of the theoretical principles of dissipative structures to the question of the emergence of a hierarchical landscape of central places, immediately exposes the similarity and difference between the "old" static approaches of Christaller and Lösch, and the new treatment by means of self-organization. In both the old and new approach, economic activities and interactions give rise to central places, which are usually urban centers. However, while in the old formulations the landscape reflects an equilibrium state which is the optimized sum of the properties of the various economic forces, the new landscape is "more than the sum of its parts" – it reflects a far-from-equilibrium situation in which the spatial hierarchical order among the central places is obtained, maintained, and then transformed, by means of an interplay between interaction and fluctuations, on the one hand, and dissipation (as in the Bénard cells), on the other.

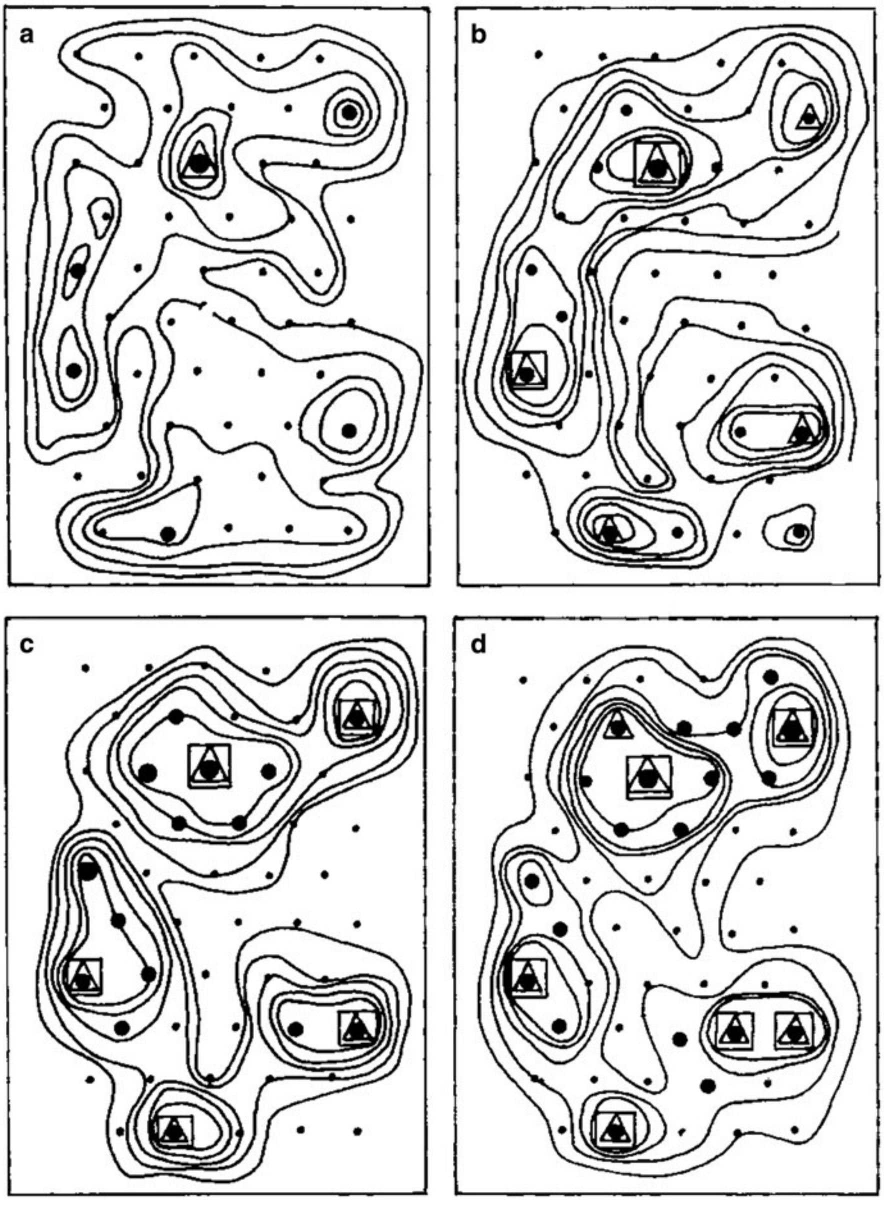

Fig. 4.3 Allen and Sanglier's (1981) simulated evolution of a dissipative system of cities. (a) at time (*t*) $t = 4$; (b) at $t = 12$; (c) at $t = 20$; (d) at $t = 34$. A small dot represents a settlement with one economic function; large dot a settlement with two economic functions; large dot inside a triangle – three functions; the latter inside a square – four functions

4.2.2 Synergetic Cities

Synergetics – the working together of many parts, individuals, subsystems, groups.. is the name assigned by Hermann Haken to his theory of self-organization (Haken 1979, 1983, 1985, 1987, 1990, 1993, 1996). As the name indicates, the emphasis here is on the interrelations, interactions and synergy among the many parts of the system and its overall structure and behavior.

Though synergetics originated in physics, it is by no means a physical theory that tries to reduce complex phenomena to the laws of matter. From the start Haken emphasized that the physical systems he was studying are similar in their behavior to phenomena of collective behavior in a variety of disciplines, and indeed, many of the notions which now form the theory were revealed and developed by a detailed investigation of case studies in a variety of domains, including sociology, psychology, cognition, AI (artificial intelligence) and also cities and urbanism. In all these studies Haken's (1996, p 39) central methodological guide was to "look for qualitative changes at macroscopic scales". Some of these case studies, in particular those of the laser, pattern formation in liquids, pattern recognition and the finger movement experiment, have become paradigm cases and a convenient way to convey the principles of synergetics. For the purpose of the present comparative discussion it will be useful to present three such cases – the laser paradigm, the pattern formation paradigm, and the pattern recognition paradigm.

4.2.2.1 The Laser Paradigm

The process that produces the laser can be regarded as the generic paradigm of synergetics. The gas laser consists of a glass tube filled with a gas of atoms or molecules. The gas tube has two mirrors at its ends, which serve to reflect the light running in axial direction rather often so that this kind of light wave can interact sufficiently strongly with the individual atoms. Because at least one of the mirrors is only semi-transparent, the laser light can eventually emerge through this mirror.

In a usual lamp, when an electric current is sent through the gas, the individual atoms may get excited and emit individual independent light waves. In the laser, the individual electrons in the atoms correlate their movements and generate a beautifully ordered coherent light wave (Fig. 4.4). This is a typical act of self-organization. There is nobody who tells the laser system how to behave in such a coherent fashion; it finds its well-ordered behavior by itself.

The interpretation of synergetics starts with Einstein's observation that when an excited atom emits a light wave, this light wave may cause other excited atoms to deliver their energy to that light wave so that this light wave is enhanced in its intensity. In the laser, initially quite a number of atoms emit their light waves independently of each other and with somewhat different wavelengths. Each of these might get support from the other excited atoms. In this way a kind of a Darwinian competition among the light waves for the energy resources of the excited

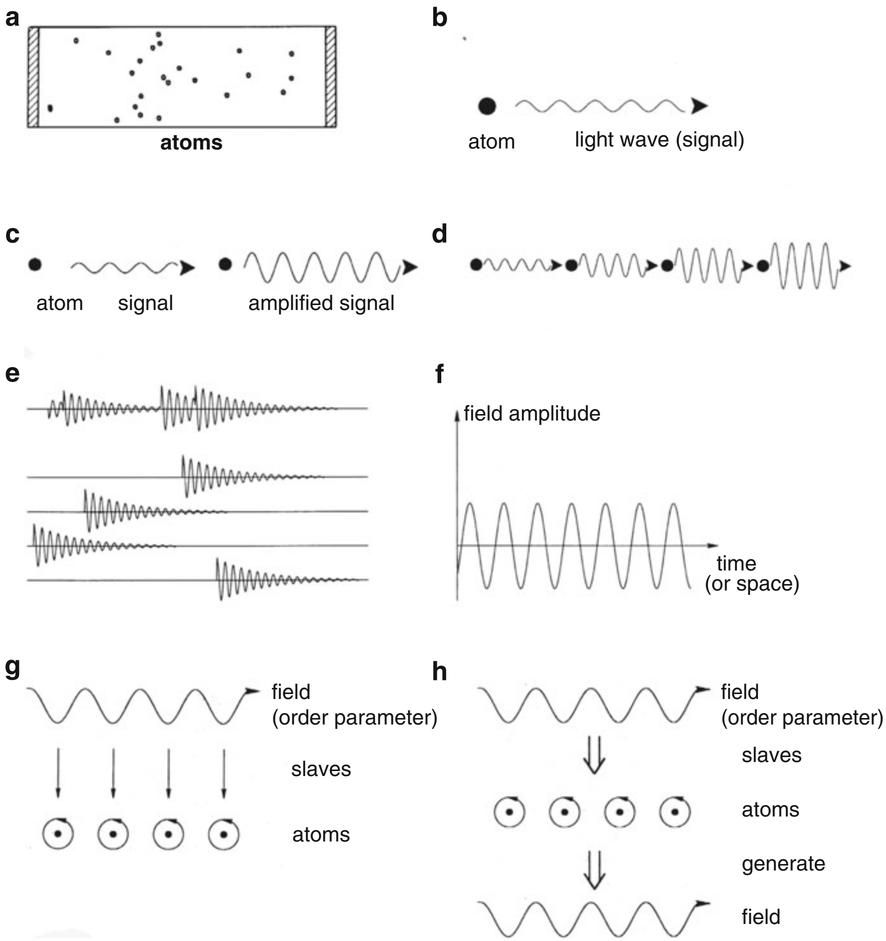

Fig. 4.4 The laser paradigm. **(a)** Typical setup of a gas laser. A glass is filled with gas atoms and two mirrors are mounted at its end faces. The gas atoms are excited by an electric discharge. Through one of the semi-reflecting mirrors, the laser light is emitted. **(b)** An excited atom emits light wave (signal). **(c)** When the light wave hits an excited atom it may cause the atom to amplify the original light wave. **(d)** A cascade of amplifying processes. **(e)** The incoherent superposition of amplified light waves produces still rather irregular light emission (as in a conventional lamp). **(f)** In the laser, the field amplitude is represented by a sinusoidal wave with practically stable amplitude and only small phase fluctuations. The result: a highly ordered, i.e. coherent, light wave is generated. **(g)** Illustration of the slaving principle. The field acts as an order parameter and prescribes the motion of the electrons in the atoms. The motion of the electrons is thus "enslaved" by the field. **(h)** Illustration of circular causality. On the one hand, the field acting as order parameter enslaves the atoms. On the other hand, the atoms by their stimulated emission generate the field (after Haken 1988/2000)

atoms begins. This competition is won by an individual light wave, which grows fastest. The winning light wave describes and prescribes the order in the laser and it is thus called the *order parameter*. It dominates the movement of the individual electrons as if by enslaving them and forcing them to move in its own rhythm. In the language of synergetics this is called the *slaving principle*.

The transition from the state of a lamp with its microscopically chaotic light field and the state of the laser with its well-ordered light field is quite sharp and occurs at a critical strength of the power input by the current into the laser. Thus the change of a single, rather unspecific, parameter, the power input in the case of the laser, may cause a systemic phase transition. This parameter is termed *control parameter*.

When the laser wave is perturbed, it adjusts rather slowly compared to the adjustment time of the individual electrons. This is quite a general feature: the order parameters are slowly varying quantities compared to the enslaved subsystems. Consequently, when excited atoms are shot into the tube, they are enslaved by the order parameter (i.e. the light field in that tube) first by delivering their own energy to the order parameter, and then by acquiring its rhythm. This interplay between the rhythms of the system and its subsystems is analogous to many cases in human life: Language can be regarded as a slowly moving order parameter. When a baby is born, it is subjected to the language of his or her parents and the other people. The baby learns the language and in technical terms is thus enslaved by the language. But by doing so, he and she eventually "emit" their personal energy into the language and in this respect support the language. As we shall see in some detail below, the same happens in city dynamics: the individual who immigrates to a new city has to learn the city and adapt to its rhythm. The individual is thus enslaved by the city's order parameter. But by adapting to the global movement of the city the individual's energy enters into, and supports, the order parameter of the city. This phenomenon is called *circular causality*.

Quite clearly, the concept of order parameters and their relationships to the individual parts of the system, a relationship governed by circular causality, applies to a great variety of phenomena in society. On the one hand, the individuals are the parts of a human society and determine its macroscopic manifestations, such as language, religion, form of government, culture, educational system or city structure. On the other, the behavior of the individuals is determined by these macroscopic manifestations or institutions, which play the role of order parameters. Order parameters may compete with each other, or may coexist, or may cooperate. Such phenomena are well known in laser physics with respect to the electric field strength acting as order parameters. In social life these phenomena also occur, e.g., languages may compete and one language may win the competition as it happened in the United States, where two main streams were competing with each other, namely English and German. In other nations languages may coexist as in Switzerland and in Israel. Finally, in a way languages may cooperate, e.g., when terms originally generated in one language are adopted by another language. This happens quite often with technical terms taken from English, for example.

4.2.2.2 The Paradigm of Pattern Formation

The case study here is the Bénard instabilities. As we've seen, as the temperature difference between the heated layer at the bottom and the cool layer on top exceeds a critical value, quite suddenly a macroscopic motion of the liquid becomes visible (Fig. 4.1 above). The temperature difference thus controls the macroscopic behavior of the system; in the language of synergetics it is thus called *control parameter*.

As the control parameter grows, the liquid starts its motion, rolls are created, their rolling speed increases, and the initial resting state becomes unstable. *Instability* thus shows up. Slightly above the instability point, the system may undergo quite different collective motions of role configurations (Fig. 4.5). At the beginning the amplitudes of these role configurations are small and independent of each other. When they grow further, they start to influence each other – in some cases they compete until one configuration suppresses the others, in others, they co-exist and even stabilize each other. "The amplitudes of the growing configurations are called *order parameters*. They describe … the macroscopic structure of the system" (Haken 1996, p 39).

The order parameters not only determine the macroscopic structure of the system, but also govern the space-time behavior of its parts. By winning the competition the order parameters enslave the many parts of the system to their specific space-time motion. This is a basic theorem of synergetics and as noted above, it is called *the slaving principle*.

In some cases, for example when the fluid is enclosed in a circular vessel, all directions for roll systems are then possible, each being governed by a specific order parameter. Which pattern will eventually be realized, depends on initial conditions. It is as if the system internally stores many patterns. This repertoire of patterns is not stored in a static fashion, but is dynamically generated anew each time. This property is termed *multistability*.

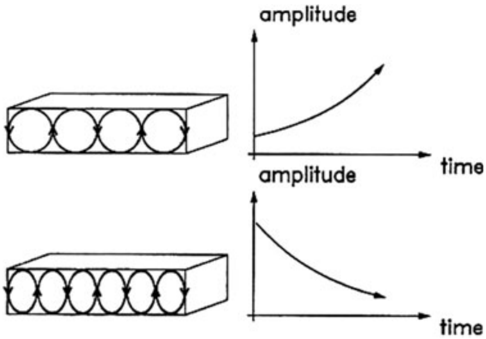

Fig. 4.5 *Left*: two different role configurations in a fluid. *Right*: The behavior of the amplitudes of these configurations in the course of time. While in one case the amplitude increases, in the other it decays

4.2.2.3 The Paradigm of Pattern Recognition

A typical experiment of pattern recognition can start as follows: a test person (or a computer) who has many patterns, of faces, city maps, etc., stored in memory, is offered a portion of one of the patterns. The task is to recognize the pattern – to decide what face, city map, etc. it is. According to synergetics, what happens is a process analogous to pattern formation as described above: at the start, the cognitive system of the person (or computer) is in a state of multistability as it enfolds many patterns, which coexist. When a few features or part of the pattern is offered, several pattern configurations and their order parameters are formed by means of *associative memory*. The order parameters enter into competition and when a certain order parameter wins the competition and enslaves the cognitive system, the task of recognition is implemented. This analogy, which is illustrated in Fig. 4.6, was first demonstrated by Haken in 1979 and has since become the basis for an intensive study of synergetics of cognition and of brain development and activity (Haken ibid 1979, 1990, 1996). Figure 4.7 is a typical implementation with respect to face recognition, by means of the so-called *synergetic computer*.

As suggested by Portugali and Haken, the above conceptualization offers also an appropriate framework for the study of cognitive maps of cities, regions, and large-scale environments (Portugali 1990; Portugali and Haken 1992). The basic idea is that cities, regions etc. can be regarded as large-scale patterns, which can never be seen in their entirety. As a consequence, the cognitive system constructs a whole cognitive map on the basis of only a partial set of features available to it. Thus it can be said that a partial set of environmental features offered to a person triggers a competition between several configurations of features and their emerging order parameters, until one (or a few) wins and enslaves the system so that a cognitive map is established.

Synergetics have developed from the start two approaches to the study of phase transition and qualitative change in self-organizing systems. The first approach study phase transition and qualitative change by means of probability distributions and direct numerical solutions. This line of research has used the conceptual framework of order parameters and the slaving principle in a rather implicit manner; its main instrument was the so-called *master equation*. The second approach

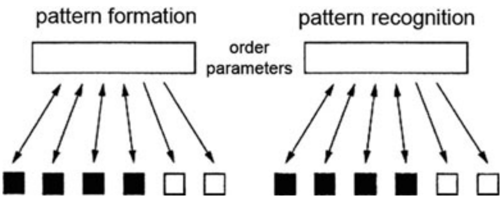

Fig. 4.6 Haken's (1979, 1991) analogy between pattern formation and pattern recognition. *Left:* a configuration of some parts of a system gives rise to an order parameter which enslaves the rest of the parts and brings the whole system to an ordered state. *Right:* a few features of a pattern shown to a person (or a computer) generate an order parameter which enslaves, and thus complements, the rest of the features, so that the whole pattern is recognized

Fig. 4.7 Pattern recognition of faces by means of the synergetic computer. *Top*: examples of prototype patterns stored in the computer, with family names encoded by letters. *Bottom*: when part of a face is used as an initial condition for the pattern recognition equations, their solution yields the complete face with the family name

developed by focusing on the state variables of the system and by an explicit consideration of the order parameters, the slaving principles and the other tenets of synergetics as presented above. These two lines of research are characteristic also of the approaches of synergetic cities. One approach, led by Weidlich and coworkers has developed sociological and economic applications of synergetics by employing the master equation, and the other approach, by Haken and Portugali, was inspired by Haken's elaboration of synergetics in the domains of cognition, pattern recognition and brain activities.

4.2.2.4 Slow Cities and Fast Regions

One way to look at Haken's synergetics and its slaving principle is in terms of interplay between slow and fast processes:

> *If in a system of nonlinear equations of motion for many variables these variables can be separated into slow ones and fast ones, a few of the slow variables … are predestined to become "order parameters" dominating the dynamics of the whole system on the macro-scale* (Weidlich 1998).

This perspective stands at the basis of Weidlich's and coworkers studies on sociodynamics and more recently on cities and urbanism (Weidlich 1987, 1994, 1997, 1998).

According to this perspective, fast and slow processes are easily identifiable in processes of settlement and urbanism. The fast ones typify the local urban micro-level of building sites, streets, subways, etc., whereas the slow processes typify the macrolevel of whole regions, which are often described as systems of cities. The relations between the slow and the fast processes are described by the slaving principle: on the one hand, the regional system

> *serves as the environment and the boundary condition under which each local urban microstructure evolves. On the other hand, the . . . regional macrostructure is . . . the global resultant of many local structures* (Weidlich 1998).

This circular causality between the local and the global, allows one to study global regional systems by assuming that local processes adapt to the slow regional ones, and to study local urban processes by treating the regional context as given, and of course to study the complex interplay between the local and the global. In all three cases Weidlich (1999) has prescribed a four stages approach: stage 1 concerns the configuration space of the variables; stage 2, measures the utility of each configuration; stage 3, defines transition rates between configurations which are in fact utility differences; stage 4 derives stochastic or quasi-deterministic evolution equations for the system under consideration. The central evolution equation is the master equation, which defines the probability that the configuration under examination is realized at a certain time.

The above theoretical procedure has been used to study the role of population pressure in "fast and slow processes in the evolution of urban and regional settlement structures", and in urban evolution. Figure 4.8 illustrates some results from these studies, in which the city capacity for building and development is related to population pressure. Figure 4.8a shows the evolving city capacity when the urban plain is uniform, and Fig. 4.8b, when it is disturbed in one of its sites.

4.2.2.5 Pattern Formation and Pattern Recognition in the City

Thus far, most applications of synergetics to the domain of cities and regions were in line with Weidlich's approach as described above. That is, they have employed mainly the very early and basic notions of synergetics – the order parameters and slaving principle – and usually in a rather implicit way. On the other hand, much of the advance made in the last four decades in synergetics was done in connection with issues of cognition and brain functioning. In fact, the above analogy between pattern formation and pattern recognition provided the foundation for these advances.

Haken and Portugali (1995) suggested that the above analogy is specifically attractive to the study of cities. The latter can be perceived as complex, self-organizing systems, which are both physical and cognitive: individuals' cognitive maps determine their location and actions in the city, and thus the physical structure of the city, and the latter simultaneously affects individuals' cognitive maps of the city. In their preliminary mathematical model Haken and Portugali construct the city as a hilly landscape which is evolving, changing and moving as a consequence of the

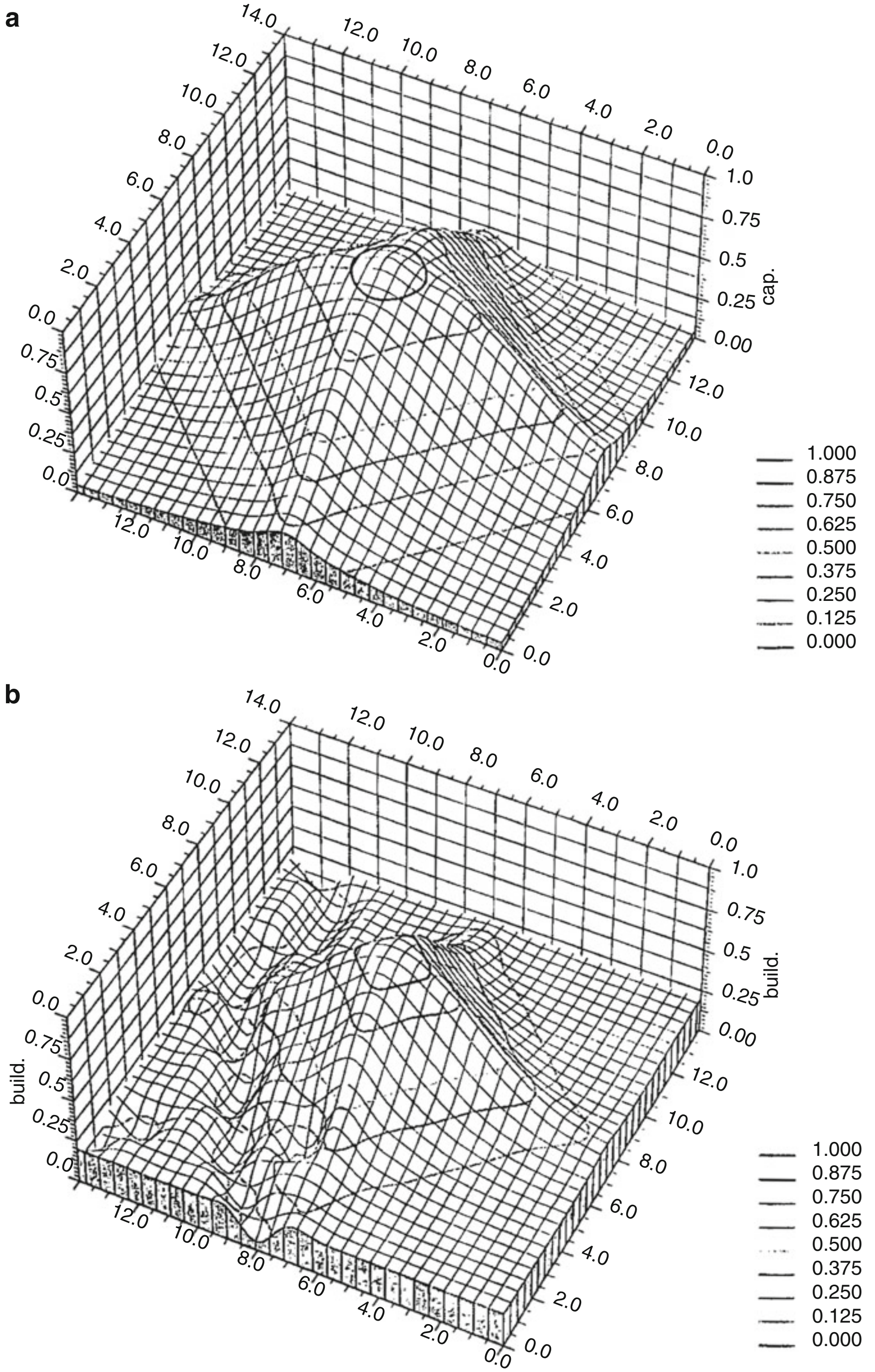

Fig. 4.8 Building and development under population pressure (Weidlich 1999). *Top*: on a uniform urban plain. *Bottom:* on an urban plain with disturbances

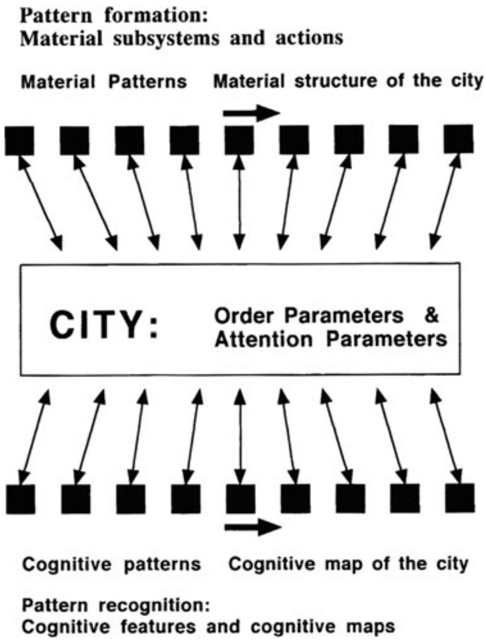

Fig. 4.9 The city as a self-organizing system that is at the same time both physical and cognitive. Its emerging order- and attention-parameters enslave the city's cognitive and material patterns

movement and actions of individuals (firms etc.). The latter give rise to the order parameters, which compete and enslave the individual parts of the system and thus determine the structure of the city. The significant and new feature of this exposition is that the order parameters enslave and thus determine, two patterns: one is the material pattern of the city, and the other is the cognitive pattern of the city, that is to say, its cognitive map(s). This is exemplified diagrammatically in Fig. 4.9.

One of the more interesting outcomes of the model is the set of *attention parameters*, which emerge by means of self-organization. The latter can be seen as the order parameters of specific subsystems composing the pattern. In a state of multistability, or in case of an ambivalent pattern (e.g., 'vase or faces?' in Fig. 19.4) they determine which aspect of the pattern is seen (i.e., attracts attention) first. This is of outmost importance in city dynamics. The city is full of patterns, yet individuals are attentive to only a few of them. The latter form the cognitive maps of the city and it is according to them that individuals and firms behave, take decisions and act in the city. In the model we investigate cases were one attention parameter dominates the dynamics and cases where no cross-attention is paid; that is, when two or more urban communities are cognitively not aware of each other. This situation entails the emergence and persistence of an urban cultural or socio-economic mosaic where a few coexisting attention parameters govern the dynamics. In Part IV (Chaps. 19, 20), we present the above approach in full and elaborate it in connection with city dynamics and decision making in the context of urban and regional planning.

4.2.3 Sandpile Cities

Imagine building up a sand pile by slowly adding particles, as in Fig. 4.10, for instance:

> *As the pile grows, there will be bigger and bigger avalanches. Eventually a statistical stationary state is reached in which avalanches of all sizes occur, that is the correlation length is infinite. Thus, in analogy with equilibrium thermodynamical systems, the state is 'critical'. It is self-organized because no fine-tuning of external fields was needed to take the system to the critical state: the criticality is unavoidable.*
> *The sandpile has two incongruous features: the system is unstable in many different locations; nevertheless the critical state is absolutely robust. On the one hand, the specific features, such as the local configurations of sand, change all the time because of the avalanches. On the other, the statistical properties, such as the size distribution of the avalanches, remain essentially the same* (Bak, Chen, and Creutz 1989).

Applied metaphorically to the domain of cities the scenario can go like this:

> *Imagine a growing city – demographically by a steady inflow of population and spatially by the new locations (parcels of land) they occupy. As more and more people come to the city, locations of various sizes (avalanches) will be occupied. Eventually a statistical stationary state is reached in which occupation of locations of all sizes (avalanches) occur. . . . . Thus, in analogy with equilibrium thermodynamical systems, the state is 'critical'. It is self-organized because no fine-tuning of external fields was needed to take the system to the critical state: the criticality is unavoidable.*
>
> *The emerging city has two incongruous features: it is unstable in many different locations; nevertheless the critical state is absolutely robust. On the one hand, the specific features, such as the local configurations of locations, change all the time because of the avalanches. On the other, the statistical properties, such as the size distribution of locations (the avalanches), remain essentially the same.*

The sandpile is the canonical example of Bak's (Bak 1996; Bak, Chen, and Creutz 1989; Bak and Chen 1991) *self-organized criticality* (SOC). As just illustrated, it can naturally be, and indeed has been, applied to cities (Batty 1995, 1996; Batty and Xie 1999; Batty 2005). It adds to the "grand" complexity and self-organization theories, such as synergetics and dissipative structures, a kind of a zooming-into the internal dynamics of self-organized systems in their steady-state

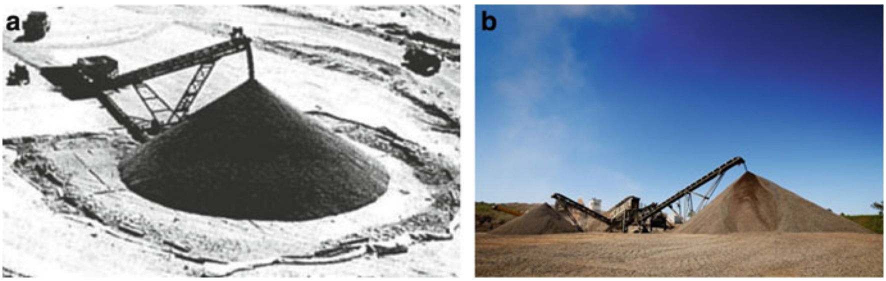

Fig. 4.10 The sandpile is the canonical example of self-organized criticality

periods – when the system is governed by what in synergetics is called order parameters. Similarly to synergetics, for instance, SOC refers to systems in their growing process: In synergetics one speaks of a growing control parameter (e.g., slowly adding particles to the sandpile). As in synergetics so with SOC, as the control parameter crosses a certain threshold – a critical point in the language of SOC, the system (sandpile) enters a steady state. The interesting part of SOC starts here: it shows how complex and rich the internal dynamics of a steady-state situation can be. – It demonstrates that during steady state the system that is subject to continuous growth maintains its overall structure by means of temporally unpredictable avalanches whose size distribution takes the form of the power law.

With respect to cities, SOC suggests, first, that when the population of a city, or a system of cities, is growing, the morphology of this growth shows up in locations of various sizes. Second, that when this growth crosses a certain critical threshold, the city, or system of cities, enters a steady state. Thirdly, when looking into the dynamics of this steady state it can be seen that this growth advances by means of avalanches of locations of various size. Fourthly, while the temporal distribution of these morphological locational avalanches is chaotic and unpredictable, their size distribution remains robust and takes the form of the power law.

For students of urbanism the statistical observation that "the size distribution of avalanches remain essentially the same" and takes the form of the power law, implies an immediate link to the century-old 'rank-size rule', according to which the size-distribution of cities remains essentially the same under circumstances of ongoing population growth (Chap. 2, Sec. 2.4 above). COC in this respect provides a theoretical foundation for the rank-size rule, which was often criticized for not having any theoretical basis. As we'll see below the same can be said about fractal and network theories that are closely linked with SOC.

However, the essence of self-organized criticality is not the final statistical size-distribution, but the process behind it. The problem here is that in the human domain of cities there is no sufficiently detailed data to describe this dynamics. As a consequence, most applications of COC to the domain of cities took the form of "computer simulations of what are essentially idealized systems" (Batty 2005, p 433). In *Cities and Complexity* Batty (ibid) indeed presents such a computer simulation of an idealized urban growth process. By means of the latter he demonstrates how the dynamics of the sandpile model "is consistent with the maintenance of a stable urban form". Commencing with a hypothetical city with its central business district (CBD) and subcenters, and with the assumption that population is distributed in that city according to the power law, he simulates a sandpile urban growth process (Fig. 4.11): Successive units of development – the urban agents – enter the various areas of the city. When the capacities at the various urban areas exceed their critical threshold, relocations ("avalanches") of urban agents/population occur – similarly to avalanches in the sandpile. As can be seen in Fig. 4.11, the avalanches occur with increasing frequency as the city builds up.

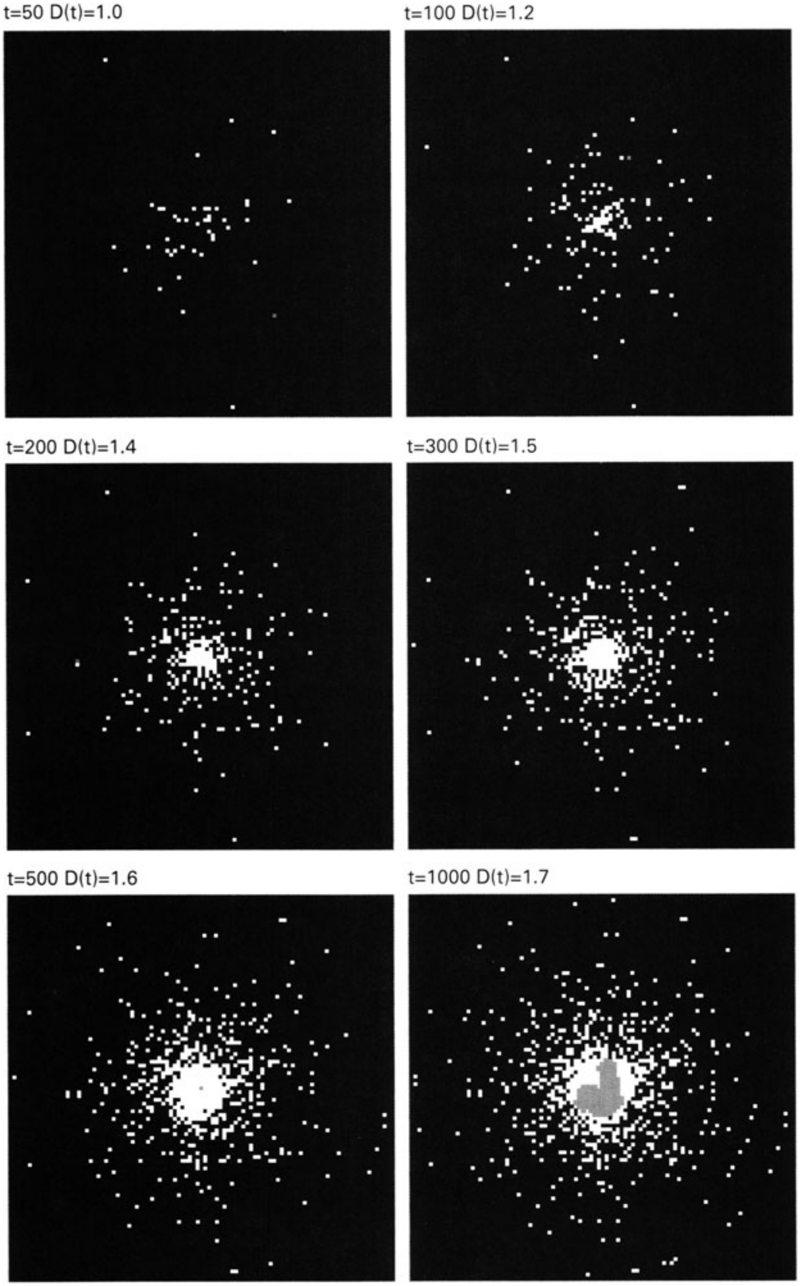

Fig. 4.11 Batty's (2005, Fig. 10.1) simulation of a hypothetical urban growth model in terms of COC. "The model is constructed as a simple agent-based structure with units developments acting as agents responding to two features of the landscape: density constraints . . . and .. the sand pile rules" (ibid 434). The white dots/areas represent populated urban areas; the lighter gray tone in Fig. 4.11 (*bottom, right*) "indicates the extent of an avalanche that leads to relocation of . . . agents at the time when the simulation reaches 1000 units" (ibid). D(*t*) is the fractal dimension at time (*t*)

4.3 Short-Term Complexity Theories of Cities

4.3.1 Chaotic Cities

While the origins of chaos theory can be traced back to Henri Poincaré in the 1880s and Jacques Hadamard in 1890s, its modern use is due to Edward N. Lorenz (1963) and Mitchell Feigenbaum (1978), among others, as well as to the emergence in the 1960s of complexity and self-organization theories as theories about "order out of chaos". Chaos theory, which has since become one of the leading complexity theories, refers to one form of chaos (Haken 1996, Chap. 13), namely, *global, macroscopic*, *deterministic chaos* (by contrast to *local microscopic chaos*).

Local chaos stems from the irregular motion or behavior of the very many individual parts of a complex system. Examples might be the motion of the molecules of a gas, the movement of cars on an uncrowded freeway, or of people in an uncrowded piazza, and so on. In all such cases the individuals are moving in an irregular and uncoordinated trajectories.

Deterministic chaos refers to systems that are *sensitive to initial conditions*. The famous example here is Lorenz *strange attractor* that describes a system (point x, y, z) that jumps irregularly from one region of space to the other as a result of small changes in initial conditions (Fig. 4.12). As is well recorded, Lorenz has found this property accidently when as a matter of shortcut he was running his weather equations with the decimal .506 instead of .506127; the result was a completely different weather forecast. This effect was at a latter stage described by Lorenz as *the butterfly effect*.

In terms of synergetics, deterministic chaos arises when as a consequence of self-organization, for example, the many individual parts are enslaved by a few order parameters, and as a consequence exhibit a coordinated motion. On the face of it this new state is the exact opposite of chaos; yet, it is not. Quite often in these cases, the system is dominated by order parameter(s) which are macroscopically chaotic: for some time one order parameter dominates the system, then suddenly another. Such jumps occur irregularly in a chaotic manner due to the fact that these systems are *sensitive to initial conditions*.

Similarly to fractal theory (below), the development of deterministic chaos theory is computer-dependent: Starting from a deterministic situation the theory shows how by means of an iterative process the system moves from order to chaos. A commonly used example to convey chaos is by reference to population dynamics: As illustrated in Fig. 4.13, a slight change in initial conditions (increasing b from 3.56 to 3.56999) entails an infinite number of solution, that is to say, chaos. Fractal theory looks at the reverse process by which iterative chaotic processes give rise to highly structured fractal patterns. This is nicely illustrated by the so-called *chaos game* (Fig. 4.14). The two theories are in this respect two facets of a single phenomenon and thus complement each other: chaos theory looks at "the way to chaos", while fractal theory at "the way to order".

Fig. 4.12 Two famous notions associated with Lorenz's work: *Left*: Photograph of a butterfly as a reference to his "butterfly effect". *Right*: Trajectories of Lorenz's *strange attractor*. While the two notions refer to quite different phenomena, the "butterfly effect" is internally related to the strange attractor

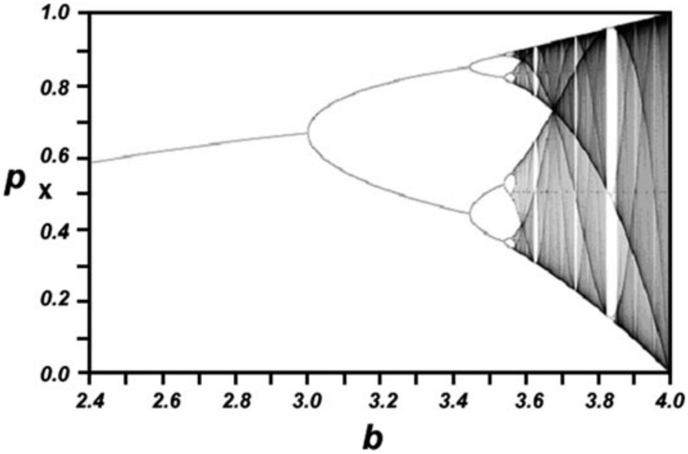

Fig. 4.13 The way to chaos. A simple population dynamics can be described by: $P(n+1) = b.P(n)$, that is, population P at year $n+1$ is P at year n, multiplied by b – the rate of population growth. According to Pierre François Verhulst's (1845) this equation can be normalized to $P(n+1) = b.P(n)(1–P(n))$. Now, when b is small, say 1.00, this equation yields one attractor; when $b = 3.0 \rightarrow 2$ attractors; $b = 3.44 \rightarrow 4$ attractors; $b = 3.56 \rightarrow 8$ attractors; $b = \ldots$; but then when $b = 3.56999 \rightarrow$ infinity of attractors, that is to say, chaos

On the face of it, this tension between chaos and order is specifically and intuitively appropriate for the study of cities for their dual image; namely, that cities are sometimes seen as chaotic entities and sometimes symbols of order: They are chaotic, when one considers the fact that they emerged (and still re-emerge) in an urban revolution(s), or when one faces winding streets of old towns, traffic jams, congestion, pollution, and the like; and they are symbols of planned order when one is attentive to the regularity of urban systems, to city walls, the iron-grid pattern of ancient and modern cities, grand boulevards and so on. Given this image of the city as at once chaotic and ordered, one would expect a multiplicity of studies on chaos

Fig. 4.14 The chaos game. Coined by British mathematician Michael Fielding Barnsley (1993), the chaos game refers to an iterative random process that when repeated a large number of times, might often (not always) give rise to a fractal such as the Sierpinski triangle or a leaf

and the city; yet this is not the case: there are only few applications of chaos theory to cities and urbanism and even those are highly theoretical and with no explicit links to the real dynamics of cities. Some, such as Dendrinos and Sonis (1990) study – *Chaos and Socio-Spatial Dynamics* – refers to socio-spatial dynamics in general with no specific relation to cities, while others that will be discussed next consider cities in a rather conceptual or theoretical manner.

4.3.1.1 Global Chaos in Ancient Urbanism

Global or deterministic chaos shows itself in the long-term evolution of cities and urbanism. A case in point is my interpretation of the archaeological record of the first 3000 years of urbanism in the region of Israel/Palestine in terms of a sequence of three urban revolutions (Portugali 2000, Chap. 15 and further bibliography there): the first, around 3000 B.C. gave rise to some 700 years of the first (Early Bronze) urban era; the second around 2000 B.C. gave rise to the second urban era (Middle Bronze), and a third around 1000 B.C. gave rise to the Iron Age urban culture (Fig. 4.15). The three urban eras are characterized by hierarchical settlement systems (Gofna and Portugali 1988) and as can be seen in Fig. 4.15, each of the three relatively stable urban steady states was preceded by short and unstable-chaotic periods of nonurban nomadic society. From the archaeological record we further know that the first and second urban cultures ended abruptly when the urban system collapsed and large segments of the urban population underwent a process of *nomadization* (which can be regarded the reverse of urbanization).

4.3.1.2 Deterministic Chaos and Urbanism

A very recent example of deterministic chaos in relation to urbanism is Yanguang Chen's (2009) suggestion that "Spatial interaction creates period-doubling and chaos of urbanization". Chen starts from the process of spatial interaction that as we've seen above (Chap. 2, Sec. 27) is central to urban dynamics at the local and

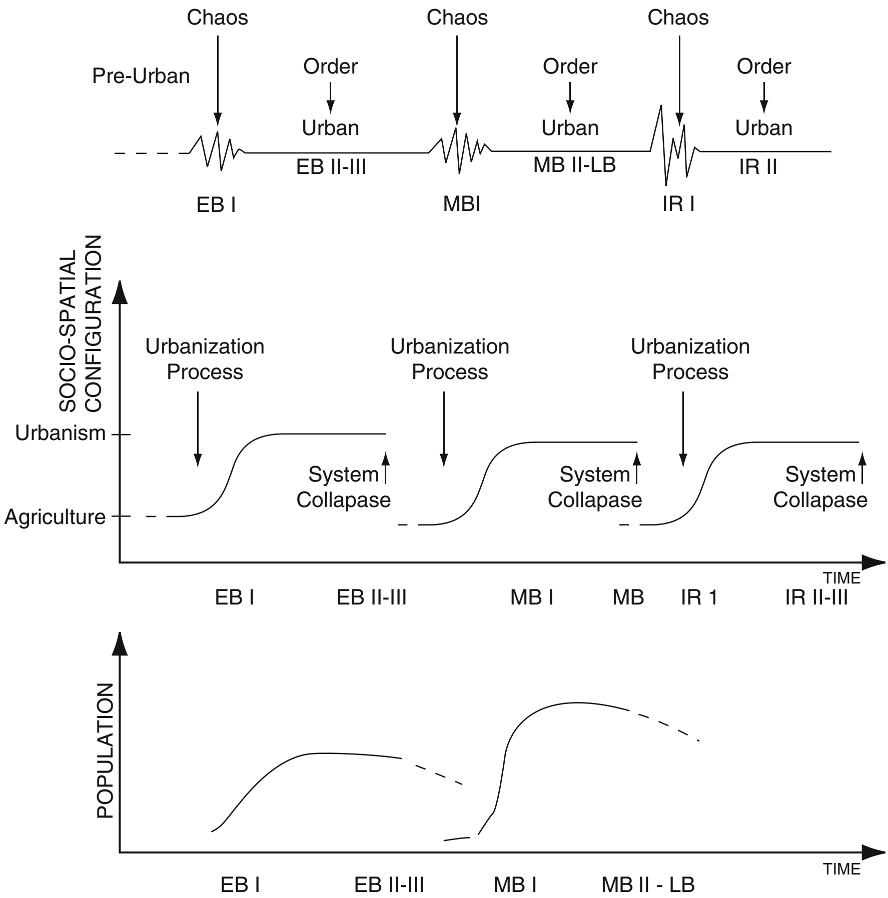

Fig. 4.15 Ancient urbanization and chaos. *Top*: The evolution of the settlement system in Palestine, from the Early Bronze Period to the Iron Age, exhibits long periods of urban steady state that are interrupted by short, nonurban periods characterized by system collapse, nomadization, strong fluctuations and chaos. *Center*: A description of the process as a rhythm between agriculture and urbanism, interrupted by global collapses of the urban system. *Bottom*: The above rhythm between urban steady state and nonurban chaos shows itself also in the calculated population changes (by Gophna and Portugali 1988) in the Early Bronze and Middle Bronze periods. Adapted from Portugali 2000, Fig. 15.8

regional scales. He refers specifically to spatial interaction as it takes place in rural-urban population migration – a process that dominates much of the urban dynamicsvh of China in recent decades. Defining the level of urbanization as "the percentages of urban population" in a given closed system/region, he demonstrates mathematically the title of his paper, namely, that the urban process of spatial interaction can give rise to steady-state behavior, to two-cycle oscillation, to four-cycle oscillation, and finally to chaotic behavior (Fig. 4.16). To this urbanization growth process corresponds the "classical" mark of chaos, namely, sensitivity to initial conditions (Fig. 4.17). However, as Chen is careful to emphasize, this urban chaos is created by the manipulation of the parameters and thus reflects "the possible world rather than the real world. . . . Whether or not urbanization in the real world can exhibit bifurcation and chaos is still a pending question." In a subsequent paper (Chen 2009b) he presents a chaotic attractor produced by the rural-urban interaction model (Fig. 4.18). Commenting on this attractor he notes the following:

> *Clearly, the trajectory [in Fig. 4.18] is infinitely enlaced in the limited phase space, but never repeats itself. This kind of strange attractor can be named rural-urban interaction attractor, whose box-counting dimension is about 1.5, and the correlation dimension is around 0.75. However, as will be illuminated later, it can only appear in an imaginary world instead of the real world.*

While Chen's is a purely theoretical study, his previous studies on the process of urbanization in contemporary China hint that the latter urban process provides the context for this more theoretical account (Chen 2009c and further bibliography there).

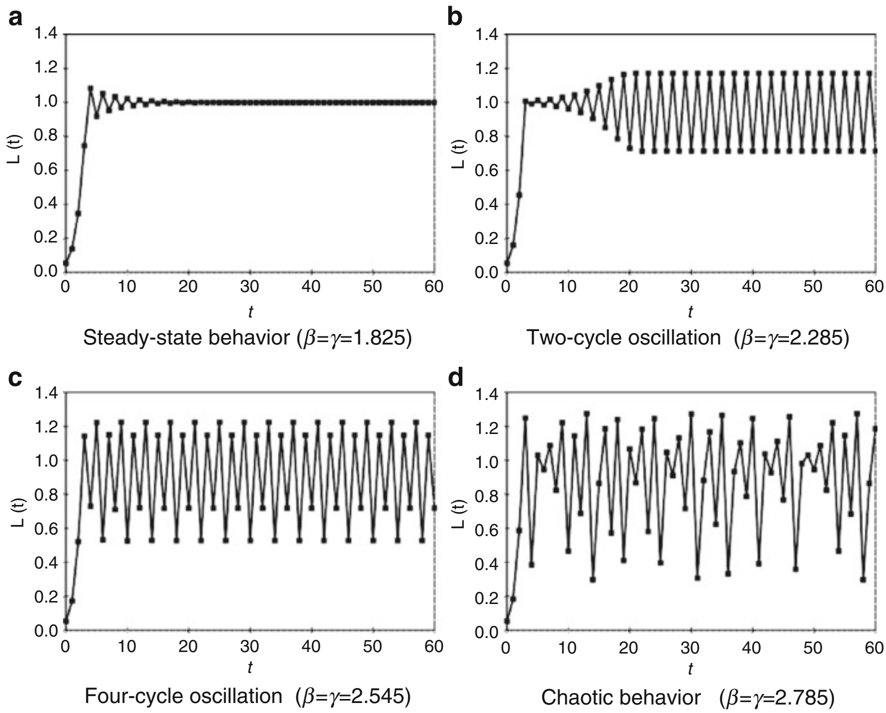

Fig. 4.16 According to Chen (2009, Fig. 1), an urban process driven by spatial interaction can give rise to steady-state behavior, to two-cycle oscillation, to four-cycle oscillation, and finally to chaotic behavior

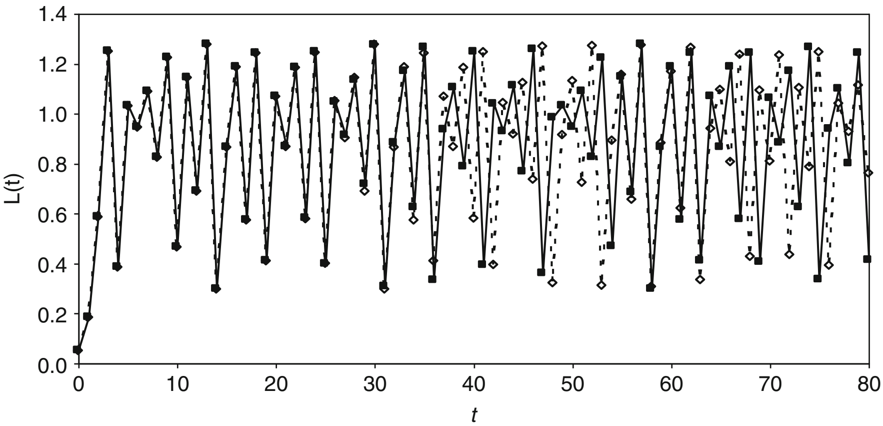

Fig. 4.17 Sensitive dependence on initial conditions of the level of urbanization in chaotic state (Chen ibid, Fig. 2)

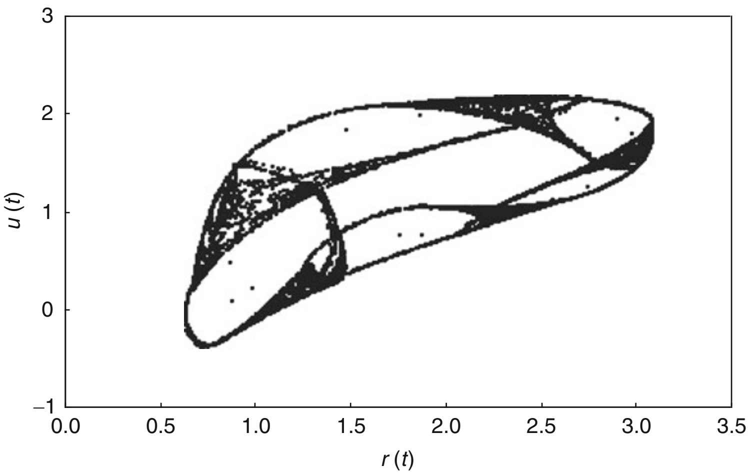

Fig. 4.18 The chaotic attractor produced by the rural-urban interaction model of after 10,000 iterations (Chen 2009a, Fig. 1)

4.3.1.3 Deterministic Chaos in Cities

In the *End of Certainty* Prigogine with Stengers (1997) exemplify deterministic chaos and the sensitivity to initial conditions by assuming two types of motion denoted as – or + within the phase space illustrated in Fig. 4.19. This leads to two situations represented in Fig 4.20. In Fig. 4.20 *Left* there are two regions, one corresponding to motion – and the other to motion +. Given Fig. 4.20 *Left*, they write the following:

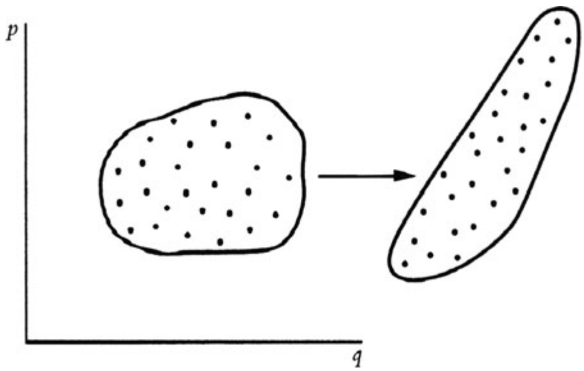

Fig. 4.19 Prigogine with Stengers' (ibid Fig. 1.4, p 34) illustration of ensembles in phase space: "Gibb's ensemble is represented by a cloud of particles differing according to their initial conditions. The shape of the cloud changes over time"

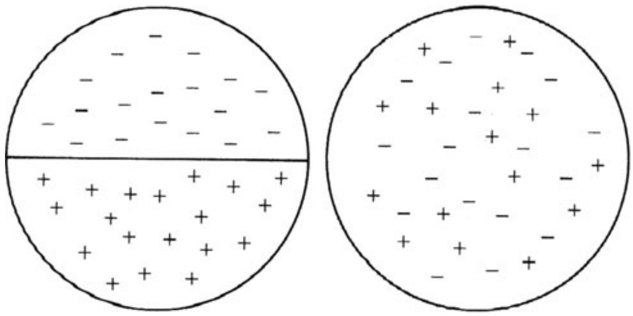

Fig. 4.20 *Left*: A stable dynamical system in which "the motions denoted as + or – lie in distinct regions of phase space". *Right*: An unstable dynamical system in which "each motion + is surrounded by – and vice versa". (Prigogine with Stengers ibid p 36, Figs. 1.5 and 1.6)

> *If we discard the region close to the boundary, each – is surrounded by –, and each + by +. This corresponds to a stable system. Slight changes in initial conditions do not alter the result.* (Prigogine with Stengers ibid, pp 35–6).

Then they turn to the diagram in Fig. 4.20 *Right* and continue (ibid, p 36):

> [In Fig. 20 *Right*], *instead, each + is surrounded by –, and visa versa. The slightest change in initial conditions is amplified, and the system is therefore unstable.*

A similar situation emerged from our cellular automata urban simulation model called *City* (Portugali 2000, Chap. 5). Here we've simulated a scenario by which agents of two cultural groups (Greens and Blues) come to a city when each agent has a tendency to reside among its own people – Green agents among Greens and Blue agent among Blues. At the beginning the city is highly unstable/chaotic and sensitive to initial conditions (as in Fig. 4.21 *top, left* and Fig. 4.22), but then as a consequence of this sensitivity, one can observe a process of self-organization that ends with a highly stable urban landscape with Greens and Blues segregated in different regions of the urban landscape (as in Fig. 4.21 *bottom, right*, Fig. 4.22 and Fig. 4.23 *top*).

However, there is an important difference between Prigogine with Stengers' example and ours: In the case of Fig. 4.20 Prigogine with Stengers (1977, p 35) ask

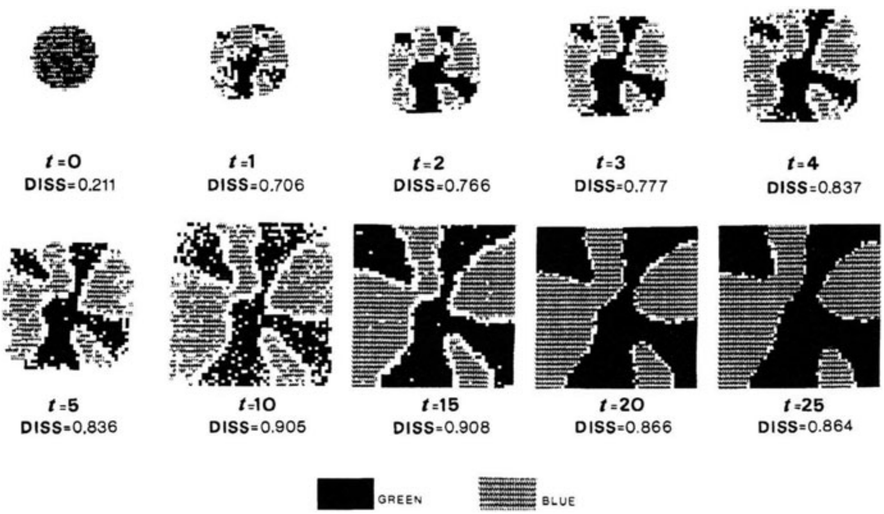

Fig. 4.21 Time evolution of segregation in a city with two cultural groups (Portugali 2000, Fig. 5.2)

us to "discard the region close to the boundary" (between the homogeneous – and + areas). In our case the boundary is an emergent property of the dynamics. Furthermore, when we zoom-in into the boundary we see that it remains chaotic (Figs. 4.22, 4.23), that is, the boundary is a dynamic entity that is constantly changing and moving. Our interpretation is that this chaotic boundary is necessary in order to keep the rest of the city stable. It is as if the city maintains its global structure by socio-spatially imprisoning local chaotic elements that threaten its global stability. We have termed this phenomenon *the captivity principle* and suggested that it might play a supplementary role to Haken's slaving principle (Portugali 2000, Chap. 5.8; Haken and Portugali, in preparation).

The play between chaos and order might show up not only in the long-term evolution of the city, but also in its daily routines. The movements of cars on the roads, of pedestrians on pavements and the like, are characterized by shifts between instable and stable motions and as such have been studied by reference to chaos theory.

4.3.2 Fractal Cities

Fractal Cities is the title of Batty and Longley's (1994) book in which they show how Mandelbrot's (1983) theory of fractal geometry can be applied to the study of cities, their structure and evolution. In that book, as part of their contribution to the issue, they have also summarized the literature on cities and fractals. Since it first appeared, there have been may applications that are discussed in Batty's (2005)

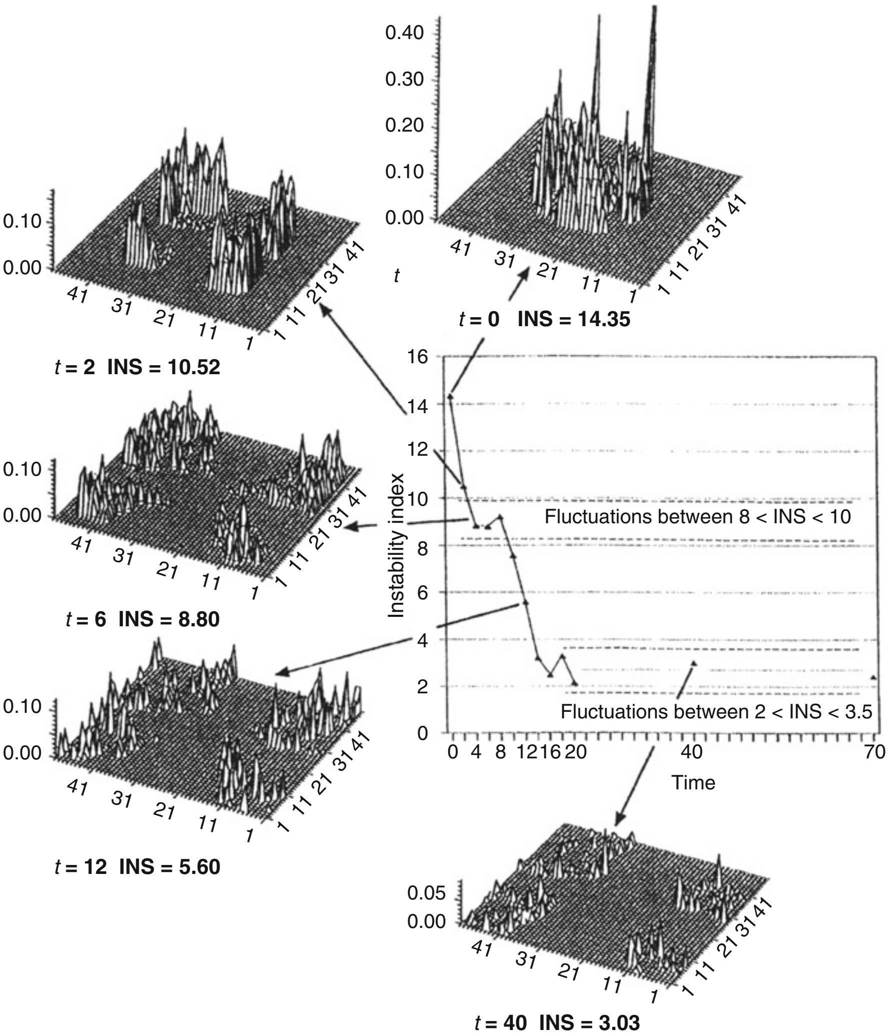

Fig. 4.22 Time evolution of SIS (stability-instability surface) in a city with two cultural groups when 33% of one of the groups (Greens) are neutrals, that is, indifferent as to their neighbors (Portugali ibid Fig. 5.8)

recent book in which fractals are seen as an important medium in the understanding of *Cities and Complexity*.

Mandelbrot's theory – to my mind, visually the most beautiful complexity theory – is based on two interrelated notions known as *self-similarity* and the *fractal dimension*, and, on the idea that a rather simple iterative process might produce highly complex geometrical forms. The notion of self-similarity has a long history that goes back to Leibnitz in the 17th century who discussed

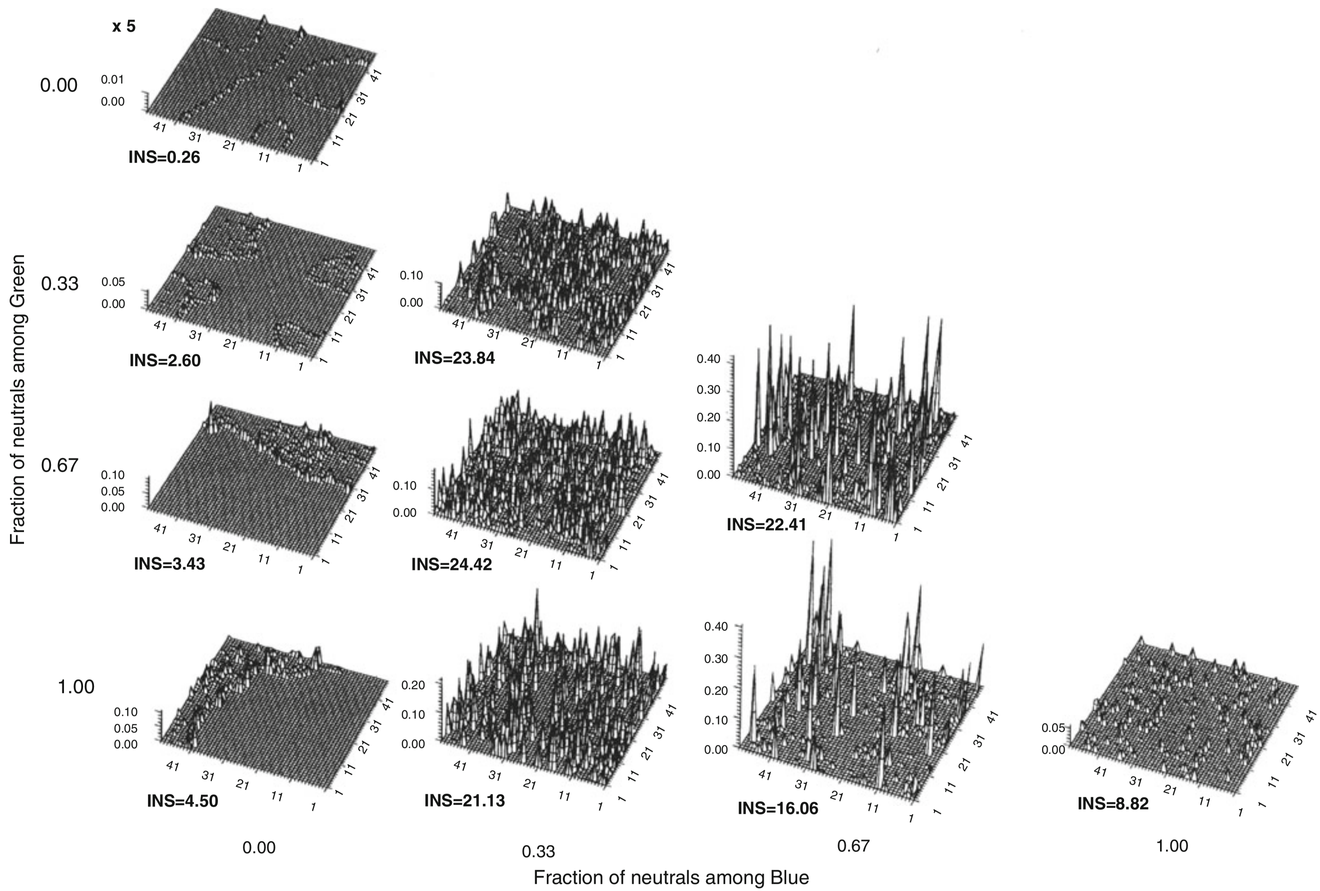

Fig. 4.23 Time evolution of SIS in a city with two cultural groups when the fraction of neutrals in one of the groups (Greens) is growing from zero to 33%, 67%, and 100%. (Portugali ibid Fig. 5.6)

recursive self-similarity. Some famous landmarks along the way are the *Cantor set* from 1883, *Koch curve* or *Koch snowflake* from 1904 (Fig. 4.24), *Sierpinski triangle* and carpet from 1915, and *Levy C curve* from 1938. In 1960 Mandelbrot explored self-similarity in a famous paper "How long is the coast of Britain: Statistical self-similarity and fractional dimension" and in 1975 he coined the notion *Fractal*. While Mandelbrot came to the idea of fractals by studying market behavior (Barcellos 1984), his theory of fractals became famous and popular after he published in 1982, *The Fractal Geometry of Nature*. In the 1980s, he also introduced the so-called *Mandelbrot set* that appeared on the cover page of *Scientific American* from August 1985 and became the icon for the whole theory (Fig. 4.24).

The notion of fractal dimension is somewhat counter-intuitive – it says that fractals do not have the conventional dimension of 0 (point), 1 (line), 2 (plane), 3 (cube) but rather broken dimensions such as 0.35 (an object that is more than a point but less than a line), or 1.6 (more than a line but less than a plane), and so on. This notion follows directly from the property of self-similarity: Thus the fractal dimension of the Koch curve, for instance, is 1.26. A nice illustration to the usefulness of the broken dimension is Mandelbrot's (1967) paper "How long is the coast of Britain?" mentioned above. The answer: its length is infinite because due to self-similarity the finer the scale of the measuring device the longer becomes the line. How then can we compare the coast of Britain to that of, say, Israel? By their fractal dimension: Both are fractals whose fractal dimension is more than a line but less than a plane, however, the coastline of Britain is more indented than that of Israel and therefore its fractal dimension will be closer to 2.

The development of Fractal geometry was associated with the development of computers and the possibility they offered to simulate sequential iterative processes; so much so that some observers have referred to fractals as computer-made artifacts. And indeed, all computer-made fractals mentioned above were created by such an iterative process in which simple relationships and rules gave rise to complex forms. The bold claim of Mandelbrot was that these computer simulations authentically

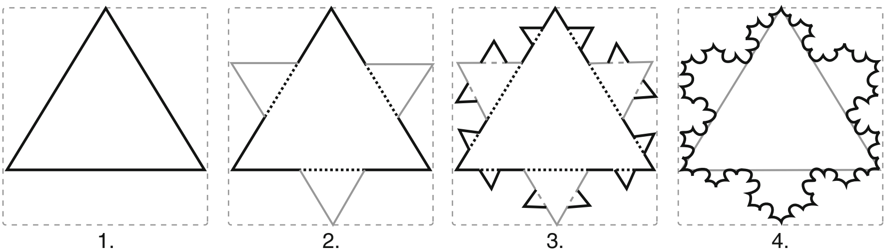

Fig. 4.24 *Koch snowflake*. First described by Swedish mathematician Niels Fabian Helge von Koch in 1904, the building of this fractal starts with an equilateral triangle and continues with the removal of the inner third of each side, building another equilateral triangle at the location where the side was removed, and then repeating this iterative process indefinitely

mimic the way nature produces its own complex, self-similar forms such as trees, leaves, coastlines, mountains or lakes.

As shown by Batty and Longley (ibid) in great details, the above properties of self-similarity, broken dimension and iterative processes together with their corollaries such as the power law size distribution, were for years implicit in the study of cities: In the theoretical central place systems of Christaller and Lösch, in the size distribution of systems of cities of Auerbach, Zipf and others, in studies about the morphology of cities and more. In their book, Batty and Longley have explicated these properties and added to them their own new studies and other studies such as Beguigui (1995) about the fractal structure of Paris' Metro/train system and Frankhouser (1994) about the fractality of urban structures. In their book they introduce and elaborate the various models by which fractal structures can be generated and simulated and show how such models can, on the one hand, simulate the growth of a tree, while on the other, the urban growth of a city and/or a metropolitan area (Fig. 4.25).

As a theory about complex self-organizing systems Fractal geometry is closely related to chaos theory. As we've seen above (Fig. 4.14), the so-called *chaos game* gives rise to the fractal structure of a leaf. As shown by several authors (e.g., Mandelbrot, ibid), an important property of the Boltzmann's strange attractor is that the trajectories of its many parts are self-similar and can be described by a fractal whose dimension is between 2 and 3. More generally, some of the attractors or order parameters, which govern a self-organized system in its steady-state, might be

Fig. 4.25 The DLA (Diffusion limited aggregation) model that is often used to simulate a plant like fractal (*left*), enabled Batty and Longley (1994, Fig. 7.16) to simulate the evolving morphology of the town of Tauton in Somerset, South West England

fractals. That is to say, they have a fractal dimension, and they generate complex, self-similar shapes by means of simple iterative rules.

The above property is significant to our intuitive understanding of the meaning of steady-state and order parameters in self-organizing cities: To say that a city is in a steady state and that it is governed by one or more order parameters does not mean equilibrium and stability, as is the case of Christaller's and Lösch's central place theories, for example, but rather a rich and complex evolution and change according to a given ordering principle. A case in point is the paper by Benguigui et al. (2000): Studying the morphological evolution of the Tel Aviv metropolitan region from 1931 to 1991 they show how this metropolitan area grew first to the north with a fractal dimension of about 1.5 to 1.7 and then from 1985 onwards also to the south with a fractal dimension of 1.667 (Fig. 4.26).

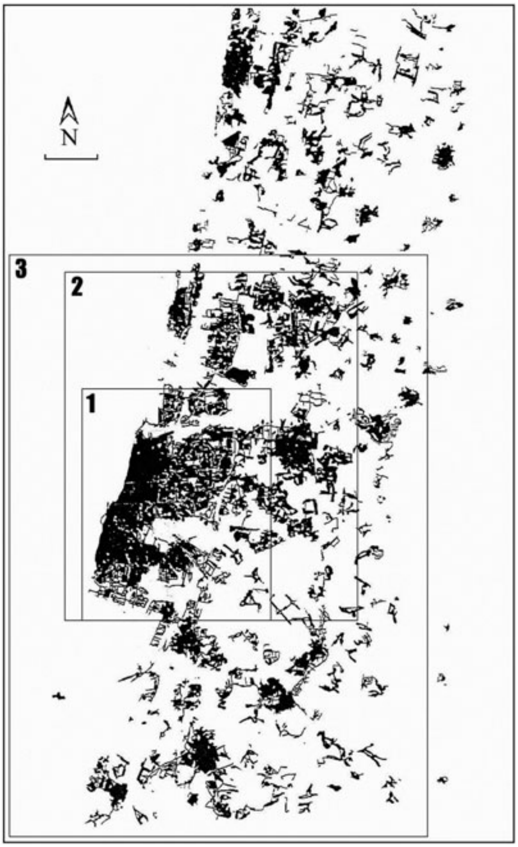

Fig. 4.26 Map of the Tel Aviv metropolitan area divided into three study regions: 1- central part; 2 - northern part; 3 - entire ensemble. From 1931 to 1991 this metropolitan region evolved, morphlogically, as a fractal structure. The central parts 1 and 2 were fractal during the entire period, while their fractal dimension increased with time. The entire metropolitan area became fractal only after 1985. In 1991 the fractal dimension of the Tel Aviv metropolitan area was found to be 1.667 with error of 0.037 (*Source*: Benguigui et al. 2000, Fig. 5)

4.4 Complexity Models of Cities

4.4.1 Cellular Automata and Agent-Based Cities

A standard two-dimensional cellular automata (CA) model is a lattice of cells, where each individual cell can be in one of several possible states (empty, occupied, etc.) and have one out of several possible properties (developed, underdeveloped, poor, rich, and on the like). The dynamics of the model is generated by an iterative process in which for every iteration the state of each cell is determined anew by some transformation rule(s). The rules are local and they refer to the relations between the cell and its nearest neighbors. The name of the game is to see how, what, and in what circumstances, local interrelations and interactions between cells entail global structures, behaviors and properties of the system as a whole. Fig. 4.27 is an illustration.

Agent Base (AB) models can be seen as an extension and elaboration of CA. In place of, or in addition to, the cells of CA, in AB simulation models the focus is on the agents: Each agent is seen as a decision maker that behaves, takes decisions and interacts with other agents and the environment according to a set of pre-determined rules of the game (similar in nature to the transformation rules of CA). However, unlike cells that cannot move and thus have local relations only, agents can move, see and know beyond their local neighboring areas. They thus have nonlocal mezzo and/or global relations that are determined by the above-noted rules of the game. These rules might include also feedback rules that affect the further behavior and action of the agents. This latter property allows agents to “learn”, “change their mind” and behavior and thus adapt to changing social, cultural and/or environmental conditions; Fig. 4.28 is an illustration.

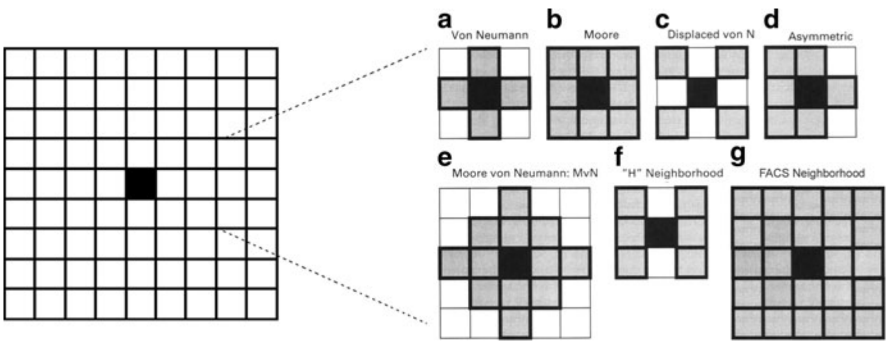

Fig. 4.27 The dynamics of the CA model is generated by an iterative process in which for every iteration the state of each cell is determined anew by transformation rule(s) that refer to the relations between the cell and its nearest neighbors (*left*). Different types of “neighbors” have been, and can be, employed (*right*). Neighbors’ configurations **a-c** and **e** were named after their “inventors” (e.g., von Neuman 1951, 1961, 1966; Moore 1970, or combinations thereof – M&N), while configurations **d**, **f**, after their shape. The FACS neighbors (**g**) were used by Portugali and coworkers in their FACS USM (Portugali 2000, Part II and Sect. 4.4.2 below)

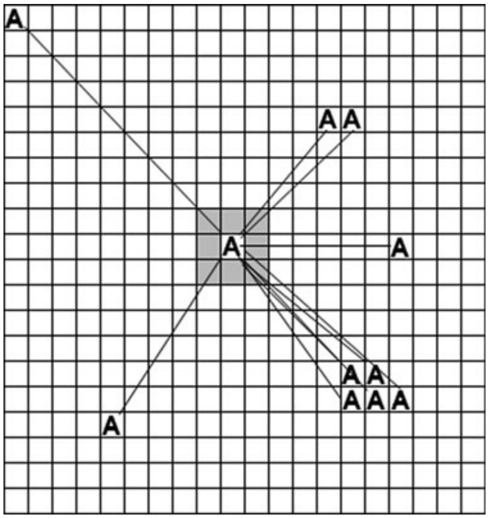

Fig. 4.28 The dynamics of the AB model is generated by an iterative process in which, for every iteration, the state and properties of every agent are determined anew by transformation rule(s) that refer to the relations between the agent and other agents and the properties of the landscape

The origin of CA and AB models goes back to Alan Turing and his ideas concerning self-reproducing machines, to John von Neumann‘s elegant demonstration that such machines, or *automata*, are in principle possible, and then to John Conway‘s *game of life* which was an explicit CA game (Gardner 1971). As it turned out, simple CA and AB games are capable of generating very complex global structures and behaviors – a property which made this kind of models a very attractive research tool.

An early, pre-computer, version of agent-based models (that are directly related to cities) was suggested by Thomas Shelling (1971) in his paper "Dynamic Models of Segregation". More recently, CA and AB models have been used quite extensively to simulate and study complex systems and processes of self-organization in a variety of domains. For example, Manneville et. al. (1989) in physics with respect to fluid dynamics and turbulence theory, Demongeot et. al. (1985) in the domain of neurobiology and computer sciences, Langton (1986) in the study of Artificial Life. From a wider perspective it is important to mention Eigen and Winkler‘s (1983) book *Laws of the Game* and Wolfram‘s (2002) *A New Kind of Science* (see also Toffoli and Margolus 1987).

The attractivity of CA and AB models to the study of self-organization stems from the fact that the more conventional tendency to use differential equations in models of self-organization, often makes computation

> *very tedious .. In this situation (especially if we are interested mainly in qualitative results) we can abstain from a numerical integration of exact differential equations and turn to the analysis of much simpler systems represented by networks of cellular automata* (Mikhailov 1990, p 40)

The main difference between studies of self-organization by means of differential equations and by means of CA/AB, is that in the first, the basic concepts of self-organization, such as attractors or order parameters, are explicit mathematical elements in the models, whereas in the second case they are implied or derived from the simulation as its interpretation concepts: you set up a CA/AB game, observe its evolving scenario, and then, post factum you derive a phase-space that describes the evolution of an attractor or an order parameter.

The attractiveness of CA models to the study of cities is almost self-evident. Similarly to cities that are built of discrete spatial units such as houses, lots, city-blocks and the like, CA models are built of discrete spatial units – the cells. In real cities the properties of local spatial units (e.g., land value) are determined, to a large extent, in relation to their immediate neighbors; so are the properties of the cells in CA models. These resemblances make CA models, intuitively and mathematically, natural tools to simulate urban processes. However, the dynamics of cities is dominated not only by local relations between its infrastructural physical elements but also by local and nonlocal relations between the many *agents* that are active in the city; that is, human individuals, families, households, firms and public agencies. It is here that *Agent Based* simulation models come in; their aim is to mimic the behavior and action of the many urban agents.

In the last decade CA and AB urban simulation models have become the most dominant media to study cities as complex, self-organizing systems. This shows up in the subtitle of Batty‘s (2005) book *Cities and Complexity: Understanding cities with cellular automata, agent-based models and fractals* (where the edition of "fractals" reflects Batty‘s personal taste). The same applies to Benenson and Torrens‘ (2004) book *Geosimulation: Automata based modelling of urban phenomena*. The main body of the two books is devoted to the various uses of CA and AB models in simulating the many facets of urban dynamics such as land-use, social and cultural segregation, urban morphology, urban spatial economy, movement in cities and more. To some extent *Self-Organization and the City* (Portugali 2000) belongs to this group too as Part II of the book that forms its core presents a family of CA and AB urban simulation models. However, it differs from this group in that the book as a whole and its models make explicit links to social theory and cognitive elements of human behavior in cities. They are described in the next section.

An example of a CA urban simulation model has already been introduced above in the discussion about deterministic chaos in cities (Sect. 4.3.1.3, Figs. 4.21–4.23). Despite its simplicity (or maybe because of that) this model has produced some important insights about the dynamics of cities. As we‘ve seen above, it has exposed the role of (captive) chaos in keeping the city in a steady state. Another interesting outcome of this model concerns the non-correspondence between the global properties of the city as a whole and the local-"personal" properties and tendencies of single urban agents. As illustrated in Fig. 4.29, at the beginning we‘ve run the model with agents belonging to two cultural groups – greens and blues – when all agents are *segregatives*, that is, prefer to live among their own kind; blue agents in blue neighborhood and green in their neighborhoods. But then we introduced neutrals – agents that are indifferent as to their neighbors and we run the model several times with increasing proportion of neutrals. The interesting outcome and insight was that the city remains highly segregative in face of increasing proportions of neutrals with the implication that a small number of segregatives is sufficient to turn the whole city

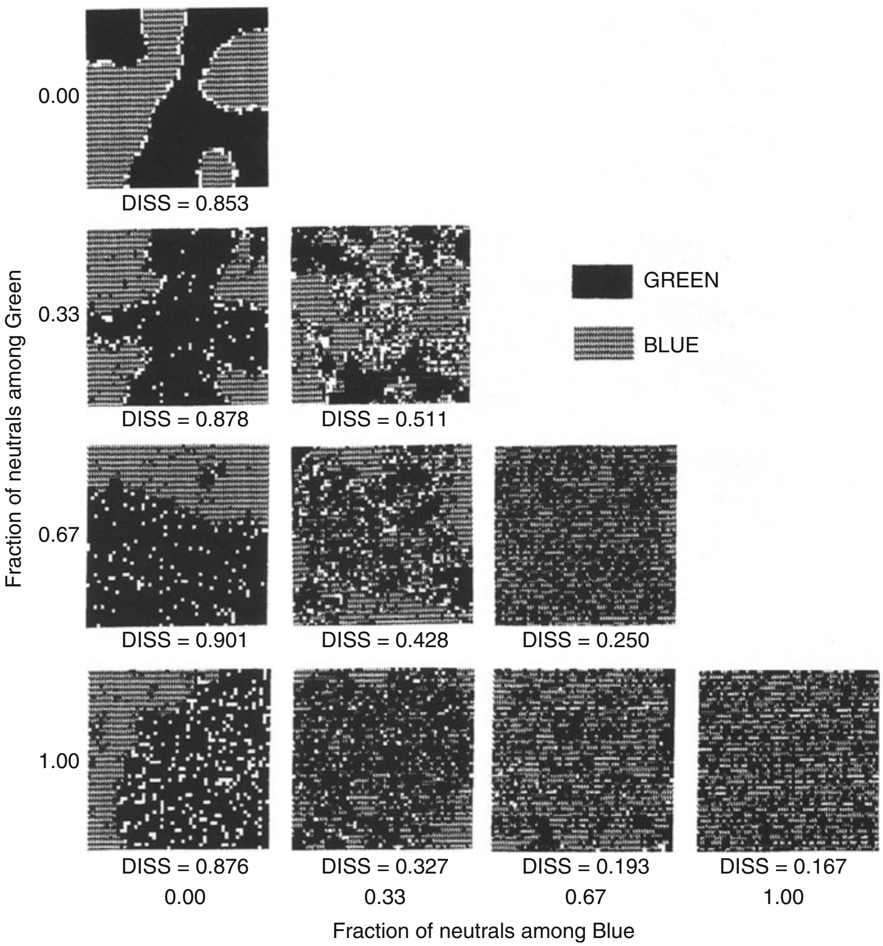

Fig. 4.29 Spatial distribution of two cultural groups with increasing proportions of neutrals and segregatives in both groups (Portugali 2000, Fig. 5.3)

into a segregative city – a finding that is in line with Shelling's (1971) segregation model. The next section introduces a special kind of CA/AB urban simulation modes, whereas in Part IV below two of these models are described in details. For further discussion and information on CA and AB models see the above books by Batty (2004) and Benenson and Torrens (2004).

4.4.2 FACS (Free Agents on a Cellular Space) Cities

FACS – free agents on a cellular space – is a family of simulation models specifically designed to deal with urban dynamics in general and with social and cultural urban segregation in particular. Their central idea is that looking at three sets of relationships can capture the essence of urban dynamics: the interrelationships between infrastructural urban elements such as buildings, parks, roads, etc.; the interrelationships between the various urban agents, and, the interrelationships between urban agents and urban elements.

Computationally – or model-wise, FACS models are built as a superposition between two layers corresponding to two kinds of models: AB and CA. That is an infrastructure layer, which is a usual CA urban simulation model with its 2-dimensional cellular space, and on top, a superstructure layer of individual free agents (Fig. 4.30). They are 'free agents' in that they can move from one cell to the other all over the "city", they have past, they have plans for the future, and they act intentionally; they are capable of learning and can thus change their "mind", action and behavior; they can see beyond local situations, their seeing is subjective and is captured by the notion of cognitive maps. The latter determine their actions and behavior in the city. In short, each free agent is a self-organizing system, each is a virtual human individual, family, firm, planning team and the like.

The link between the two layers is created, on the one hand, by the fact that the properties of each of the cells in the CA is determined by the properties of the agent that occupies it, while on the other, by the fact that the agents are capable of learning. That is, agents constantly observe the state of their neighboring cells and agents, as well as the state of the city as a whole, and evaluate the validity of their internally represented urban perceptions and behavioral tendencies in light of

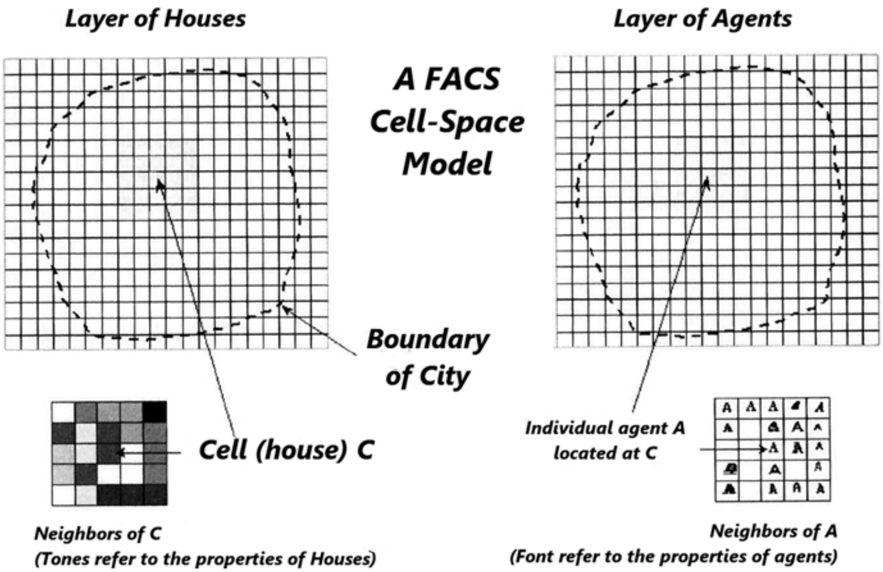

Fig. 4.30 A typical FACS model is constructed of two-layers: an AB population layer of human agents describing the migratory and interaction activities of individuals (*right*), superimposed on a CA infrastructure describing the urban landscape (*left*). (Portugali 2000, Fig. 4.6)

this externally observed situation; and if the dissonance between the externally represented information as it comes from the city and the internally represented information crosses a certain threshold, they change their minds and their corresponding behavioral patterns. The theory behind this kind of modeling is termed SIRN (synergetic inter-representation networks) and it is introduced in some detail in Part II, specifically in Chap. 7. The essence of the SIRN process and FACS models is a circular causation between two-scales self-organizing systems, forming a single network of internal and external representations: the individual free agents determine the city which can thus be seen as the external representation of their actions and behavior; and the city in its turn determines the internal representations (e.g., cognitive maps) of individuals and through these their action and behavior in the city, in a circular causality.

In Part IV, we use FACS models to study how, by means of self-organization, the city dynamics entails the emergence of a new urban cultural group – a phenomenon that is typical of current postmodern and hypermodern cities. We start the game with two groups of individuals, Greens and Blues, belonging to two cultural groups. They enter the city as immigrants, intending to find a proper location in it, where the Greens' intention is to live among greens, and the Blues, among blues. If such a free agent finds a satisfying location, it will live there and will become an inhabitant, if not, it will try to move to another location in the city, thus participating in creating the city's intra-urban migration. Some agents who cannot find a location according to their intentions, leave the city and thus create its outmigration, and some get stuck in a location that they do not like. Because of various systemic situations they cannot migrate and are thus enforced to behave counter their intentions. The outcome is that they enter a situation of *spatial cognitive dissonance*. With time, in order to resolve this cognitive conflict, some might change their intention, and in certain situations this change of intention gives birth to a new cultural identity in the city.

What we try to achieve by this new kind of modeling is to be able to examine, simultaneously, self-organization at the local level of the individual, and self-organization at the global level of the city: to see how the city dynamics might create a self-organization process at the individual level, and how the latter might entail self-organization at the city level. This approach departs from the usual procedure in self-organization studies which tend to study macro-scale city self-organization by ignoring micro-scale self-organization at the level of the individual, and to study micro-scale cases, as in cognitive studies concerning the behavior of individuals in the city, by ignoring the global self-organization process of the city, or by assuming it as fixed.

4.4.3 Small World Cities

In 1967, social psychologist Stanley Milgram published a paper entitled "the small world problem" in the popular magazine *Psychology Today* (Milgram 1967). Two years later he published (with Travers) a rigorous paper on this issue in the journal

Sociometry (Travers and Milgram 1969). In these papers Milgram presented results from a set of experiments he conducted on “the small world problem”, namely, on the probability that two randomly selected people in the United States would know each other. The basic assumption was that the population of the US forms a social network and the aim of the experiments was to count the number of ties needed to connect any two people. In more formal language the aim was to find the average *path length* between any two nodes in the network. Milgram has found that the average path length is about six.

Milgram was not the first to study this issue. In fact he started his experiments in the US after coming from Paris where he was working with other scholars on this very issue. Also, the notion of *six degrees of separation*, which is often attributed to Milgram was in fact coined by the American playwrite John Gare.

For several decades Milgram‘s experimental results had the fate of most scientific studies, namely, they attracted a small community of scientists interested in this topic. However, at the end of the 1990s the small world phenomenon became popular again when a link was made between graph theory and the study of complex systems and networks. From this conjunctional perspective Watts and Strogatz (1998) demonstrated that complex networks have small world characteristics (Fig. 4.31) while Barabasi and Albert (1999) demonstrated that, depending on their underlying construction or growth principles, complex networks can be *scale free* thus following the *power law* (Fig. 4.32). These pioneering studies of Watts and Barabasi entailed an interdisciplinary wave of a large number of papers and several books in physics, mathematics, computer science, biology, economics, and sociology (Barabasi 2002). The result of this wave became known as the *New Science of Networks* (Watts 2004; Newman et al. 2006; Barrat et al. 2008).

The notion *network* is implicit in all theories of complexity. What Watts, Barabasi and the others did in their new science of networks was to explicate this link. The new science of networks thus became a new approach to, or a theory of, complexity. In particular, Barabasi has demonstrated that the scale-free property of complex systems and the entailed power law distribution are markers of the process of self-organization that typifies complex systems.

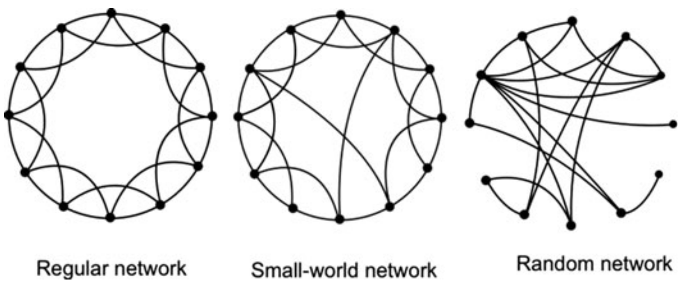

Fig. 4.31 A small world network (*center*) as a superposition of a regular (*left*) and random (*right*) networks

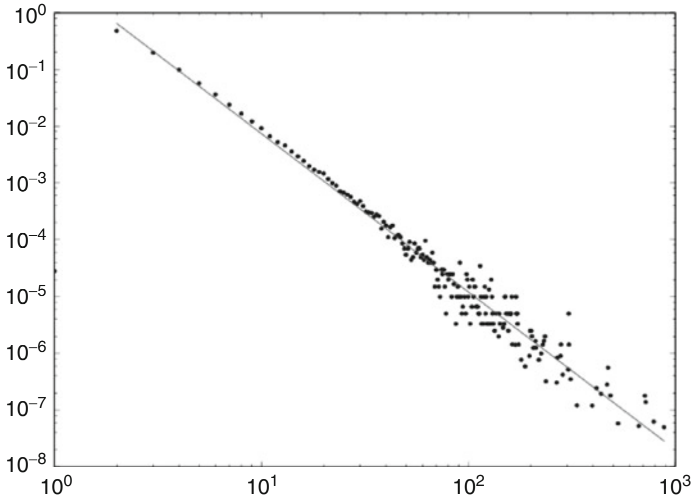

Fig. 4.32 Complex networks are *scale free* in the sense that the size distribution of their vertices and hubs follows the *power law*

The rank-size, scale-free and power law reminds one of the long history of looking at cities in these terms and of the fractal cities described above. It was therefore just a matter of time until the link to the study of cities would be made. This link was first made by Batty (2001) in an editorial entitled "Cities as small world". Batty‘s editorial note was followed by a large number of studies that applied the new science of networks to a variety of urban domains. Thus, in the domain of transportation one can mention Bin Jiang‘s (2006, 2007) studies that characterize roads‘ traffic dynamics in Gävle, Sweden in terms of scale-free networks; the same was found for the transit system in Beijing (Wu et al. 2007), pedestrian movement (Jiang 2006) and for the canal networks of Venice (Blanchard and Volchenkov 2007). Andersson et al. (2003) showed that the market dynamics generates land values that can be represented as a growing scale-free network. Finally, Batty (2005) in his *Cities and Complexity* has suggested viewing cities and their dynamics from the integrative perspectives of networks, fractals, self-organized criticality and AB modeling.

Viewing cities as networks reminds one also of Alexander‘s (1965) classic "a city is not a tree", that demonstrated that cities are typified not by a simple *tree network*, but rather by a complex *semi-lattice network* (Fig. 4.33). Alexander‘s view was recently reformulated by Salingaros (2005, 2006) in terms of the new science of networks. Another example is Hillier‘s (1999; Hillier and Hanson 1984) *space syntax* that analyzes the morphology of urban spaces in terms of networks. Space syntax exposes the way society determines the urban morphology and the way the latter feeds back and re-shapes society. The link between space syntax and network analysis has already produced several useful results (e.g., Hillier and Lida 2005).

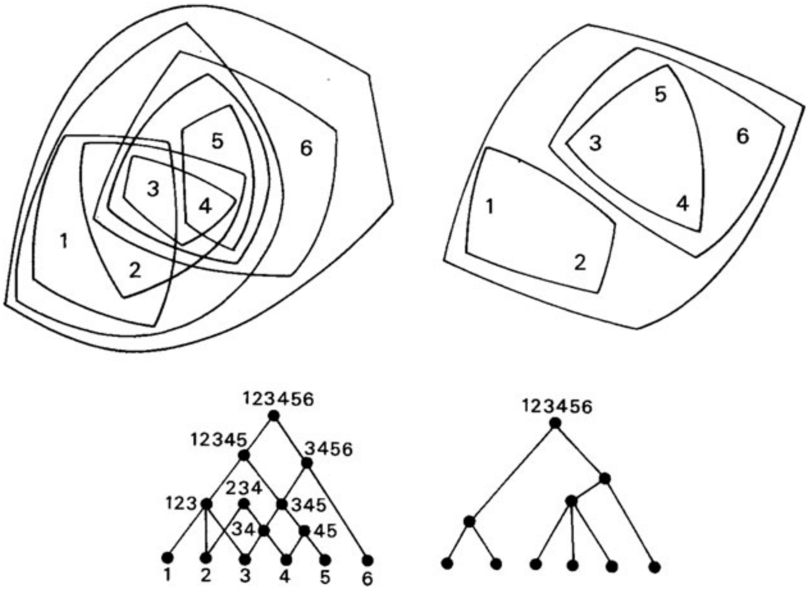

Fig. 4.33 The distinction between a *tree* structure (*right*) and a *semi-lattice* structure (*left*) according to Alexander (1965)

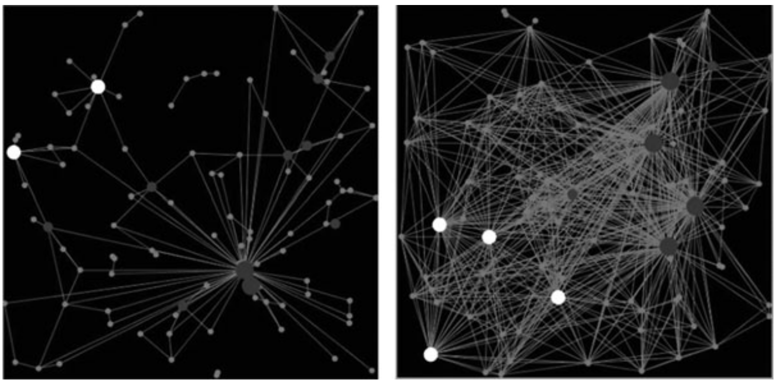

Fig. 4.34 Two examples for urban commuting networks simulated by the model. The sizes and colors of the nodes represent different sizes of urban economic centers

In a recent study Blumenfeld and Portugali (2010) have devised a network simulation model that is built as a superposition between the AB urban simulation model and a network model. A typical scenario of the model starts with a set of spatially independent nodes that represent cities in a region, for instance. The novel property of the model is that the probability for interaction between the nodes follows the logic of the gravity-interaction model, namely, it is directly related to the size of the nodes and inversely to the distance that separate them. Interaction might refer to commuters, and/or migration flows between the cities. When the level of interaction crosses a certain threshold, a link between the nodes is created.

Employing the model Blumenfeld and Portugali simulate and study the evolution and dynamics of urban networks and their different scales. They begin with networks in their intra-city level (i.e. within the city itself) and follow their evolution until they exist in the intercity (i.e. between cities). They show several scenarios that correspond nicely to size distribution of urban networks as observed in reality. Figure 4.34 illustrates two snapshots from the evolving urban landscape.

4.5 Concluding Notes: CTC – First, Second, or Third Culture of Cities?

CTC as we‘ve just seen, having originated in the sciences and by scientists, were introduced to cities by physicists such as Allen and Weidlich and at a later stage were welcomed by quantitative regional and urban scientists. Some thus see CTC as the new science of cities – as a new and more sophisticated version of the first culture of cities. On the other hand, as indicated above, complexity theories themselves form a new science or a new scientific approach, among other things because they have found in matter properties hitherto assigned to life, humans and humanity such as history, evolution, unpredictability, irreversibility, nonlinearity, uncertainty and the like (Portugali 1985). This is so also with respect to CTC and the second science of cities, namely, SMH and PPD cities. As I‘ll elaborate below, both domains are critical of the first science of cities and both share similar views as to the dynamics of cities; and there are differences of course. Based on the explicit links between CTC and the first science of cities and the implicit and subtle connections between CTC and the second culture of cities, my suggestion is that CTC has the potential to become a third culture of cities that bridges the gap between the two cultures of cities and reconciles their seemingly irreconcilable standpoints. This option will be discussed in some length below.