Chapter 5
Bed-Load Transport

5.1 General

The transport of sediment in rivers, by which the river morphological changes are closely related, is an important aspect in fluvial processes. The term *load*, as often used to define the sediment transport, refers to the quantity of sediment that is transported in a stream. More specifically, it is used to define the rate (volume or weight per unit time and width) at which the sediment is transported.

When the bed shear stress τ_0 induced by the flow exceeds the threshold bed shear stress τ_{0c} for the initiation of sediment motion, the sediment particles forming the bed are set in motion. The *bed-load transport* is the mode of sediment transport where the sediment particles slide, roll, or travel in succession of low jumps, termed *saltation*, but belong close to the bed, from where they may leave temporarily. The dislodgment of the sediment particles is rather intermittent, as turbulence (velocity fluctuations) interacts with the bed particles randomly to play an important role in transporting them. It is, however, convenient to distinguish the modes of sediment transport as *bed load* (*slide*, *roll*, and *saltation*) and *suspended load*. Figure 5.1 presents a schematic of different modes of sediment transport.

At relatively small *excess bed shear stress* ($\tau_0 - \tau_{0c}$), the bed-load transport takes place in a sliding and/or rolling mode. It therefore describes a sediment motion generally in contact with the bed; while individual sediment particles have intermittent motion, but substantially continuous. The bed-load transport in this mode is known as *contact load*. With an increase in excess bed shear stress, increasingly sediment particles are driven streamwise in a short succession of jumping or bouncing mode of motion, as the particles lose contact with the bed for a short while to attain a mean height in water of a number of particle diameters. The bed-load transport in this mode is called *saltation*. According to Einstein (1942, 1950), the bed-load transport is defined as the transport of sediment particles within a thin layer having a thickness of two particle diameters above the bed by sliding, rolling, or traveling in succession of jumps with a streamwise distance of a few particle diameters. On the other hand, Bagnold (1956) defined the

S. Dey, *Fluvial Hydrodynamics*, GeoPlanet: Earth and Planetary Sciences,
DOI: 10.1007/978-3-642-19062-9_5,

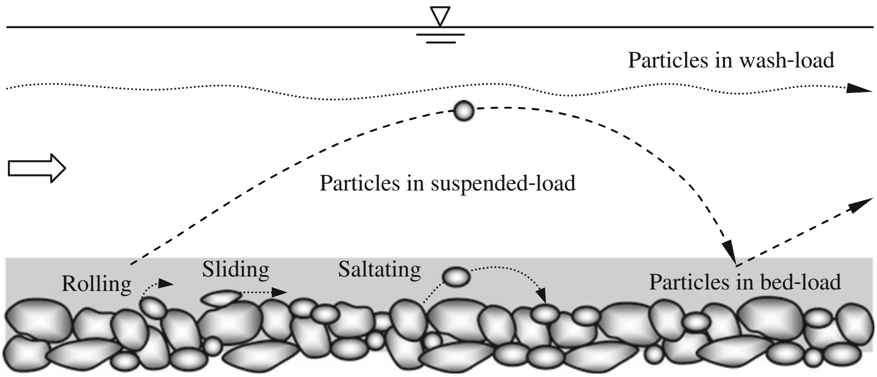

Fig. 5.1 Schematic of different modes of sediment transport

bed-load transport that takes place by successive contacts of the particles with the bed being limited by the gravity effect.

With a further increase in excess bed shear stress, the production of turbulence near the bed and its diffusion in upward direction lift up relatively finer sediment particles from the bed keeping them in suspension, as they are transported by the flow. The upward diffusion of turbulence retains the particles in the fluid domain against the gravity; while relatively coarser particles are still transported as bed load. In reality, the particles stay occasionally in contact with the bed and are displaced by making more or less large jumps to remain often surrounded by the fluid. The sediment transport in suspension mode is termed *suspended load*. Bagnold (1956) defined the suspended-load transport that takes place by balancing submerged weight of the particles with upward diffusion of turbulent eddies. In both bed-load and suspended-load transports, the sediment transport is established by the action of gravity on the fluid phase driving the sediment particles by the induced drag.

It is useful to provide approximate limiting values to separate different modes of sediment transport:

$$6 > w_s/u_* \geq 2 \text{ contact-load, bed-load} \tag{5.1a}$$

$$2 > w_s/u_* \geq 0.6 \text{ saltation, bed-load} \tag{5.1b}$$

$$0.6 > w_s/u_* \text{ suspended-load} \tag{5.1c}$$

where u_* is the shear velocity and w_s is the settling or terminal velocity of particles. Generally, the amount of bed load transported through a large deep river is approximately 5–25 % of the suspended load.

In natural stream, *wash load* is the portion of sediment that is carried by the flow such that it always remains close to the free surface. It is in near-permanent suspension and transported without deposition, essentially passing straight through the stream. It consists of very fine sediment particles, such as silt and clay. The composition of wash load is distinct because it is almost entirely made up of particles that are only found in small quantities in the bed. Nevertheless, wash-load particles are also brought in by the overland flow or from the cohesive stream banks. As the wash-load particles tend to be very fine, they have a small settling velocity, being easily kept in suspension by the turbulence in flow. A physical characterization of the wash load is a difficult proposition, as the wash load, by definition, cannot be determined by the given flow characteristics of a river.

5.2 Definition of Bed-Load Transport

The term *bed-load transport* is defined as the sediment particles, such as silt, sand, gravel, etc., carried by the stream flow in the streamwise direction immediately above the bed as sliding, rolling, and/or saltating at a velocity less than that of the stream flow. The bed-load transport rate q_b is generally expressed as the solid volume of sediment transported per unit time and width. It is also expressed as the weight of sediment transported per unit time and width, denoted by g_b, or the submerged weight of sediment transported per unit time and width, denoted by g_{bs}. However, in nondimensional form, the bed-load transport rate is designated as *bed-load transport intensity* and denoted by Φ_b. The bed-load transport intensity Φ_b is related with q_b, g_b, and g_{bs} as follows:

$$\Phi_b = \frac{q_b}{(\Delta g d^3)^{0.5}} = \frac{g_b}{\rho_s g (\Delta g d^3)^{0.5}} = \frac{g_{bs}}{\Delta \rho g (\Delta g d^3)^{0.5}} \tag{5.2}$$

where Δ is the submerged relative density ($= s - 1$), s is the relative density of sediment ($= \rho_s/\rho$), ρ_s is the mass density of sediment, ρ is the mass density of water, g is the acceleration due to gravity, and d is the representative sediment size, that is the median or weighted mean diameter.

The bed-load transport rate q_b can be defined as the product of the particle velocity u_b in streamwise direction, the volumetric concentration C of particles transported as bed-load, and the thickness δ_b of bed-load transport layer. It is therefore given by

$$q_b = u_b C \delta_b \tag{5.3}$$

The bed-load transport rate q_b can also be defined as the product of the particle velocity u_b in streamwise direction, the number of particles in motion N_b per unit area, and the volume of particles V_b. It is thus

$$q_b = u_b N_b V_b \tag{5.4}$$

Further, by defining the particle velocity u_b as the ratio of saltation or step length λ_s to saltation or step period t_e, that is $u_b = \lambda_s/t_e$, Eq. (5.4) can be rewritten as

$$q_b = \frac{\lambda_s}{t_e} N_b V_b = \lambda_s E_b = \lambda_s D_b \quad \wedge \quad E_b = D_b = \frac{N_b V_b}{t_e} \tag{5.5}$$

where E_b and D_b are the degraded or aggraded volume of particles per unit time and area.

Another way of defining bed-load transport rate is the *pickup rate*. It is, in fact basically, defined as the number of particles picked up per unit time and area. Later, the definition of pickup rate E_p has been modified to the mass of particles picked up per unit time and area. The nondimensional pickup rate, known as the *sediment pickup function* Φ_p, is defined according to Einstein (1950) as

$$\Phi_p = \frac{E_p}{\rho_s (\Delta g d)^{0.5}} \tag{5.6}$$

Although different researchers studied pickup rate (Einstein 1950; Fernandez Luque 1974; Yalin 1977; Nakagawa and Tsujimoto 1980; de Ruiter 1982, 1983; van Rijn 1984b; Dey and Debnath 2001), it, however, remains almost unclear whether contact load or saltation contributes to pickup rate.

5.3 Bed Shear Stress Concept for Bed-Load Transport

5.3.1 du Boys' Approach

The pioneering attempt to predict the bed-load transport rate was due to MP du Boys in 1879, who was a French engineer. His analysis was based on the force balance between the force applied to the top layer of sediment bed by the flowing fluid and the frictional resistance between the top layer of sediment particles and the layers beneath it.

du Boys (1879) assumed that the sediment particles move in series of superimposed layers of individual thickness $\Delta\varepsilon$ by the tractive force offered by the uniform flow as given by the bed shear stress $\tau_0 = \rho g h S_0$ applied to the surface of the top layer; where h is the flow depth and S_0 is the streamwise bed slope. The mean velocity of the successive layers that are sliding over each other increases linearly toward the bed surface. It implies that the velocity is highest at the top layer forming the bed surface and zero (minimum) at the lowest layer at a depth of $\Delta\varepsilon \cdot m$; where m is the number of layers. Figure 5.2 illustrates the definition sketch of du Boys model. Under the equilibrium condition, the top layer is one where the tractive force balances the frictional resistance force between these layers.

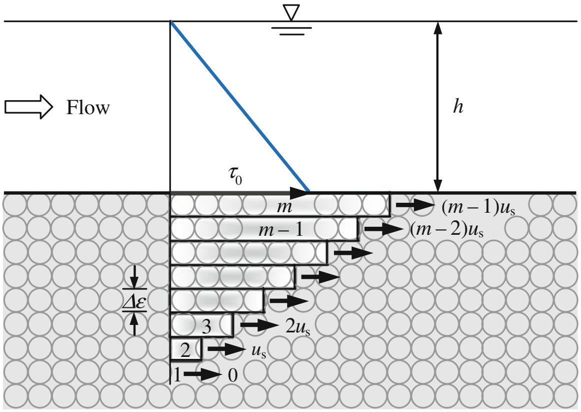

Fig. 5.2 Definition sketch of du Boys' bed-load model

The coefficient of frictional resistance μ_f between successive layers is assumed to be constant, such that the force balance is

$$\tau_0 = \rho g h S_0 = \mu_f \cdot \Delta\varepsilon \cdot m(\rho_s - \rho)g \tag{5.7}$$

The fastest moving layer being the top layer moves with a velocity of $(m - 1)u_s$, where u_s is the velocity of the second lowest layer. As the layers between the first and the m-th move according to a linear velocity distribution, the sediment transport rate (in volume per unit time and width, that is, $m^3\ s^{-1}\ m^{-1}$) is given by

$$q_b = \Delta\varepsilon \cdot m \frac{(m-1)u_s}{2} \tag{5.8}$$

The threshold condition at which sediment motion is just about to begin can be obtained by setting $m = 1$. Then, from Eq. (5.7), threshold bed shear stress τ_{0c} can be determined, and thus, m is obtained as the ratio of applied bed shear stress to threshold bed shear stress as follows:

$$\tau_{0c} = \mu_f \cdot \Delta\varepsilon(\rho_s - \rho)g \;\Rightarrow\; m = \frac{\tau_0}{\tau_{0c}} \tag{5.9}$$

It is introduced into Eq. (5.8) and then

$$q_b = \left(\frac{\Delta\varepsilon \cdot u_s}{2\tau_{0c}^2}\right)\tau_0(\tau_0 - \tau_{0c}) \tag{5.10}$$

du Boys referred the first term within the parenthesis in right-hand side of Eq. (5.10) as a characteristic of sediment coefficient and denoted by χ. Thus, the equation becomes

$$q_b = \chi \tau_0 (\tau_0 - \tau_{0c}) \tag{5.11}$$

The sediment coefficient χ was determined from the experimental data obtained by Schoklitsch (1914). According to Graf (1971), it is

$$\chi = \frac{0.54}{\Delta \rho g} \text{ (in metric units)} \tag{5.12}$$

Straub (1935) related χ with the particle size d (in mm) ($0.125 < d < 4$ mm) as

$$\chi = \frac{6.89 \times 10^{-6}}{d^{0.75}} \text{ (in SI units)} \tag{5.13}$$

5.3.2 du Boys Type Equations

du Boys equation that is characterized by the excess bed shear stress is one of the classical equations of bed-load transport. Later, investigators have tried to put forward improved version of bed-load transport equations, known as *du Boys type equations*, based on excess bed shear stress. They are discussed below:

Shields (1936) obtained the threshold bed shear stress that had a value for which the extrapolated sediment flux (bed-load transport) became zero. Therefore, he basically studied the flow conditions corresponding to the bed-load transport rate greater than zero. He obtained an empirical equation of bed load as

$$q_b = \frac{10 q S_0}{s \Delta^2 \rho g d}(\tau_0 - \tau_{0c}) \Rightarrow \Phi_b = \frac{10U}{s(\Delta g d)^{0.5}}(\Theta - \Theta_c)\Theta \tag{5.14}$$

where Θ and Θ_c are the Shields and threshold Shields parameters, respectively, q is the flow rate per unit width ($= Uh$), and U is the depth-averaged flow velocity. The Shields parameter is given by $\Theta = \tau_0/(\Delta \rho g d)$ and Θ_c corresponds to τ_{0c}.

Meyer-Peter and Müller (1948) gave the following equation of bed load including the effects of particle roughness:

$$q_b = \frac{8}{\Delta \rho^{1.5} g}\left[\left(\frac{C_R}{C'_R}\right)^{1.5} \tau_0 - \tau_{0c}\right]^{1.5} \Rightarrow \Phi_b = 8(\eta_C \Theta - \Theta_c)^{1.5} \quad \wedge \quad \eta_C = \left(\frac{C_R}{C'_R}\right)^{1.5} \tag{5.15}$$

where C_R is the total Chézy coefficient due to effective bed roughness k_s, that is $18\log(12\ h/k_s)$ or $U/(R_b S_0)^{0.5}$, R_b is the hydraulic radius, and C'_R is the Chézy coefficient due to particle roughness d_{90}, that is $18\log(12\ h/d_{90})$. In Eq. (5.15), Meyer-Peter and Müller recommended the value of $\Theta_c = 0.047$. Their formula corresponded well with the experimental data for coarse sands and gravels. The η_C was reported to vary from 0.5 to 1 that corresponds to coarse sand and a small form drag. Considering $k_s \approx d_{90}$, the η_C becomes unity; and the Meyer-Peter and Müller formula can be simplified to

$$\Phi_b = 8(\Theta - \Theta_c)^{1.5} \tag{5.16}$$

Subsequently, Frijlink (1952) proposed a formula that can approximate Meyer-Peter and Müller formula, but it is not a du Boys type equation. It is

$$\Phi_b = 5(\eta_C \Theta_c)^{0.5} \exp\left(-\frac{0.27}{\eta_C \Theta}\right) \tag{5.17}$$

However, Chien (1954) showed that the Meyer-Peter and Müller formula can be replaced by

$$\Phi_b = (4\Theta - 0.188)^{1.5} \tag{5.18}$$

Further, Wong and Parker (2006) reanalyzed the experimental data used by Meyer-Peter and Müller and found a better fitting for the Meyer-Peter and Müller formula with the following equation:

$$\Phi_b = 3.97(\Theta - 0.0495)^{1.5} \tag{5.19}$$

For the high bed-load transport rate, Wilson (1966) put forward an empirical equation as

$$\Phi_b = 12(\Theta - \Theta_c)^{1.5} \tag{5.20}$$

Chang et al. (1967) suggested that the bed-load transport can be determined from the following relationship:

$$\Phi_b = K_t \frac{\Delta}{s} \cdot \frac{U}{(\Delta g d)^{0.5}} (\Theta - \Theta_c) \quad \wedge \quad K_t = K_b \frac{s}{\Delta} \cdot \frac{1}{\tan \phi} \tag{5.21}$$

where K_b is a constant and ϕ is the angle of repose of the sediment. In the above, K_t represents a constant defining the bed-load transport and can be determined using Fig. 5.3.

Ashida and Michiue (1972) analyzed micro-mechanical particle collision with the bed, but not considered the saltation. They obtained the following equation of bed-load transport intensity for the range of particle size $0.3 \le d \le 7$ mm:

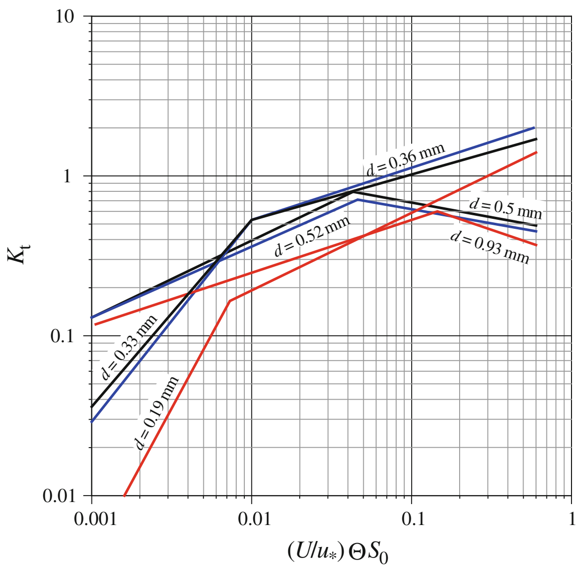

Fig. 5.3 Variation of K_t with $(U/u_*)\Theta S_0$ for different sediment sizes (Chang et al. 1967)

$$\Phi_b = 17(\Theta - \Theta_c)(\Theta^{0.5} - \Theta_c^{0.5}) \tag{5.22}$$

In the above, Ashida and Michiue recommended the value $\Theta_c = 0.05$.

Fernandez Luque and van Beek (1976) used laboratory experimental data to suggest bed-load transport intensity as

$$\Phi_b = 5.7(\Theta - \Theta_c)^{1.5} \tag{5.23}$$

They considered a range of Θ_c within $0.05 \leq \Theta_c \leq 0.058$ for $0.9 \leq d \leq 3.3$ mm.

For gravel-bed rivers, Parker (1979) proposed

$$\Phi_b = 11.2\frac{(\Theta - 0.03)^{4.5}}{\Theta^3} \tag{5.24}$$

Smart (1984) measured bed-load transport rate in steep channels ($0.03 \leq S_0 \leq 0.2$) for the gravel sizes $2 \leq d \leq 10.5$ mm. Based on his data and the data of Meyer-Peter and Müller, he proposed

$$\Phi_b = 4\frac{C_R}{g^{0.5}}\left(\frac{d_{90}}{d_{30}}\right)^{0.2} S_0^{0.6}(\Theta - \Theta_c)\Theta^{0.5} \tag{5.25}$$

The bed-load transport intensity equation derived by van Rijn (1984a) for $0.2 \leq d \leq 2$ mm is

$$\Phi_b = \frac{0.053}{D_*^{0.3}}\left(\frac{\Theta}{\Theta_c} - 1\right)^{2.1} \tag{5.26}$$

where D_* is the particle parameter, that is $d(\Delta g/\upsilon^2)^{1/3}$, and υ is the kinematic viscosity of water.

Graf and Suszka (1987) gave a bed-load transport intensity formula for steep bed slopes as

$$\Phi_b(\Phi_b \le 10^{-2}) = 10.4\left(1 - \frac{0.045}{\Theta}\right)^{2.5}\Theta^{1.5} \tag{5.27a}$$

$$\Phi_b(\Phi_b > 10^{-2}) = 10.4\Theta^{1.5} \tag{5.27b}$$

Madsen (1991) recommended

$$\Phi_b = K_b(\Theta - \Theta_c)(\Theta^{0.5} - 0.7\Theta_c^{0.5}) \tag{5.28}$$

where $K_b = 8/\tan\phi$ for sliding and rolling sand particles, and $K_b = 9.5$ for saltating sand particles in water. However, Niño and García (1998) proposed a similar equation with $K_b = 12/\mu_d$ for saltating particles. They determined a dynamic coefficient of friction $\mu_d = 0.23$.

Nielsen's (1992) equation for sand and gravel ($0.69 \le d \le 28.7$ mm) transport is

$$\Phi_b = 12(\Theta - 0.05)\Theta^{0.5} \tag{5.29}$$

Damgaard et al. (1997) conducted experiments for the wide variation of streamwise bed slope (–32° $\le \theta \le$ 32°; where θ is the streamwise bed angle with the horizontal). They introduced a correction factor f_θ to Meyer-Peter and Müller formula as

$$\Phi_b = 8(\Theta - \Theta_c)^{1.5} f_\theta \tag{5.30}$$

where

$$\begin{aligned} f_\theta(-32^\circ < \theta \le 0) &= 1 + 0.8\left(\frac{\Theta_c}{\Theta}\right)^{0.2}\left(1 - \frac{\Theta_{c\theta}}{\Theta_c}\right)^{1.5+\frac{\Theta}{\Theta_c}} \\ f_\theta(0 < \theta \le 32^\circ) &= 1 \end{aligned} \tag{5.31}$$

where $\Theta_{c\theta}$ is the threshold Shields parameter on streamwise bed slope.

Lajeunesse et al. (2010) suggested

$$\Phi_b = 10.6(\Theta - \Theta_c)(\Theta^{0.5} - \Theta_c^{0.5} + 0.025) \tag{5.32}$$

5.3.3 *Other Empirical Relationships Involving Bed Shear Stress*

Kalinske (1947) emphasized on the near-bed turbulence that plays an important role in analyzing bed particle motion. The time-averaged bed-load transport rate q_b was expressed as a product of three quantities: volume of a particle, number of particles in motion per unit area, and time-averaged particle velocity $\bar{u}_\mathrm{b}$. It is

$$q_\mathrm{b} = \frac{\pi d^3}{6} \cdot \frac{4 p_\mathrm{n}}{\pi d^2} \cdot \bar{u}_\mathrm{b} \tag{5.33}$$

where p_n is the fraction of moving particles. The time-averaged particle velocity $\bar{u}_\mathrm{b}$ can be obtained as

$$\bar{u}_\mathrm{b} = c_0 \int_{u_\mathrm{cr}}^{\infty} (u_\mathrm{d} - u_\mathrm{cr}) f(u_\mathrm{d}) \mathrm{d}u_\mathrm{d} \tag{5.34}$$

where c_0 is the constant of proportionality, u_d is the instantaneous flow velocity at the particle level, u_cr is the threshold velocity (at the particle level) for the particle motion, and $f(u_\mathrm{d})$ is the frequency distribution of u_d. The $f(u_\mathrm{d})$ is given by

$$f(u_\mathrm{d}) = \frac{1}{(2\pi)^{0.5} \sigma_u} \exp\left[-\frac{(u_\mathrm{d} - \bar{u}_\mathrm{d})^2}{2\sigma_u^2}\right] \tag{5.35}$$

where σ_u is the standard deviation of u_d. Assuming $\tau_{0\mathrm{c}}/\tau_0 = (u_\mathrm{cr}/\bar{u}_\mathrm{d})^2$, where $\bar{u}_\mathrm{d}$ is the time-averaged value of u_d, the following functional relationship is obtained:

$$\frac{\bar{u}_\mathrm{b}}{u_*} = f\left(\frac{\tau_{0\mathrm{c}}}{\tau_0}\right) \tag{5.36}$$

Using Eq. (5.36), Eq. (5.33) can be expressed a functional relationship as

$$\frac{q_\mathrm{b}}{u_* d} = f_1\left(\frac{\tau_{0\mathrm{c}}}{\tau_0}\right) \tag{5.37}$$

Figure 5.4 shows this relationship.

Frijlink (1952) formula, as already given by Eq. (5.17) that can approximate Meyer-Peter and Müller formula, was one that falls under the category to involve bed shear stress. Further, the bed-load transport formula that was widely used by Engelund and Hansen (1967) for sand transport in terms of bed shear stress is

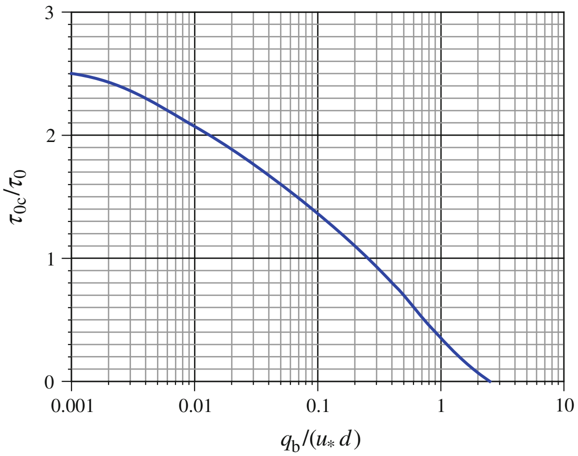

Fig. 5.4 Variation of τ_{0c}/τ_0 with $q_b/(u_*d)$ (Kalinske 1947)

$$\Phi_b = 0.05\frac{U^2}{\Delta g d}\Theta^{1.5} \tag{5.38}$$

In case of weak bed-load transport rate, Paintal (1971) obtained a bed-load transport formula for $1 \leq d \leq 25$ mm as

$$\Phi_b(0.007 < \Theta < 0.06) = 6.56 \times 10^{18}\Theta^{16} \tag{5.39}$$

The relationships proposed by Misri et al. (1984) to involve bed shear stress due to particle roughness are as follows:

$$\Phi_b(\Theta' \leq 0.065) = 4.6 \times 10^7 \Theta'^8 \tag{5.40a}$$

$$\Phi_b(\Theta' > 0.065) = \frac{0.85\Theta'^{1.8}}{(1 + 5.95 \times 10^{-6}\Theta'^{-4.7})^{1.43}} \tag{5.40b}$$

where Θ' is the Shields parameter due to particle roughness, that is $\tau'_0/(\Delta\rho g d)$, and τ'_0 is the bed shear stress due to particle roughness.

On the other hand, Cheng (2002) gave a relationship for moderate bed-load transport rate as

$$\Phi_b = 13\Theta^{1.5}\exp\left(-\frac{0.05}{\Theta^{1.5}}\right) \tag{5.41}$$

The above equation yields results similar to those obtained from Meyer-Peter and Müller formula for moderate transport rate and Paintal formula for weak transport rate.

For high bed-load transport rate, Rickenmann (1991) reported that the particles transport like a sheet flow, when $\Theta > 0.4$. Hanes (1986) suggested that under a sheet flow type transport, the intense bed-load transport can be approximated as

$$\Phi_b = 6\Theta^{2.5} \tag{5.42}$$

5.4 Discharge Concept for Bed-Load Transport

Schoklitsch (1934) was the pioneer to use discharge for the estimation of bed load. He used the data of Gilbert (1914) with his own to propose a bed-load transport rate formula for particle size $0.305 \leq d \leq 7.02$ mm as

$$g_b = \frac{7000}{d^{0.5}} S_0^{1.5} (q - q_c) \tag{5.43}$$

where g_b is the bed-load transport rate in mass per unit time and width (kg s^{-1} m^{-1}), d is in mm, and q_c is the discharge per unit width corresponding to sediment threshold. Schoklitsch determined $q_c = 1.944 \times 10^{-5}/S_0^{4/3} (m^2\, s^{-1})$ by plotting a curve of bed-load transport rate versus bed slope. He then extrapolated the curve to zero transport rate ($g_b = 0$) to determine the corresponding value of q as q_c. Schoklitsch later modified the equation for $d \geq 6$ mm as

$$g_b = 2500 S_0^{1.5} (q - q_c) \tag{5.44}$$

He redefined the threshold discharge as $q_c = h_c^{5/3} S_0^{0.5}/n = 0.26\Delta^{5/3} d^{1.5}/S_0^{7/6}$ ($m^3\, s^{-1}\, m^{-1}$); where d is in m, n is the Manning coefficient, and h_c is the flow depth corresponding to sediment threshold.

5.5 Velocity Concept for Bed-Load Transport

Donat (1929) used the Chézy equation in Eq. (5.11) and obtained the following equation of bed-load transport using average flow velocity:

$$q_b = \chi \frac{(\rho g U)^2}{C_R^4} (U^2 - U_{cr}^2) \quad \wedge \quad U^2 = C_R^2 \frac{\tau_0}{\rho g} \quad \vee \quad U_{cr}^2 = C_R^2 \frac{\tau_{0c}}{\rho g} \tag{5.45}$$

where U_{cr} is the average threshold velocity.

Barekyan (1962) proposed bed-load transport equation using average flow velocity as

$$q_b = 0.187 \rho_s g \frac{q S_0}{\Delta} \left(\frac{U}{U_{cr}} - 1 \right) \tag{5.46}$$

Based on the stream power concept, Dou (1964) established an empirical equation of bed-load transport for sand as

$$g_b = 0.01 \frac{s}{\Delta} \tau_0 (U - U_{cr}) \frac{U}{w_s} \tag{5.47}$$

5.6 Bedform Concept for Bed-Load Transport

Bedforms are discussed comprehensively in Chap. 8. Note that the bed load is the mode of sediment transport in lower flow regime when the bed is covered by ripples and/or dunes. The particles transport up the face of the mild slope of the ridge of a bedform and then drop down the steep slope being deposited on the downstream face and in the trough. As a result of sediment removal from the upstream and deposition on the downstream slope, the bedforms move downstream (Fig. 5.5). The bed-load transport can therefore be calculated directly from the movement of the bedforms. The continuity equation of sediment transport resulting in a change of bed level was given by Exner (1925) as

$$(1 - \rho_0) \frac{\partial \eta}{\partial t} + \frac{\partial q_b}{\partial x} = 0 \tag{5.48}$$

where η is the elevation of the sand-bed with respect to a horizontal reference, t is the time, x is the horizontal distance from a reference point, and ρ_0 is the porosity of sediment.

Assuming that the bedforms migrate with a velocity of U_b being independent of time, the following transformation can be used:

$$\xi = x - U_b t \tag{5.49}$$

By using Eq. (5.49), Eq. (5.48) yields

$$(1 - \rho_0) \frac{\partial \eta}{\partial \xi} \cdot \frac{\partial \xi}{\partial t} + \frac{\partial q_b}{\partial \xi} \cdot \frac{\partial \xi}{\partial x} = 0 \;\Rightarrow\; -(1 - \rho_0) U_b \frac{\partial \eta}{\partial \xi} + \frac{\partial q_b}{\partial \xi} = 0 \tag{5.50}$$

Integrating Eq. (5.50) yields

$$q_b = (1 - \rho_0) U_b \eta + A \tag{5.51}$$

Assuming that the simplified bedforms are triangular shaped with an average height or pick-to-pick amplitude of a_m and noting that the constant of integration $A = 0$ for the initial boundary condition, Eq. (5.51) becomes

$$q_b = (1 - \rho_0) U_b \frac{a_m}{2} \tag{5.52}$$

The above equation can be used to determine the bed-load transport rate from the information of the bedform migration velocity and its height.

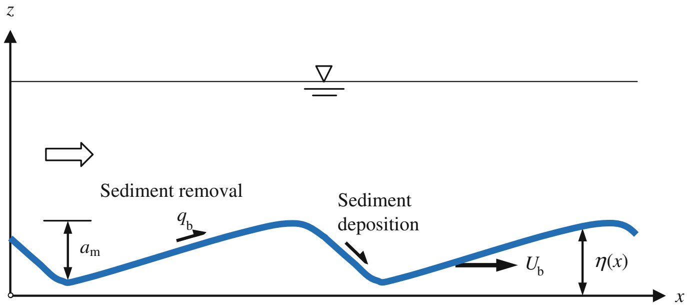

Fig. 5.5 Bed-load transport with migration of bedforms

5.7 Probabilistic Concept for Bed-Load Transport

5.7.1 Einstein's Approach

Einstein (1942, 1950) was the pioneer to develop a bed-load transport model based on the probabilistic concept. Primarily, he had two fundamental considerations that departed from the then earlier concepts. Firstly, the threshold criterion was avoided, as it is always a difficult proposition to define, if not impossible. Secondly, the transport of sediment particles was related to the velocity fluctuations instead of the time-averaged velocity. As a result of which, the beginning and the ceasing of sediment motion are expressed with probabilistic concept that relates to the ratio of submerged weight of the particle to instantaneous hydrodynamic lift induced to the particle. Some of the key issues toward the bed-load transport of sediment particles, as experimentally observed by Einstein, are as follows:

- A rigorous, but steady, exchange of sediment particles is prevalent between the bed surface and mobile bed-load layer.
- The particles travel along the bed in a series of quick steps. A particle does not, however, remain in motion continuously, but temporarily deposited on the bed after some steps with comparatively long intermediate resting periods.
- The average step, which is always the same and about 100 times the particle diameter, is simply proportional to the particle diameter and independent of the hydraulic condition and the transport rate.
- The transport rate is dependent on the average time period between two steps and the thickness of the mobile bed-load layer.

Einstein's (1942, 1950) bed-load transport model was based on the aforementioned aspects. He first presented an empirical relationship in 1942, which was then replaced by a semitheoretical approach in 1950.

Dynamic equilibrium during the bed-load transport is established by exchanging the particles from the bed within the bed-load transport layer. Thus, the conservation of sediment mass is maintained balancing the number of particles removal (washed out by the flow) per unit time and area by those deposited (put down by the flow) per unit time and area.

Rate of deposition: The average traveling distance L_x of a particle is defined by the distance that a particle travels from its starting point until it is deposited on the bed. The single step length of a particle having diameter d can be expressed as $\lambda_s d$ and for spherical particles, $\lambda_s = 100$. As a particle travels a step by a brief jump (Fig. 5.6), it goes down on the bed at a location where a local lift force exceeds the submerged weight of the particle. Thus, the particle does not stop moving but travels for a second step and so on until it is temporarily deposited on the bed with comparatively long intermediate resting periods. In this way, the sediment particles passing a section (across the flow) per unit time deposit within a length of the channel that is equal to L_x, regardless from where they have started to move. If g_b represents the bed-load transport rate in dry weight and i_{bs} is the fraction of bed load to be deposited of a given sediment size d, then the rate at which the particles of a size d are deposited per unit time and width is $g_b i_{bs}$. Therefore, the number of particles N_d deposited per unit time and area is given by

$$N_d = \frac{g_b i_{bs}}{L_x(\rho_s g k_1 d^3)} \tag{5.53}$$

where k_1 is the factor related to particle volume. The term within the parenthesis in the denominator defines the weight of a particle.

If p is the probability of lift force to exceed the submerged weight of the particles, then $n(1-p)$ particles deposit on the bed after traveling a step length, where n is the number of particles in motion. Thus, only np particles continue to move. Subsequently, the $np(1-p)$ more particles deposit and only np^2 particles remain in motion after traveling the second step length, and so on. In this way, all n particles deposit on the bed after elapsing some time. The average traveling distance[1] can therefore be determined as

$$L_x = \sum_{n=0}^{\infty}(1-p)p^n(n+1)\lambda_s d = \frac{\lambda_s d}{1-p} \tag{5.54}$$

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[1] The probability of a particle performing $(n+1)$ number of jumps is $(1-p)p^n(n+1)$. Then,

$$\sum_{n=0}^{\infty}(1-p)p^n(n+1) = 1 + p + p^2 + p^3 + + p^n = (1-p)^{-1}.$$

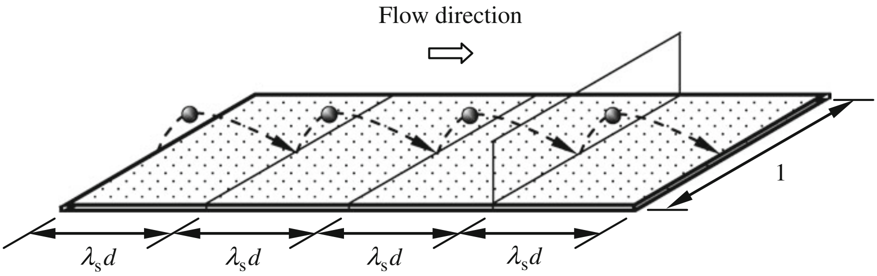


Fig. 5.6 Sketch of a particle traveling along the bed in a series of steps

Using Eq. (5.54) into Eq. (5.53), the number of particles deposited per unit time and area becomes

$$N_d = \frac{g_b i_{bs}(1-p)}{\lambda_s \rho_s g k_1 d^4} \tag{5.55}$$

Rate of removal: Depending on the availability of the particles and the flow conditions, a particle of a given size d is removed. If the fraction of sediment of a given size d to be removed is i_{br}, then the number of such particles per unit area can be given by $i_{br}/(k_2 d^2)$; where k_2 is the factor related to the projected area of the particle. If p is the probability of a particle to begin to move at any location, then p/t_e is the probability of removal per unit time. Here, t_e is the time consumed by each exchange. Therefore, the number of particles removed N_r per unit time and area is given by

$$N_r = \frac{i_{br}}{k_2 d^2} \cdot \frac{p}{t_e} \tag{5.56}$$

The exchange time t_e or the time for a particle to remove is assumed to be proportional to the time for a particle to fall a height of one diameter with a terminal velocity w_s in a still water. Thus, it is

$$t_e \sim \frac{d}{w_s} = k_3 \left(\frac{d}{\Delta g}\right)^{0.5} \tag{5.57}$$

where k_3 is a constant for time scale. Using Eq. (5.57) into Eq. (5.56), the number of particles removed per unit time and area is

$$N_r = \frac{i_{br}}{k_2 d^2} \cdot \frac{p}{k_3} \left(\frac{\Delta g}{d}\right)^{0.5} \tag{5.58}$$

Equilibrium of bed-load transport: Sediment transport is in equilibrium if the rate of sediment deposition on the bed is balanced by the rate of sediment removal from the bed. Thus, equating Eqs. (5.55) and (5.58), the equation of dynamic equilibrium is obtained as

$$N_{\mathrm{d}} = N_{\mathrm{r}} \;\Rightarrow\; \frac{g_{\mathrm{b}} i_{\mathrm{bs}}(1-p)}{\lambda_{\mathrm{s}}\rho_{\mathrm{s}} g k_1 d^4} = \frac{i_{\mathrm{br}}}{k_2 d^2}\cdot\frac{p}{k_3}\left(\frac{\Delta g}{d}\right)^{0.5} \tag{5.59}$$

The bed-load transport equation is therefore obtained from Eq. (5.59) as

$$\frac{p}{1-p} = A_*\left(\frac{i_{\mathrm{bs}}}{i_{\mathrm{br}}}\right)\Phi_{\mathrm{b}} = A_*\Phi_{\mathrm{b}*} \quad \wedge \quad A_* = \frac{k_2 k_3}{\lambda_{\mathrm{s}} k_1} \quad \vee \quad \Phi_{\mathrm{b}*} = \left(\frac{i_{\mathrm{bs}}}{i_{\mathrm{br}}}\right)\Phi_{\mathrm{b}} \tag{5.60}$$

The parameter $\Phi_{\mathrm{b}*}$ is called *bed-load transport intensity*, and the probability p of rate of sediment removal is given by

$$p = \frac{A_*\Phi_{\mathrm{b}*}}{1 + A_*\Phi_{\mathrm{b}*}} \tag{5.61}$$

Probability determination: The probability p of a sediment particle removal is a function of the ratio of submerged weight F_{G} of the particle to instantaneous hydrodynamic lift F_{L} induced to the particle. The condition of removal is therefore $p(F_{\mathrm{G}}/F_{\mathrm{L}}) < 1$. It can therefore be expressed as

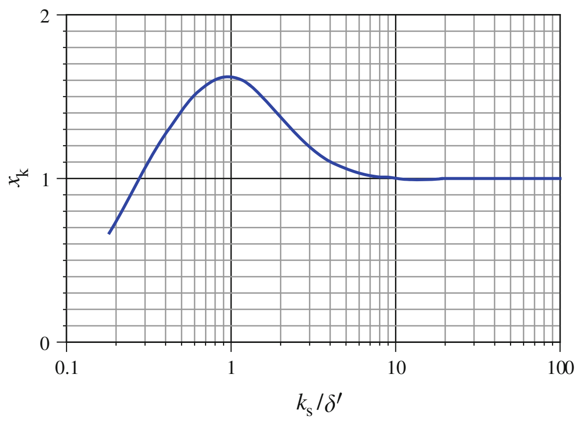
$$p = p\left(\frac{F_{\mathrm{G}}}{F_{\mathrm{L}}}\right) = p\left(\frac{\Delta g k_1 d}{C_{\mathrm{L}} k_2 u_{\delta'}^2/2}\right) \quad \wedge \quad F_{\mathrm{G}} = \Delta\rho g k_1 d^3 \quad \vee \quad F_{\mathrm{L}} = C_{\mathrm{L}}\frac{\rho}{2}k_2 d^2 u_{\delta'}^2 \tag{5.62}$$

where C_{L} is the lift coefficient and $u_{\delta'}$ is the effective instantaneous flow velocity at the edge of the viscous sublayer. Einstein and El-Samni (1949) observed that for uniform sediment particles, if the flow velocity at an elevation $z = 0.35X$ is taken as the effective flow velocity $u_{\delta'}$, the distribution of lift force fluctuations follows the Gaussian distribution with a standard deviation equaling half of the mean value and the lift coefficient as $C_{\mathrm{L}} = 0.178$ (a constant value). Here, X is the characteristic size of the bed sediment particles. The random function parameter $\eta_t(t)$ represents the lift force fluctuations with time t being distributed according to the normal error law, where the standard deviation η_0 is a universal constant having a value $\eta_0 = 0.5$. Using a nondimensional number η_* that represents the lift force fluctuations, it can be written as $\eta_t = \eta_0\eta_*$.

The effective instantaneous flow velocity $u_{\delta'}$ is expressed as

$$\frac{u_{\delta'}}{u'_*} = \frac{1}{\kappa}\ln\left(\frac{0.35X}{\Delta_{\mathrm{k}}/30.2}\right) \quad \wedge \quad \begin{aligned} X\left(\frac{\Delta_{\mathrm{k}}}{\delta'} \geq 1.8\right) &= 0.77\Delta_{\mathrm{k}} \\ X\left(\frac{\Delta_{\mathrm{k}}}{\delta'} < 1.8\right) &= 1.39\delta' \end{aligned} \tag{5.63}$$

Fig. 5.7 Variation of correction factor x_k with k_s/δ', where Nikuradse's equivalent sand roughness $k_s = d_{65}$



where κ is the von Kármán constant, Δ_k is the apparent roughness ($= k_s/x_k$), x_k is a correction factor, u'_* is the shear velocity due to particle roughness, that is $(gR'_b S_0)^{0.5}$, R'_b is the hydraulic radius due to particle roughness, and δ' is the viscous sublayer thickness ($= 11.6\upsilon/u'_*$). Einstein (1950) considered Nikuradse's equivalent sand roughness as $k_s = d_{65}$. The correction factor x_k can be obtained from the curve given by Einstein (1950) (Fig. 5.7), and thus, apparent roughness Δ_k ($= k_s/x_k$) can be determined.

Hence, the lift force can be expressed as

$$F_L = (1 + \eta_0 \eta_*) 0.178 \frac{\rho}{2} k_2 d^2 \frac{1}{\kappa^2} g R'_b S_0 \ln^2 \left(\frac{10.6X}{\Delta_k} \right) \tag{5.64}$$

The probability p of sediment removal is expressed as the probability of the ratio of the submerged weight F_G to the instantaneous lift F_L. The ratio has to be smaller than unity, that is

$$1 > \frac{F_G}{F_L} = \frac{1}{1 + \eta_0 \eta_*} \cdot \frac{\Delta d}{R'_b S_0} \cdot \frac{2 k_1 \kappa^2}{0.178 k_2} \cdot \frac{1}{\beta_x^2} \quad \wedge \quad \beta_x = \ln \left(\frac{10.6X}{\Delta_k} \right) \tag{5.65}$$

Using symbols, Eq. (5.65) can be reduced to

$$1 > \frac{1}{1 + \eta_0 \eta_*} \cdot \frac{\Psi'_b B}{\beta_x^2} \quad \wedge \quad \Psi'_b = \frac{\Delta d}{R'_b S_0} \quad \vee \quad B = \frac{2 k_1 \kappa^2}{0.178\, k_2} \tag{5.66}$$

In the above, Ψ'_b is known as *flow intensity parameter* due to particle roughness, which is reciprocal of the Shields parameter.

Einstein (1950) proposed two correction factors ξ and Y termed *hiding factor* and *lift correction factor*, respectively, which were determined experimentally (Figs. 5.8 and 5.9). Particles in the sediment mass smaller than X likely to hide

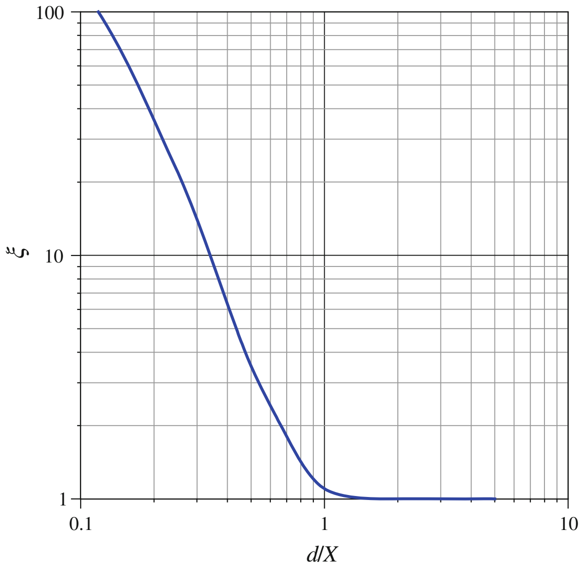


Fig. 5.8 Variation of hiding factor ξ with d/X (Einstein 1950)

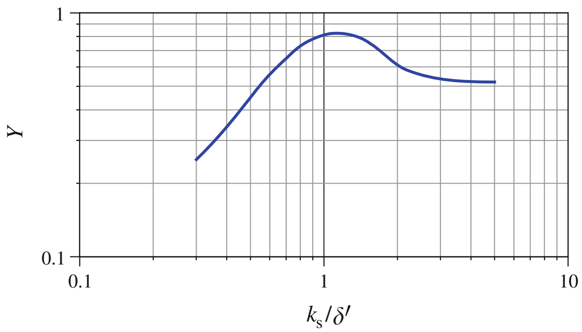


Fig. 5.9 Variation of lift correction factor Y with k_s/δ' (Einstein 1950)

between larger ones or within the viscous sublayer, as such the lift experienced by the smaller particles is to be corrected by a factor ξ^{-1}. Einstein gave a curve for the hiding factor ξ as a function of d/X (see Fig. 5.8). The lift correction factor Y takes care of the change of lift coefficient in the sediment mass due to different roughness and is expressed as a function of k_s/δ' (see Fig. 5.9).

The fluctuations of lift force are caused by the velocity fluctuations. The lift force is always positive regardless of the velocity fluctuations to be positive or negative. Thus, the inequality for the lift force can be modified as

$$\left|\eta_* + \frac{1}{\eta_0}\right| > B_* \Psi_{b*} \quad \wedge \quad B_* = \frac{B}{\eta_0 \ln^2(10.6)} \quad \vee \quad \Psi_{b*} = \Psi'_b \xi Y \frac{\ln^2(10.6)}{\beta_x^2} \tag{5.67}$$

Therefore, the threshold condition for the bed particle motion is as follows:[2]

$$\eta_* = \pm B_* \Psi_{b*} - \frac{1}{\eta_0} \tag{5.68}$$

It implies that between these two values of η_*, no sediment transport takes place. Therefore, the probability p of sediment motion, as the lift force fluctuations follow Gaussian distribution, is

$$p = 1 - \frac{1}{\pi^{0.5}} \int\limits_{-B_*\Psi_{b*} - \eta_0^{-1}}^{B_*\Psi_{b*} - \eta_0^{-1}} \exp(-t^2)\mathrm{d}t \tag{5.69}$$

Using Eq. (5.69) into Eq. (5.60), Einstein's bed-load transport equation is

$$\Phi_{b^*} = \frac{1}{A_*} \cdot \frac{1 - \frac{1}{\pi^{0.5}} \int\limits_{-B_*\Psi_{b*} - \eta_0^{-1}}^{B_*\Psi_{b*} - \eta_0^{-1}} \exp(-t^2)\mathrm{d}t}{\frac{1}{\pi^{0.5}} \int\limits_{-B_*\Psi_{b*} - \eta_0^{-1}}^{B_*\Psi_{b*} - \eta_0^{-1}} \exp(-t^2)\mathrm{d}t} \tag{5.70}$$

Einstein experimentally obtained the values of the constants that are $\eta_0 = 0.5$, $A_* = 43.5$ and $B_* = 1/7$. The variation of Ψ_{b*} with Φ_{b*} from Eq. (5.70) is shown in Fig. 5.10.[3] The $\Psi_{b*}(\Phi_{b*})$ curve matches well with the experimental data of Gilbert (1914), Meyer-Peter et al. (1934) and Chien and Wan (1999).

5.7.2 Empirical Refinement of Einstein Formula

Brown (1950) refined the Einstein formula by curve fitting and showed that the majority of flume data of Gilbert and Meyer-Peter et al. could be expressed by the following relationships:

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[2] To minimize the errors, the standard deviation of lift force fluctuations given by Eq. (5.68) should be small, then $B_* \to \infty$ as $\eta_0 \to \infty$. Hence,

$$-B_*\Psi_{b*} - \frac{1}{\eta_0} = -\infty \text{ and } B_*\Psi_{b*} - \frac{1}{\eta_0} \neq 0$$

[3] The use of Einstein's $\Psi_{b*}(\Phi_{b*})$ curve as shown in Fig. 5.10 is as follows:

Step 1: From the given bed sediment and flow conditions, compute Ψ_{b*} from Eq. (5.67). Then, the correction factors ξ and Y can be obtained from Figs. 5.8 and 5.9, respectively. The other parameters required to be computed are B_* from Eq. (5.67), $\Delta_k = k_s/x_k$, x_k from Fig. 5.7, $u'_* = (gR'_b S_0)^{0.5}$, $\delta' = 11.6\upsilon/u'_*$, and Ψ_b and B from Eq. (5.66).

Step 2: From Fig. 5.10, determine Φ_{b*} for the computed Ψ_{b*}. Thus, q_b or g_b can be obtained from Eq. (5.2).

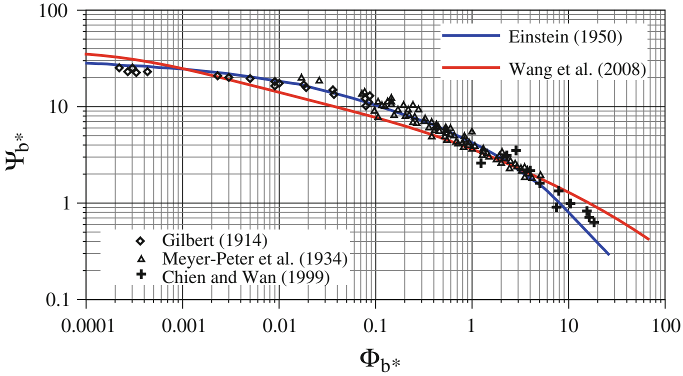

Fig. 5.10 Variations of Ψ_{b*} with Φ_{b*} obtained from the models of Einstein (1950) and Wang et al. (2008)

$$\Phi_b(1.92 < \Psi_b \leq 5.56) = 40K_f \frac{1}{\Psi_b^3} \quad \wedge \quad K_f = \left(\frac{2}{3} + \frac{36\upsilon^2}{\Delta g d^3}\right)^{0.5} - \left(\frac{36\upsilon^2}{\Delta g d^3}\right)^{0.5} \tag{5.71a}$$

$$\Phi_b(\Psi_b > 5.56) = 2.15K_f \exp(-0.391\Psi_b) \tag{5.71b}$$

For the sediment transport at higher Shields parameter ($\Psi_b \leq 1.92$), Julien (1998) suggested

$$\Phi_b(\Psi_b \leq 1.92) = 15K_f \frac{1}{\Psi_b^{1.5}} \tag{5.72}$$

In the above equations, the parameter K_f that appears in Rubey (1933) formula for terminal fall velocity was introduced by Brown to account for the effects of fall velocity of the sediment particles.

5.7.3 Modified Einstein's Approach

The derivation of Einstein's bed-load formula involves some oversimplified assumptions concerning the step length of a particle, exchange time, and probability of particle removal. Later, Wang et al. (2008) proposed a modification of the Einstein formula.

They argued that conceptually, the step length of a particle increases with the magnitude of the lift force exerted by the flow, but decreases with the submerged weight of the particle. The step length can thus be given by $\lambda_s d/\Psi_b$. The rate of particle deposition g_{dep} per unit area is obtained as

$$g_{dep} = \frac{g_b}{L_x} = \frac{g_b}{\lambda_s d}(1-p)\Psi_b \quad \wedge \quad L_x = \frac{\lambda_s d}{(1-p)\Psi_b} \tag{5.73}$$

The number of particles per unit area can be estimated as $1/(k_2 d^2)$, and their total weight is $k_1 \rho_s g d^3/(k_2 d^2)$. If p is the probability of a particle to begin to move, sediment with a total weight of $(k_1/k_2)\rho_s g d p$ is removed from the bed per unit time and area.

Based on the finding by Hu and Hui (1996) that the upward velocity of a particle is approximated by a linear relationship of shear velocity u_*, the time for a particle to be removed from the bed is inversely proportional to u_*. Wang et al., therefore, suggested that the exchange time t_e can be expressed as

$$t_e \sim \frac{d}{u_*} = k_3 \frac{d}{u_*} \tag{5.74}$$

The rate of particle removal g_{rem} per unit area is obtained as

$$g_{rem} = \frac{1}{t_e} \cdot \frac{k_1}{k_2} \rho_s g d p = \frac{k_1}{k_2 k_3} \rho_s g p u_* \tag{5.75}$$

Equilibrium is reached when the rate of sediment removal from the bed equals the rate of deposition on the bed. Equating Eqs. (5.73) and (5.75) yields

$$p = \frac{A_* \Phi_b}{\Psi_b^{-1.5} + A_* \Phi_b} \quad \wedge \quad A_* = \frac{k_2 k_3}{\lambda_s k_1} \tag{5.76}$$

Wang et al. assumed that a particle is removed only if the lift force exceeds the submerged weight of the particle, that is

$$1 + \eta_0 \eta_* > B' \Psi_b \tag{5.77}$$

where B' is the coefficient. The probability p of particle removal is given by

$$p = \frac{1}{\pi^{0.5}} \int_{(B'\Psi_b - 1)/\eta_0}^{\infty} \exp(-t^2)\mathrm{d}t \tag{5.78}$$

Combining Eqs. (5.76) and (5.78) and introducing nonuniformity of sediments, the following relationship is obtained

$$\frac{1}{\pi^{0.5}} \int_{(B'\Psi_b - 1)/\eta_0}^{\infty} \exp(-t^2)\mathrm{d}t = \frac{A_* \Phi_{b*}}{\Psi_b^{-1.5} + A_* \Phi_{b*}} \quad \wedge \quad \Phi_{b*} = \left(\frac{i_{bs}}{i_{br}}\right)\Phi_b \tag{5.79}$$

Based on the measured data used by Einstein (1950), the values of the constants were determined as $B'/\eta_0 = 0.07$, $\eta_0 = 0.5$ and $A_* = 20$. The variation of Ψ_b (read Ψ_{b*} as Ψ_b) with Φ_{b*} obtained from Eq. (5.79) is shown in Fig. 5.10. The $\Psi_b(\Phi_{b*})$ curve departs to some extent from the experimental data plots of Gilbert (1914), Meyer-Peter et al. (1934) and Chien and Wan (1999), and the curve of Einstein (1950).

5.7.4 Engelund and Fredsøe's Approach

Engelund and Fredsøe (1976) developed a bed-load transport model for the flow conditions close to the threshold of sediment motion. In this type of flow, the superficial bed particles are only transported. The model is based on the concept of Fernandez Luque and van Beek (1976), who hypothesized that the transported bed particles are to reduce the maximum fluid bed shear stress to its threshold value for the bed particle motion by exerting an average reaction force on the ambient fluid.

If the particles are transported with a mean velocity $\bar{u}_b$, when they are in motion, the hydrodynamic drag force F_D acting on a transported particle is given by

$$F_D = \frac{1}{2}\rho C_D \frac{\pi}{4} d^2 (\alpha u_* - \bar{u}_b)^2 \tag{5.80}$$

where C_D is the drag coefficient and αu_* is the flow velocity at the bed particle level. If the particle is at a distance of one to two particle diameters above the mean bed level, then $\alpha = 6$–10.

The stabilizing resistance F_R on the moving particle is

$$F_R = \Delta \rho g \frac{\pi}{6} d^3 \mu_d \tag{5.81}$$

where μ_d is the dynamic coefficient of friction for the bed particles.

At dynamic equilibrium, the hydrodynamic drag force is balanced by the stabilizing resistance ($F_D = F_R$). Thus, equating Eqs. (5.80) and (5.81) and then simplifying yield

$$\frac{\bar{u}_b}{u_*} = \alpha\left[1 - \left(\frac{\Theta_{0c}}{\Theta}\right)^{0.5}\right] \quad \wedge \quad \Theta_{0c} = \frac{4\mu_d}{3\alpha^2 C_D} \tag{5.82}$$

where Θ_{0c} is the threshold Shields parameter for a particle protruding from the bed surface. In fact, Θ_{0c} differs from Θ_c, which is the conventional threshold Shields parameter for the initiation of particle motion in a compactly arranged bed. As a particle lying on the bed is easier to move than a particle within the bed, it implies that $\Theta_c > \Theta_{0c}$. From the experimental data, Fernandez Luque and van Beek (1976) found $\Theta_{0c} = 0.5\Theta_c$. Thus, Eq. (5.82) becomes

$$\frac{\bar{u}_b}{u_*} = \alpha\left[1 - 0.7\left(\frac{\Theta_c}{\Theta}\right)^{0.5}\right] \tag{5.83}$$

For a sandy bed, $\alpha \approx 9.3$. Engelund and Fredsøe treated sediment particles as spheres of diameter d, so that the number of spherical particles per unit area of bed surface is approximately $1/d^2$. For a given flow intensity, the probability of the particles on the bed surface to move is p. Hence, the bed-load transport rate g_b is

$$g_b = \frac{\pi}{6} d^3 \rho_s g \frac{p}{d^2} \bar{u}_b \tag{5.84}$$

Using Eq. (5.83) into Eq. (5.84) yields

$$g_b = 9.3 \frac{\pi}{6} d \rho_s g p \left[1 - 0.7\left(\frac{\Theta_c}{\Theta}\right)^{0.5}\right] u_* \tag{5.85}$$

According to Bagnold, the applied bed shear stress τ_0 by the flow is composed of dispersive particle bed shear stress τ_{0b} and interfacial (intergranular) fluid bed shear stress τ_{0f}. Furthermore, he suggested that during bed-load transport, the interfacial fluid bed shear stress τ_{0f} equals the threshold bed shear stress τ_{0c} for the initiation of particle motion. This phenomenon is further discussed in the following section using a shear stress diagram. The estimation of probability p of surface bed particle removal is based on the assumption that only τ_{0c} of the applied bed shear stress τ_0 by the flow is transmitted directly to the immobile-bed particles as a skin frictional stress; whereas the residual fluid bed shear stress $(\tau_0 - \tau_{0c})$ is directly transmitted to the mobile particles as a drag induced bed shear τ_{0b} $(= nF_D)$ and indirectly transmitted to the bed by intermittent surface creep. Hence,

$$\tau_0 = \tau_{0c} + nF_D \tag{5.86}$$

where n is the number of particles moving per unit area of bed surface. As $F_D = F_R$, inserting Eq. (5.81) into Eq. (5.86) leads to an estimation of p as

$$\Theta = \Theta_c + \frac{\pi}{6}\mu_d(nd^2) = \Theta_c + \frac{\pi}{6}\mu_d p \;\Rightarrow\; p = \frac{6}{\pi\mu_d}(\Theta - \Theta_c) \quad \wedge \quad p = nd^2 \tag{5.87}$$

Using Eq. (5.87) into Eq. (5.85), the bed-load transport rate, expressed as bed-load transport intensity Φ_b, is obtained as follows:

$$\Phi_b = \frac{9.3}{\mu_d}(\Theta - \Theta_c)(\Theta^{0.5} - 0.7\Theta_c^{0.5}) \tag{5.88}$$

For an intense bed-load transport rate $\Theta \gg \Theta_c$, Eq. (5.88) can be approximated as $\Phi_b = 9.3\Theta^{1.5}/\mu_d$.

5.8 Deterministic Concept for Bed-Load Transport

5.8.1 Bagnold's Approach

Bagnold (1954) identified the limitation in Einstein's approach by revealing an inconsistency toward the stability criterion of the bed during bed-load transport. Let it be discussed with an ideal example of the flow over a plane bed formed by uniform spherical sediment particles. This situation of a streambed leads to an equal exposure of all the bed particles to the flow; and hence, the stochastic variations due to turbulence can be ignored. When the applied bed shear stress exceeds its threshold value for the particle motion, all particles in the uppermost layer are in motion simultaneously and removed by the flow. As a result, the next layer of particles comes in contact with the flow and is subsequently also removed and so on. In this way, all the subsequent underlying layers of particles are removed and equilibrium toward a stable bed never exits as long as the bed shear stress exceeds the threshold value. Bagnold, however, argued this inconsistency by decomposing the applied shear stress τ by the flow into the dispersive particle shear stress τ_b that is the shear stress transmitted due to exchange of momentum for the collision of moving particles and the interfacial fluid shear stress τ_f that is the shear stress transmitted by the interfacial fluid (Fig. 5.11). The background of the idea was that the sediment-laden flows are a result of shear that includes shear between the layers of the particles and that between the sediment and the surrounding fluid. An applied bed shear stress τ_0 induced by the fluid tractive force that acts in the streamwise direction to sustain such a shear is developed by the gravity in the streamwise direction (Fig. 5.11).

The bed shear stress decomposition is therefore

$$\tau_0 = \tau_{0b} + \tau_{0f} \tag{5.89}$$

Bagnold further argued that with the removal of a layer of particles, a dispersive pressure on the subsequent layer of particles is developed as a stabilizing force. The number of layers to be removed is governed by the interfacial fluid bed shear stress τ_{0f} until it equals the threshold bed shear stress τ_{0c} that acts on the first immobile layer. The applied bed shear stress τ_0 induced by the fluid tractive force

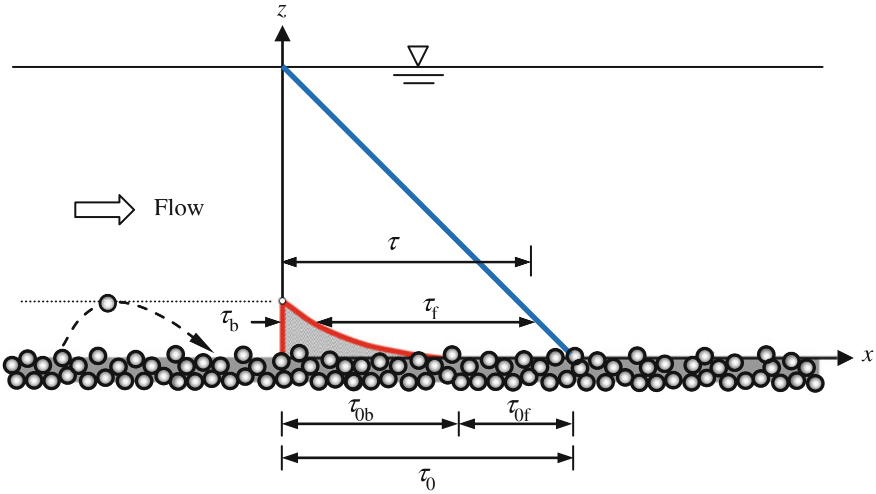

Fig. 5.11 Decomposition of applied shear stress into dispersive particle shear stress and interfacial fluid shear stress

is therefore greater than the threshold bed shear stress τ_{0c}. Hence, the τ_0 is partially transmitted to the moving particle as τ_{0b} and rest to the immobile bed as τ_{0c}.

Bagnold (1956) assumed that the saltation is the primary mode of bed-load transport. The momentum component in the streamwise direction when a saltating particle drops down to the bed is $m_G u_1$. Here, m_G is the submerged mass of the particle, and u_1 is the velocity component of the particle in the streamwise direction when it collides with the bed. The particle at the same time is acted on by a force from the bed particles producing a momentum component $m_G(-u_0)$ opposite to the streamwise direction. Here, $-u_0$ is the reduction of particle velocity component in the streamwise direction due to collision with the bed particles. To maintain the saltation of a particle, the flowing fluid therefore must act on the particle to provide a momentum component $m_G u_0$ in the time interval Δt between successive collisions of the saltating particle with the bed particles.

Therefore, the fluid flow exerts a force on the particle with a component in the streamwise direction as

$$F_x = \frac{m_G u_0}{\Delta t} = \frac{F_G u_0}{g \Delta t} \tag{5.90}$$

If $\bar{u}_b$ is the average velocity of the particle, then the work done per unit time by the flowing fluid on the particle is $F_x \bar{u}_b$. Also, the energy consumed per unit time by the flow is $F_G \bar{u}_b \tan\phi_d$; where ϕ_d is the dynamic frictional angle. Equating them and using Eq. (5.90) yield

$$\frac{F_x}{F_G} = \tan\phi_d = \frac{u_0}{g\Delta t} \tag{5.91}$$

The vertical distance z_n is the location at which the particle is acted upon by a force F_x to accelerate the particle from $u_1 - u_0$ to u_0. If the flow velocity at z_n is $\bar{u}_n$, then the $u_r(= \bar{u}_n - \bar{u}_b)$ exists at an elevation $z = z_n$. As a number of particles are in motion along the bed during bed-load transport, then

$$\tau_{bn}\bar{u}_b = F_G\bar{u}_b\tan\phi_d = g_{bs}\tan\phi_d \tag{5.92}$$

where τ_{bn} is the shear stress for maintaining sediment motion at $z = z_n$. So, the bed-load transport rate g_{bs} (in submerged weight per unit time and width) is

$$g_{bs} = \frac{\tau_{bn}}{\tan\phi_d}(\bar{u}_n - u_r) \tag{5.93}$$

Using a coefficient a, the shear stress τ_{bn} is given by

$$\tau_{bn} = a\tau_0 \tag{5.94}$$

The flow velocity is considered to follow the logarithmic law in the flow region $z \geq z_n$, and the velocity at $z = 0.4h$ is considered to be equal to the depth-averaged flow velocity U. Then,

$$\bar{u}_n = U - \frac{u_*}{\kappa}\ln\frac{0.4h}{z_n} \tag{5.95}$$

Using Eqs. (5.94) and (5.95) into Eq. (5.93) yields

$$g_{bs} = \frac{a\tau_0}{\tan\phi_d}\left[U - \frac{u_*}{\kappa}\ln\left(\frac{0.4h}{z_n}\right) - u_r\right] \tag{5.96}$$

Determination of a: Bagnold argued $a = 0$ at the threshold condition and $a \to 1$ for the high flow velocity corresponding to intense bed-load transport. It is thus given by

$$a = \frac{u_* - u_{*c}}{u_*} \tag{5.97}$$

Determination of u_r: The hydrodynamic drag force exerted by the flow on a particle is balanced by the bed resistance. It can be expressed as

$$F_x = \frac{1}{2}C_{Dx}\frac{\pi}{4}d^2\rho u_r^2 = F_G\tan\phi_d \tag{5.98}$$

where C_{Dx} is the drag coefficient for the drag force acting in the streamwise direction.

When a particle falls with a terminal fall velocity w_s in a still fluid, the drag force F_{Dz} acting on the particle is balanced by the submerged weight F_G of the particle. Then,

$$F_{Dz} = \frac{1}{2} C_{Dz} \frac{\pi}{4} d^2 \rho w_s^2 = F_G \tag{5.99}$$

where C_{Dz} is the drag coefficient for a settling particle. From Eqs. (5.98) and (5.99), the following relationship is obtained:

$$u_r = w_s \left(\frac{C_{Dz} \tan \phi_d}{C_{Dx}} \right)^{0.5} \tag{5.100}$$

It was found from the measured data that $C_{Dx} \approx C_{Dz}$ and $\tan^{0.5} \phi_d \approx 1$. Therefore, Eq. (5.100) becomes

$$u_r = w_s \tag{5.101}$$

Determination of z_n: In the absence of any bedforms, the average elevation of the saltating particles is proportional to their diameter. Thus,

$$z_n = m_1 d \quad \wedge \quad m_1 = K_1 \left(\frac{u_*}{u_{*c}} \right)^{0.6} \tag{5.102}$$

where K_1 is a coefficient. In laboratory experiments, $K_1 = 0.4$ was found by Francis (1973); but in rivers, it becomes 2.8 for sands and 7.3–9.1 for gravels (Bagnold 1977).

Equation of bed-load transport rate: Using Eqs. (5.97), (5.101) and (5.102) into Eq. (5.96), the equation of bed-load transport rate obtained by Bagnold in terms of submerged weight is given by

$$g_{bs} = \frac{u_* - u_{*c}}{u_*} \cdot \frac{\tau_0 U}{\tan \phi_d} \left[1 - \frac{1}{\kappa} \left(\frac{u_*}{U} \right) \ln \left(\frac{0.4h}{m_1 d} \right) - \left(\frac{w_s}{U} \right) \right] \tag{5.103}$$

Later, Bagnold (1966) simplified the analysis introducing an efficiency factor e_b for the bed-load transport. He balanced the available fraction of flow energy per unit time and area (that is the stream power) $\tau_0 U e_b$ with the work done required to move the bed-load particles $F_G \bar{u}_b \tan\phi_d (= g_{bs} \tan\phi_d)$. Thus, equation of bed-load transport rate is

$$g_{bs} = \frac{\tau_0 U}{\tan \phi_d} e_b \quad \wedge \quad g_b = \frac{s}{\Delta} g_{bs} \Rightarrow g_b = \frac{\tau_0 U s}{\Delta \tan \phi_d} e_b \tag{5.104}$$

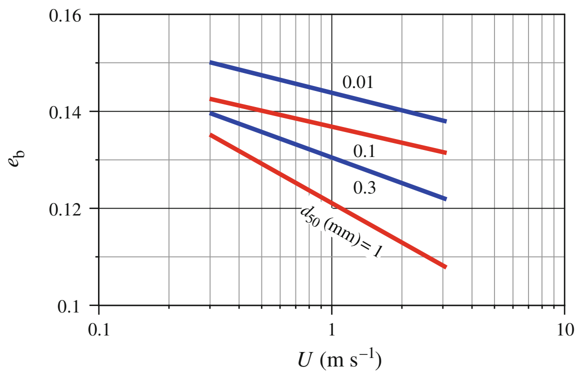

Fig. 5.12 Variation of bed-load transport efficiency e_b with U for different particle sizes d

The variation of bed-load transport efficiency e_b with U for different particle sizes d given by Bagnold is shown in Fig. 5.12. The prediction of e_b is possible for $d = 0.01$–1 mm.

5.8.2 *Yalin's Approach*

Yalin (1977) proposed a bed-load transport model based on the analysis of forces acting on a sediment particle. The equations of force acting on a moving sediment particle in the streamwise and normal directions are

$$F_x = m_G \frac{du_b}{dt} \tag{5.105a}$$

$$-F_z - F_G = m_G \frac{dw_b}{dt} \tag{5.105b}$$

where F_x and F_z are the force components of flow acting on a particle in the streamwise and normal directions, respectively, and u_b and w_b are the velocity components of a sediment particle in the streamwise and normal directions, respectively. The force components F_x and F_z are given by

$$F_x = \frac{\pi}{8} C_{Dx} \rho d^2 (u_d - u_b)^2 \tag{5.106a}$$

$$F_z = \frac{\pi}{8} C_{Dz} \rho d^2 w_b^2 \tag{5.106b}$$

where u_d is the instantaneous streamwise flow velocity at the particle level.

A particle detaches from the bed by the action of hydrodynamic lift force F_L. The difference $F_L - F_G > 0$ near the bed reduces with distance from the bed and becomes $F_L - F_G = 0$ at an elevation where the particle reaches its maximum vertical velocity component $[w_b]_{max}$. The $[w_b]_{max}$ can be determined from the following equation:

$$-F_z - F_G + F_L = m_G \frac{dw_b}{dt} \tag{5.107}$$

Equation (5.107) represents the initial condition of Eq. (5.105b). To solve these equations, Yalin made the assumptions: (1) The F_L/F_G ratio decreases with z/d according to the exponential law, that is $F_L/F_G \sim \exp(-z/d)$, (2) the drag coefficients C_{Dx} and C_{Dz} are constants, and (3) the nondimensional flow velocity u/u_* in the vicinity of the bed is constant.

As a result, he obtained an expression for u_b and then its average value $\bar{u}_b$ over the time when the particle is in motion. It is given by

$$\bar{u}_b = u_* C_1 \left\{ 1 - \frac{\Theta_c}{a_1(\Theta - \Theta_c)} \ln\left[1 + a_1\left(\frac{\Theta}{\Theta_c} - 1\right)\right]\right\} \tag{5.108}$$

where $a_1 = 2.45\Theta_c^{0.5}/s^{0.4}$ and C_1 is a constant to be determined. He determined the submerged weight of the bed-load transport per unit area W_s from the dimensional analysis. It follows

$$\frac{W_s}{\Delta\rho g d} = f_1(\Theta, R_{*d}) \tag{5.109}$$

where $\Theta = R_b S_0/(\Delta d)$, R_b is the hydraulic radius, and $R_{*d} = u_* d/\upsilon$. Equation (5.109) can be rewritten as

$$\frac{W_s}{\Delta\rho g d} = f_2\left(\Theta, \frac{\Delta g d^3}{\upsilon^2}\right) \quad \wedge \quad R_{*d} = \left(\frac{\Delta g d^3}{\upsilon^2}\Theta\right)^{0.5} \tag{5.110}$$

At the threshold of particle motion, $\Theta(W_s = 0) = \Theta_c$, and thus

$$f_2\left(\Theta_c, \frac{\Delta g d^3}{\upsilon^2}\right) = 0 \tag{5.111}$$

Equations (5.110) and (5.111) are combined to

$$\frac{W_s}{\Delta\rho g d} = f_2(\Theta, \Theta_c) \tag{5.112}$$

Yalin assumed that the left-hand side of Eq. (5.112) is linearly proportional to nondimensional excess bed shear stress. Hence,

$$\frac{W_s}{\Delta\rho g d} = C_2\left(\frac{\Theta}{\Theta_c} - 1\right) \tag{5.113}$$

where C_2 is a constant to be determined.

Substituting Eqs. (5.108) and (5.113) into Eqs. (5.105a, b) and determining the constants from the measured data, the bed-load transport rate g_b in weight per unit time and width is given by $g_b = (s/\Delta)g_{bs} = (s/\Delta)W_s\bar{u}_b$. Thus, the bed-load equation of Yalin is

$$g_b = 0.635\rho_s g d u_*\left(\frac{\Theta}{\Theta_c} - 1\right)\left\{1 - \frac{\Theta_c}{a_1(\Theta - \Theta_c)}\ln\left[1 + a_1\left(\frac{\Theta}{\Theta_c} - 1\right)\right]\right\} \tag{5.114}$$

Equation (5.114) can be expressed in nondimensional form as

$$\Phi_b = 0.635\Theta^{0.5}\left(\frac{\Theta}{\Theta_c} - 1\right)\left\{1 - \frac{\Theta_c}{a_1(\Theta - \Theta_c)}\ln\left[1 + a_1\left(\frac{\Theta}{\Theta_c} - 1\right)\right]\right\} \tag{5.115}$$

For initiation of bed-load transport, $\Theta \to \Theta_c$ and $a_1[(\Theta/\Theta_c) - 1] \approx 0$. Hence, one can write

$$\frac{\Theta_c}{a_1(\Theta - \Theta_c)}\ln\left[1 + a_1\left(\frac{\Theta}{\Theta_c} - 1\right)\right] \approx 1 - \frac{1}{2}\cdot a_1\left(\frac{\Theta}{\Theta_c} - 1\right) \tag{5.116}$$

The bed-load transport rate equation, Eq. (5.115), becomes

$$\Phi_b = 0.635 a_1\frac{\Theta^{0.5}}{2}\left(\frac{\Theta}{\Theta_c} - 1\right)^2 \tag{5.117}$$

For high intensity bed-load transport rate, $\Theta \gg \Theta_c$ and $(\Theta - \Theta_c) \to \infty$. Hence, it is given by

$$\frac{\Theta_c}{\Theta - \Theta_c} \to 0 \tag{5.118}$$

The bed-load transport rate equation, Eq. (5.115), then becomes

$$\Phi_b = 0.635\Theta^{0.5}\left(\frac{\Theta}{\Theta_c} - 1\right) \tag{5.119}$$

5.9 Equal Mobility Concept for Bed-Load Transport

Parker et al. (1982) developed a concept of equal mobility assuming that the bed-load transport of gravels can be accomplished through mobility of the particles exposed to the flow. The participation of the underneath particles in bed-load transport can only be possible up to the extent of the degradations that can result in an exposure of those particles to the flow. They referred coarser surface layer with bed-load transport as *pavement*; however, it is different from an *armor layer*. In this concept, the particle size distribution of bed load is approximated by that of underneath particles for all flows capable of mobilizing the majority of available gravel sizes.

Based on the equal mobility concept, Parker et al. (1982) developed a functional relationship between a bed-load transport function Φ_{bi}^{+} and a bed shear stress parameter Θ_i^{+} for a gravel size of d_i. The Φ_{bi}^{+} and Θ_i^{+} are given by

$$\Phi_{bi}^{+} = \frac{\Delta g_{bi}}{p_i (ghS_0)^{0.5} hS_0} \tag{5.120a}$$

$$\Theta_i^{+} = \frac{hS_0}{\Delta d_i \tau_{0i}^{+}} \tag{5.120b}$$

where g_{bi} is the bed-load transport rate per unit width for the fractional particle size d_i, p_i is the fraction by weight of size d_i, and $\tau_{0i}^{+} = 0.0875(d_{50}/d_i)$.

Due to equal mobility of all sizes, a specific particle size, termed *subpavement size* and denoted by d_{50}, is used to characterize the bed-load transport. Based on the field data of gravel-bed streams with sizes from 18 to 28 mm, Parker et al. (1982) proposed

$$\Phi_{b}^{+}(0.95 < \Theta_{50}^{+} < 1.65) = 2.5 \times 10^{-3} \exp[14.2(\Theta_{50}^{+} - 1) - 9.28(\Theta_{50}^{+} - 1)^2] \tag{5.121a}$$

$$\Phi_{b}^{+} = 11.2\left(1 - \frac{0.822}{\Theta_{50}^{+}}\right)^{4.5} \tag{5.121b}$$

where Θ_{50}^{+} is the bed shear stress parameter defined in Eq. (5.120b) corresponding to subpavement size d_{50}.

5.10 Sediment Pickup Function

Pickup rate, defined as volume rate of sediment removal per unit area, was studied by different investigators. Although the mode of bed-load transport according to the concept of pickup is not clear, there are three concepts of sediment pickup. As already discussed, Einstein (1950) hypothesized that after a period of rest, a sediment particle can only be picked up. The period of rest is longer than that of pickup. In his hypothesis, the total distance between two successive periods of rest can be traveled by a particle by performing several brief jumps. A particle covers an average step length of 100*d* by performing a jump. However, the pickup definition of Yalin (1977) is different from that of Einstein. Yalin hypothesized that a particle can be picked up when it detaches the bed surface to perform a jump. It implies that a jump by a particle involves a pickup and then deposition. According to de Ruiter (1982, 1983), the period of pickup equals the time period required to travel (from rest) by a particle over a distance of its half the diameter.

The approach of Einstein (1950) was stochastic. He assumed that a sediment particle is lifted when the instantaneous lift having a Gaussian distribution exceeds the submerged weight of the particle. His sediment pickup formula is

$$\Phi_{\mathrm{p}} = \alpha_{\mathrm{p}} p \tag{5.122}$$

where α_{p} is the coefficient and p is pickup or removal probability, that is the time fraction during which a sediment particle is picked up by the flow, which has already been discussed in Einstien's approach.

Fernandez Luque (1974) used experimental data for $0.9 \leq d \leq 1.8$ mm and proposed

$$\Phi_{\mathrm{p}}(0.05 \leq \Theta \leq 0.11) = \alpha_{\mathrm{p}}(\Theta - \Theta_{\mathrm{c}})^{1.5} \tag{5.123}$$

According to Yalin (1977), the period of pickup is proportional to the ratio of the particle diameter to shear velocity. Using a stochastic approach, he obtained a sediment pickup formula as

$$\Phi_{\mathrm{p}} = \alpha_{\mathrm{p}} p \Theta \tag{5.124}$$

Based on experimental data ($3 \leq d \leq 13.5$ mm), Nakagawa and Tsujimoto (1980) suggested

$$\Phi_{\mathrm{p}}(0.03 \leq \Theta \leq 0.2) = \alpha_{\mathrm{p}} \left(1 - \frac{0.035}{\Theta}\right)^{3} \Theta \tag{5.125}$$

They recommended $\alpha_{\mathrm{p}} = 0.02$ for spherical particles.

According to de Ruiter (1982, 1983), the pickup time period was found to be much smaller than that of instantaneous bed shear stress exceeding its threshold value. Based on stochastic approach, he proposed

$$\Phi_{\mathrm{p}} = \alpha_{\mathrm{p}} p_{\mathrm{p}} \left(\frac{\sigma_0}{\Delta \rho g d} \cdot \frac{\tan \phi}{\Theta_{\mathrm{c}}}\right)^{0.5} \tag{5.126}$$

where p_p is the pickup probability function and σ_0 is the standard deviation of instantaneous bed shear stress. The value of coefficient α_p, recommended by de Ruiter, is 0.016.

van Rijn (1984b) conducted experiments with different sand sizes ($0.13 \leq d \leq 1.5$ mm) and proposed an empirical equation of pickup function as

$$\Phi_p = 3.3 \times 10^{-4} D_*^{0.3} \left(\frac{\Theta}{\Theta_c} - 1 \right)^{1.5} \tag{5.127}$$

Dey and Debnath (2001) performed experiments with various uniform and nonuniform sand sizes ($0.24 \leq d \leq 1.55$ mm). Considering the effects of sediment nonuniformity, they proposed

$$\Phi_p = 6 \times 10^{-4} D_*^{0.24} \left(\frac{\Theta}{\Theta_c} - 1 \right) \sigma_g^{1.9} \tag{5.128}$$

where σ_g is the geometric standard deviation of particle size distribution.

5.11 Saltation

5.11.1 Characteristics of Saltation

When the bed shear stress just exceeds the threshold value for the initiation of particle motion, the particles roll and/or slide in contact with the bed. As the bed shear stress increases further, the particles move along the bed by a series of short jumps with approximately same step lengths. This phenomenon is called *saltation*. The saltation of a particle is governed by the hydrodynamic drag and lift forces and also the bed roughness. Due to the gravity, the particle begins to descend and returns to the bed when it is lifted by the hydrodynamic force to a certain height. In this way, the particle undergoes a saltation process as shown in Fig. 5.13. Subsequently, a new step of saltation may begin as a result of an impact against the bed and the lift force. According to the laboratory experimental observations by Francis (1973) and Abbott and Francis (1977), the characteristics of a saltating particle are described as follows:

The particle transport in saltation mode is limited to a maximum height of about ten times the particle diameter. The particle motion is dominated by the gravitational force, although it can be set off by the impulses of velocity fluctuations (near-bed turbulence agitations) during bursting events or by the effects of wall shear flow that a particle experiences a shear lift due to the velocity gradient in the vicinity of the bed. The hydrodynamic pressure and the viscous skin friction can also be the sources to provide momentum to the particles. In the rising stage of particle trajectory, the vertical component of the drag force and the gravitational

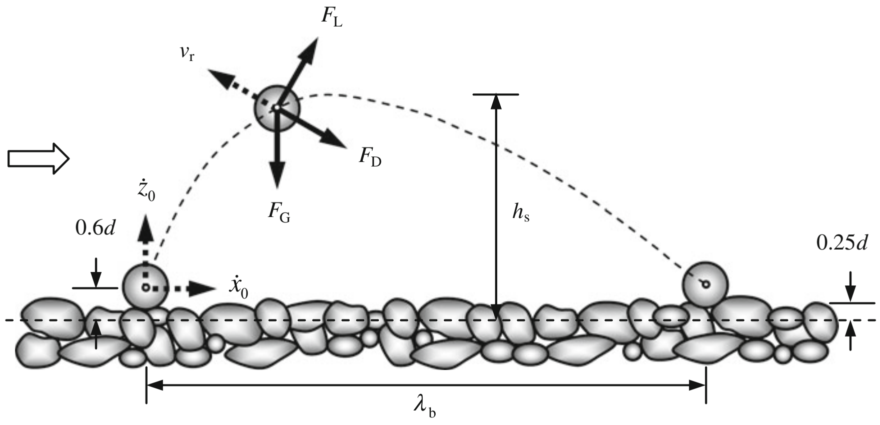

Fig. 5.13 Schematic of a particle saltation

force are together directed downwards; while in the falling stage of particle trajectory, the vertical component of drag force being directed upwards opposes the gravitational force. The lift force is always directed upwards provided the particle velocity to lag behind the fluid velocity at the saltating particle.

It is observed that some particles move in the form of series of saltations. It means that after the particles coming back to the bed performing a saltation, they immediately perform next saltation without any pause on the bed. It is obvious that the lift force is the main cause of lifting up the particles from the bed. However, the effects of the bed impact force by no means can be neglected. As a saltating particle strikes the bed particles, it may either ricochet off the bed particles or impact against them. During the impact of the particles with the bed particles, majority of the momentum, that they possess, is transferred to the bed particles in a succession of horizontal impulses. It may cause to initiate a rolling motion of the surface particles, termed *surface creep*. However, a saltating particle may cease motion, if it falls within one of the local depressions on the bed surface.

5.11.2 Particle Trajectory and Characteristic Parameters (van Rijn's Approach)

The forces acting on a saltating particle, as shown in Fig. 5.13, were analyzed by van Rijn (1984a). In fact, he analyzed the problem deterministically in the context of estimation of bed-load transport rate. In Fig. 5.13, the forces are the submerged weight of the particle F_G acting downwards and the hydrodynamic force components in the form of drag and lift. The direction of drag force F_D is opposite to the direction of the particle velocity v_r relative to the fluid flow; whereas the lift force is in the normal direction.

Equations of motion: The trajectory of a saltating particle can be determined by solving the equations of motion. Assuming a spherical saltating particle and the forces due to fluid accelerations to be of a second order, the equations of motion, according to White and Schultz (1977), can be written as

$$m_a \ddot{x} - F_L \left(\frac{\dot{z}}{v_r} \right) - F_D \left(\frac{\bar{u} - \dot{x}}{v_r} \right) = 0 \tag{5.129a}$$

$$m_a \ddot{z} - F_L \left(\frac{\bar{u} - \dot{x}}{v_r} \right) + F_D \left(\frac{\dot{z}}{v_r} \right) + F_G = 0 \tag{5.129b}$$

where m_a is the particle mass plus *added fluid mass*, v_r is the particle velocity relative to the fluid flow, that is $[(\bar{u} - \dot{x})^2 + \dot{z}^2]^{0.5}$, $\bar{u}$ is the local time-averaged flow velocity in x-direction, $\dot{x}$ and $\dot{z}$ are the streamwise and vertical velocities of particle, respectively, and $\ddot{x}$ and $\ddot{z}$ are the streamwise and vertical accelerations of particle, respectively.

The *added fluid mass* or *virtual mass* is the inertia added to a system. An accelerating or decelerating particle is to move some volume of surrounding fluid, as it moves through it, since the particle and fluid cannot occupy the same physical space simultaneously. For simplicity, this can be assumed as some volume of fluid moving with the particle, though in reality all the fluid is accelerated to various degrees. Therefore, the total mass of a spherical particle can be given by

$$m_a = \frac{1}{6}(\rho_s + \alpha_m \rho)\pi d^3 \tag{5.130}$$

where α_m is the added mass coefficient. Assuming a potential flow, the added mass of a sphere is obtained as the half of the fluid mass displaced by the sphere. However, in real fluid flow, the flow is separated from the sphere and α_m may be different from that for a potential flow. The value $\alpha_m = 0.5$ was considered by van Rijn.

The drag force F_D, which is resulted from the pressure and the viscous skin frictional effects, can be expressed as

$$F_D = C_D \frac{\rho}{2} v_r^2 \frac{\pi}{4} d^2 \tag{5.131}$$

The drag coefficient C_D can be determined from the empirical expressions given by Morsi and Alexander (1972).

The lift on a particle in the wall shear layer of flow is induced by two ways. They are due to (1) velocity gradient in the shear layer and (2) spinning motion of the particle as a *Magnus effect*. For a sphere moving in a viscous fluid flow, Saffman (1968) determined the lift F_{Ls} due to shear as

$$F_{\mathrm{Ls}} = C_{\mathrm{L}}\rho\upsilon^{0.5}d^2 v_{\mathrm{r}}\left(\frac{\partial \bar{u}}{\partial z}\right)^{0.5} \tag{5.132}$$

The Magnus lift F_{Lm} due to spinning motion in a viscous fluid flow obtained by Rubinow and Keller (1961) is given by

$$F_{\mathrm{Lm}} = C_{\mathrm{L}}\rho d^3 v_{\mathrm{r}}\omega \tag{5.133}$$

where ω is the angular velocity of the particle. The total lift force F_{L} is therefore

$$F_{\mathrm{L}} = F_{\mathrm{Ls}} + F_{\mathrm{Lm}} \tag{5.134}$$

The submerged weight of the spherical particle is given by Eq. (4.9). The velocity distribution in the wall shear layer is assumed to follow the logarithmic law given by Eq. (4.27), where the zero-velocity level can be considered as $z_0 = 0.11(\upsilon/u_*) + 0.03k_{\mathrm{s}}$.

Boundary conditions and solution scheme: The virtual bed level is assumed to be at $0.25d$ below the top of the bed particles, as shown in Fig. 5.13. The initial position of the particle lying on closely packed bed particles is $0.6d$ above the virtual bed level. Here, d is the representative particle size, assumed to be d_{50}. According to the experimental observations by Francis (1973) and Abbott and Francis (1977), $\dot{x} = \dot{z} = 2u_*$. Equations (5.129a, b) were first transformed[4] by van Rijn to a system of ordinary simultaneous differential equations of the first order. Then, he solved them by a numerical method known as automatic step-change differential equation solver. The characteristic parameters of saltating particles were computed for the range $u_* = 0.04$–0.14 m s^{-1} and $d_{50} = 0.1$–2 mm. He assumed $k_{\mathrm{s}} = 2d_{50}$ and calibrated C_{L} as $C_{\mathrm{L}}(R_{*d} \leq 5) = 1.6$, $C_{\mathrm{L}}(5 < R_{*d} < 70) = 1.6$–$20$ varying linearly, and $C_{\mathrm{L}}(R_{*d} \geq 70) = 20$.

Characteristic parameters of saltating particles: The saltation length λ_{b} and height h_{s} were first computed. Then, they are empirically correlated with the nondimensional particle parameter D_* [$= d(\Delta g/\upsilon^2)^{1/3}$] and the nondimensional excess bed shear stress $(\Theta/\Theta_{\mathrm{c}}) - 1$ as follows (van Rijn 1984a):

$$\frac{\lambda_{\mathrm{b}}}{d_{50}} = 3D_*^{0.6}\left(\frac{\Theta}{\Theta_{\mathrm{c}}} - 1\right)^{0.9} \tag{5.135a}$$

$$\frac{h_{\mathrm{s}}}{d_{50}} = 0.3D_*^{0.7}\left(\frac{\Theta}{\Theta_{\mathrm{c}}} - 1\right)^{0.5} \tag{5.135b}$$

[4] The procedure of transformation of second order differential equation to first order and the numerical solution methodology of a system of ordinary simultaneous differential equations can be found in Bose (2009).

Table 5.1 Formulas of saltation length λ_b and height h_s proposed by different investigators

References	Saltation length	Saltation height
Fernandez Luque and van Beek (1976)	$\lambda_b/d_{50} = 16$	–
Abbott and Francis (1977)	$\lambda_b = \lambda_b(\Theta)$	$h_s = h_s(d_{50}, \Theta)$
Sekine and Kikkawa (1992)	$\lambda_b/d_{50} = 3000(u_*/w_s)^{1.5} \times (u_* - u_{*c})/u_*$ where u_{*c} = threshold shear velocity	–
Niño et al. (1994)	$\lambda_b/d_{50} = 2.3\Theta/\Theta_c$	$h_s = h_s(d_{50}, \Theta/\Theta_c)$
Lee and Hsu (1994)	$\lambda_b/d_{50} = 196.3(\Theta - \Theta_c)^{0.788}$	$h_s/d_{50} = 14.3(\Theta - \Theta_c)^{0.575}$
Hu and Hui (1996)	$\lambda_b/d_{50} = 27.5\ s^{0.94}\Theta^{0.9}$	$h_s/d_{50} = 1.78\ s^{0.86}\Theta^{0.69}$
Lajeunesse et al. (2010)	$\lambda_b/d_{50} = 70(u_* - u_{*c})/w_s$	–

The above equations suggest that the saltation length and height increase with an increase in particle parameter and excess bed shear stress, but independent of flow depth. Experimental observations by Poreh et al. (1970) on saltation length and Williams (1970) on saltation height confirmed that $\lambda_b \approx 8d_{50}$ for $d_{50} = 1.35$ mm and $h_s = 5$–$40d_{50}$ for $d_{50} = 1.9$ mm. The results obtained from Eqs. (5.135a, b) are more or less in conformity with these experimental results. Besides van Rijn's Eqs. (5.135a, b), Table 5.1 furnishes the formulas of saltation length λ_b and height h_s proposed by different investigators. It is obvious that their results are quite varying from one another.

For a saltating particle, van Rijn (1984a) computed the mean velocity $\bar{u}_b$ as a function of nondimensional particle parameter and nondimensional bed shear stress as

$$\frac{\bar{u}_b}{u_*} = 9 + 2.6 \log D_* - 8\left(\frac{\Theta_c}{\Theta}\right)^{0.5} \tag{5.136}$$

Further, van Rijn (1984a) approximated Eq. (5.136) in a simpler form as

$$\frac{\bar{u}_b}{(\Delta g d_{50})^{0.5}} = 1.5\left(\frac{\Theta}{\Theta_c} - 1\right)^{0.6} \tag{5.137}$$

Besides Eqs. (5.136) and (5.137), Table 5.2 furnishes the formulas of mean velocity $\bar{u}_b$ of a saltating particle given by different investigators.

Bed-load transport rate: van Rijn (1984a) defined the bed-load transport rate q_b as a product of the particle velocity $\bar{u}_b$, the volumetric concentration C of transported particles, and the saltation height h_s. It is therefore given by

Table 5.2 Formulas of mean velocity $\bar{u}_b$ of a saltating particle given by different investigators

References	Mean velocity of particle
Fernandez Luque and van Beek (1976)	$\bar{u}_b = 11.5(u_* - 0.7u_{*c})$
Engelund and Fredsøe (1976)	$\bar{u}_b = u_*[10 - 0.7(\Theta_c/\Theta)^{0.5}]$
Abbott and Francis (1977)	$\bar{u}_b = a(u_* - u_{*c}) \quad \wedge \quad a = 13.4 - 14.3$
Sekine and Kikkawa (1992)	$\bar{u}_b = 8(u_*^2 - u_{*c}^2)^{0.5}$
Niño et al. (1994)	$\bar{u}_b = a(u_* - u_{*c}) \quad \wedge \quad a = 6.8 - 8.5$
Lee and Hsu (1994)	$\bar{u}_b = 11.53u_*(\Theta - \Theta_c)^{0.174}$
Hu and Hui (1996)	$\bar{u}_b = 11.9(u_* - 0.44u_{*c})$

$$q_b = \bar{u}_b C h_s \tag{5.138}$$

Note that if the saltation height h_s is replaced by the thickness δ_b of bed-load transport layer, then Eq. (5.138) becomes Eq. (5.3). Analysis of the experimental data by van Rijn (1981) showed that the bed-load concentration C (by volume) can be represented by

$$\frac{C}{C_0} = \frac{0.18}{D_*}\left(\frac{\Theta}{\Theta_c} - 1\right) \tag{5.139}$$

where C_0 is the maximum bed-load concentration. He determined $C_0 = 0.65$. It is interesting to note that the bed-load concentration C is inversely proportional to the nondimensional particle parameter and directly proportional to the nondimensional excess bed shear stress. Using Eqs. (5.135b), (5.137) and (5.139) into Eq. (5.138), van Rijn (1984a) obtained a bed-load transport equation, which has already been given as Eq. (5.26) as a du Boys type equation.

5.12 Fractional Bed Load of Nonuniform Sediments

Natural streams are typically made up of nonuniform sediment mixtures, whose transport phenomenon is therefore of immense importance. Unlike the transport of uniform sediment, the problems related to fractional nonuniform sediment transport are rather complex, especially when the consideration is given to the shelter–exposure interactions of bed particles of different sizes. Einstein (1950) was the pioneer to develop fractional transport rate of nonuniform sediments. Since then, Ashida and Michiue (1972), Parker et al. (1982), Patel and Ranga Raju (1996), Wu et al. (2000), and some other investigators put forward different methods to calculate the fractional bed-load transport rate of nonuniform sediments. Besides, Hsu and Holly (1992) proposed a method to determine the size fractional composition of nonuniform bed load aided by probability and availability of mobile sediments. The probabilistic approach by Einstein (1950) and the equal mobility approach by

Parker et al. (1982) taking into the fractional transport rate have already been discussed. Here, some other important methods are introduced.

Ashida and Michiue's (1972) bed-load transport formula for uniform sediment is given by Eq. (5.22). This formula was found to overestimate the individual size fractions of bed-load transport rate when compared with the experimental data for nonuniform sediment mixtures. They recommended the values of Shields parameters to be corrected for the fractional size of sediment. Thus, the equation of fractional bed-load transport intensity is given by

$$\Phi_{\mathrm{b}i} = 17(\Theta_i - \Theta_{\mathrm{c}i})(\Theta_i^{0.5} - \Theta_{\mathrm{c}i}^{0.5}) \quad \wedge \quad \Phi_{\mathrm{b}i} = \frac{q_{\mathrm{b}i}}{p_i(\Delta g d_i^3)^{0.5}} \quad \vee \quad q_{\mathrm{b}i} = i_{\mathrm{b}} q_{\mathrm{b}} \tag{5.140}$$

where Θ_i and $\Theta_{\mathrm{c}i}$ are the Shields parameter and threshold Shields parameter corresponding to a fraction p_i of size d_i, respectively, and i_{b} is the fraction of bed-load transport. Equation (5.140) thus can be used to compute total bed-load transport for the entire range of particle size distribution of the bed sediment.[5]

Hsu and Holly's (1992) method begins with the determination of the size distribution of transported sediment and then ends with the estimation of bed-load transport rate. The each fractional size d_i in the transported sediment is hypothesized to be proportional to the joint probability of its mobility under the prevailing hydraulic condition and its availability on the bed surface within the active layer. If the flow velocity fluctuations follow the Gaussian probability distribution, the probability $p_{\mathrm{r}i}$ of removal of size d_i is derived as

$$p_{\mathrm{r}i} = \frac{1}{(2\pi)^{0.5}\sigma_{\tilde{u}_{\mathrm{d}}}} \int\limits_{(u_{\mathrm{cr}i}/\bar{u}_{\mathrm{d}})-1}^{\infty} \exp\left(-\frac{\tilde{u}_{\mathrm{d}}^2}{2\sigma_{\tilde{u}_{\mathrm{d}}}^2}\right) \mathrm{d}\tilde{u}_{\mathrm{d}} = 0.5 - 0.5\mathrm{erf}\left(\frac{(u_{\mathrm{cr}i}/\bar{u}_{\mathrm{d}}) - 1}{2^{0.5}\sigma_{\tilde{u}_{\mathrm{d}}}}\right) \tag{5.141}$$

---

[5] The procedure of computation of total bed-load transport for the entire range of particle size distribution of the bed sediment is as follows:

Step 1: Compute $\Phi_{\mathrm{b}i}$ for the fraction p_i of sediment size d_i by using Einstein's approach or Ashida and Michiue's bed-load transport formula or by any other standard method given in this chapter.

Step 2: Compute $i_{\mathrm{b}} q_{\mathrm{b}}$ by using Eq. (5.140) as

$$\Phi_{\mathrm{b}i} = \frac{q_{\mathrm{b}i}}{p_i(\Delta g d_i^3)^{0.5}} \quad \wedge \quad q_{\mathrm{b}i} = i_{\mathrm{b}} q_{\mathrm{b}} \;\Rightarrow\; i_{\mathrm{b}} q_{\mathrm{b}} = \Phi_{\mathrm{b}i} \times p_i(\Delta g d_i^3)^{0.5}$$

Step 3: For each size fraction, the $i_{\mathrm{b}} q_{\mathrm{b}}$ can be computed in this way. The total bed-load transport can therefore be obtained by summing up the results over the entire range of particle size distribution.

Note: In case of a mixture of small size of sediment spread, the size d_{35} can be approximated as an effective sediment size for the approximate estimation of total bed-load.

where $\tilde{u}_\mathrm{d} = u'/\bar{u}_\mathrm{d}$, u' is the fluctuations of instantaneous streamwise velocity at the bed particle level, $\bar{u}_\mathrm{d}$ is the time-averaged streamwise velocity at the bed particle level, $\sigma_{\tilde{u}_\mathrm{d}}$ is the standard deviation of $\tilde{u}_\mathrm{d}$, and $u_{\mathrm{cr}i}$ is the near-bed threshold velocity for the initiation of motion of sediment size d_i. They employed Qin's (1980) formula given by Eq. (4.160) for the computation of $u_{\mathrm{cr}i}$ for a given size d_i, and used $\sigma_{\tilde{u}_\mathrm{d}} = 0.2$, as suggested by Yen et al. (1988).

The availability of fractional size d_i is equivalent to its fractional representation $\beta_{\mathrm{b}i}$ on the bed surface within the active layer. Therefore, the fraction p_i of size d_i in the transported sediment is

$$p_i = \frac{p_{\mathrm{r}i}\beta_{\mathrm{b}i}}{\int_{d_\mathrm{min}}^{d_\mathrm{max}} p_{\mathrm{r}i}\beta_{\mathrm{b}i}\mathrm{d}d} \tag{5.142}$$

In this way, the particle size distribution of the transported sediment in bed load is determined. Then, the weighted mean sediment size d_m and mean near-bed threshold velocity u_crm are estimated. For the estimation of bed-load transport rate for all fractional sizes, Hsu and Holly used Shamov's formula (Zhang et al. 1989):

$$g_\mathrm{b} = 12.5 g d_\mathrm{m}^{0.5}\left(\bar{u}_\mathrm{d} - u_\mathrm{cr}|_{d_\mathrm{min}}\right)\left(\frac{\bar{u}_\mathrm{d}}{u_\mathrm{crm}}\right)^3\left(\frac{d_\mathrm{m}}{h}\right)^{0.25} \quad \wedge \quad u_\mathrm{cr}|_{d_\mathrm{min}} = u_\mathrm{cr}(d_\mathrm{min}) \tag{5.143}$$

Hsu and Holly originally expressed g_b in mass of bed-load transport rate per unit width. The right-hand side of Eq. (5.143) is multiplied by g (acceleration due to gravity) to convert the unit to N m^{-1} s^{-1}.

Patel and Ranga Raju (1996) expressed the fractional bed-load transport intensity $\Phi_{\mathrm{b}i}$ as a function of $\Theta_{\mathrm{c}i}\xi_\mathrm{b}$. In fact, they corrected the threshold Shields parameter $\Theta_{\mathrm{c}i}$ corresponding to fractional size d_i by a factor ξ_b, termed *hiding–exposure correction factor*. The estimation of ξ_b is as follows:

$$\xi_\mathrm{b} = \frac{0.0713}{C_\mathrm{m}(C_\mathrm{s}\Theta_{\mathrm{c}i})^{0.75144}} \quad \wedge \quad \Theta_{\mathrm{c}i} = \frac{\tau'_{0i}}{\Delta\rho g d_i} \tag{5.144a}$$

$$C_\mathrm{m}(M > 0.38) = 1, C_\mathrm{m}(0.05 < M \leq 0.38) = 0.7092 \log M + 1.293 \tag{5.144b}$$

$$\log C_\mathrm{s} = 0.0644\tau^{*3} - 0.1949\tau^{*2} - 0.9571\tau^* - 0.1957 \quad \wedge \quad \tau^* = \log\left(\frac{\tau'_{0i}}{\tau_{0\mathrm{cg}}}\right) \tag{5.144c}$$

where M is the Kramer's uniformity parameter, $\tau'_{0i} = \rho g R'_\mathrm{b} S_0$, $R'_\mathrm{b} = (Un'/S_0^{0.5})^{1.5}$, $n' = d_{65}^{1/6}/24$, and $\tau_{0\mathrm{cg}}$ is the threshold bed shear stress for the geometric mean size d_g [$\approx (d_{84.1}d_{15.9})^{0.5}$] of the nonuniform sediment mixture as per Shields. The variation of fractional bed-load transport intensity $\Phi_{\mathrm{b}i}$ with $\Theta_{\mathrm{c}i}\xi_\mathrm{b}$ obtained by Patel and Ranga Raju (1996) is illustrated in Fig. 5.14.

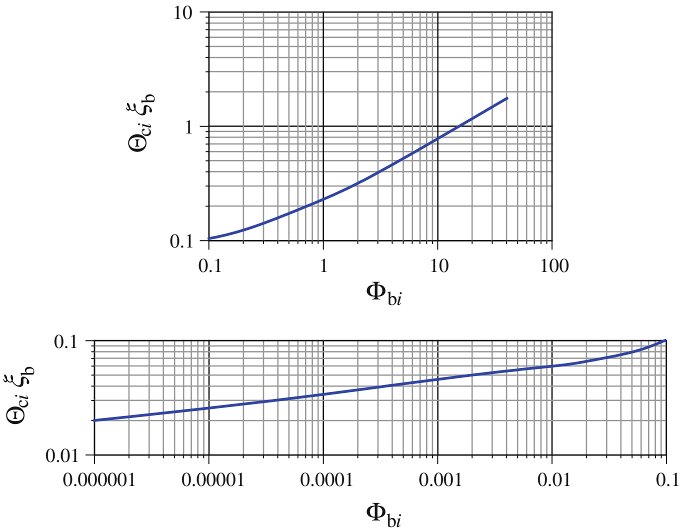

Fig. 5.14 Curve to estimate fractional bed-load transport rate (Patel and Ranga Raju 1996)

Wu et al. (2000) presented a relationship of fractional bed-load transport intensity Φ_{bi} as a function of nondimensional excess particle bed shear stress. It is

$$\Phi_{bi} = 5.3 \times 10^{-3}\left[\left(\frac{n'}{n}\right)^{1.5}\left(\frac{\tau_0}{\tau_{0ci}}\right) - 1\right]^{2.2} \tag{5.145}$$

where $n' = d_{50}^{1/6}/20$, τ_{0ci} is the threshold bed shear stress corresponding to size d_i, $\tau_0 = \rho g R_b S_0$, R_b is the total hydraulic radius, and n is the Manning roughness coefficient $(= R_b^{2/3} S_0^{0.5}/U)$.

Note that one can use Meyer-Peter and Müller (1948) formula for the estimation of fractional bed-load transport intensity Φ_{bi} (van Rijn 1993). The threshold bed shear stress is corrected to account for the nonuniformity effect as $\xi_i \Theta_c$. Then,

$$q_{bi} = 8 p_i (\Delta g d_i^3)^{0.5}\left[\left(\frac{C_R}{C_R'}\right)^{1.5}\Theta_i - \xi_i \Theta_c\right]^{1.5} \Rightarrow \Phi_{bi} = 8(\eta_C \Theta_i - \xi_i \Theta_c)^{1.5} \tag{5.146}$$

In the above, the correction factor ξ_i given by Egiazaroff (1965) is

$$\xi_i = \frac{\log^2 19}{\log^2(19 d_i/d_m)} \quad \wedge \quad d_m = \sum p_i d_i \tag{5.147}$$

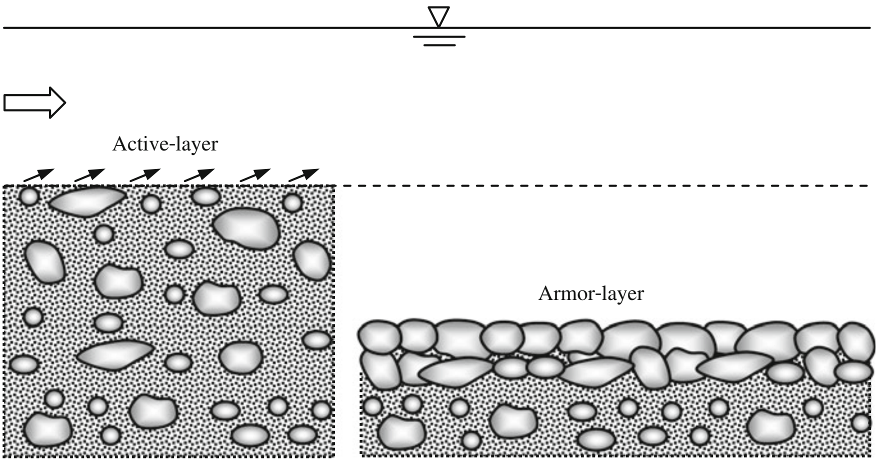

Fig. 5.15 Definition sketch of streambed armoring

5.13 Sediment Sorting and Streambed Armoring

The time-dependent transport rate of nonuniform sediment mixture is a complicated process due to sorting of a sediment bed in addition to sediment diffusion as suspension. In a sediment mixture, the resistance to an individual particle motion depends upon particle size and shape, as well as sheltering and exposure to the flow. *Sediment sorting* is defined as a selective transport of different fractional sizes of sediment particles. When the sediment transport rate of a bed exceeds the rate of sediment supply by the approaching flow, the sediment bed starts to degrade. *Active layer* refers to the surface layer of the sediment bed from which the sediment can be entrained to the flow. Because of the nonuniformity of the sediment, typically, exposed finer particles are transported easily at a faster rate than the coarser ones, and the remaining bed particles are being coarsened. Thus, the size of particles' sorting takes place. The weakly entrained or unentrained coarse particles tend to accumulate in the surface layer, forming a band of coarser particles. Gradually, this coarsening process stops until a layer of coarse particles is completely developed to cover the streambed protecting the underneath finer sediment particles from being transported. Once this process is completed, the streambed is called *armored* and the layer of coarsest particles is called the *armor layer* (Fig. 5.15).

Due to variable nature of flow condition of a natural streambed, typically one or more than one layers of armor particles are required to protect the underneath finer sediment particles (Fig. 5.16). Borah (1989) and Froehlich (1995) reported that the natural armor-layer thickness is one to three times the armor-particle sizes. However, the thickness, porosity, and particle size distribution of an armor layer

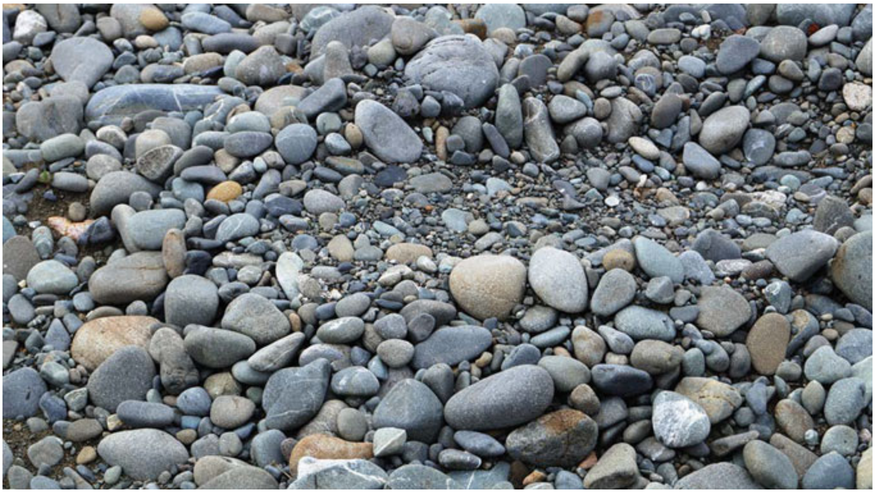

Fig. 5.16 Natural streambed armoring. Photograph by the author

vary with flow and bed evolution. Importantly, fine sediment can still be winnowed at a very feeble rate through the interstices of armored particles.

Borah et al. (1982) considered an active layer to be homogeneous within itself at any given time and proposed estimation for the thickness of the active layer t_a from the volumetric consideration as

$$t_a = \frac{1}{\sum_{i=M}^{n} p_i} \cdot \frac{d_M}{1 - \rho_{0M}} \tag{5.148}$$

where M is the fraction of the size d_M or larger than d_M, that cannot be transported by the flow, d_M is the size for the fraction M, and ρ_{0M} is the porosity for the fraction M. Thus, the fractional size d_M and larger sizes contribute to an armor layer.

In an active layer, the rate of transport from the bed surface decreases with time but does not truly go to zero even after a long time (several days). It implies that the development of an armor layer is an asymptotic process. When the bed shear stress increases, the finer particles are transported and coarser ones stay in place. Eventually, an upper limiting condition of the streambed is reached, which is called the *threshold armoring condition*. The corresponding bed shear stress is used to define the *threshold bed shear stress for armoring* τ_{0ca}. Hence, a sediment mixture has a range of bed shear stress over which its bed surface can be armored. The armor layer is thus now formed by the near-coarsest particles d_{90} or even coarser than d_{90}, which are found in a particle size distribution curve of nonuniform sediment. However, for a higher flow rate, when $\tau_0 > \tau_{0ca}$, the armor layer becomes unstable and subsequently is destroyed. Correia and Graf (1988) suggested the median size of armor particles: $d_{50a} \approx 1.4 d_{50}$ and $d_{50a} \leq 0.6 d_{90}$.

Raudkivi (1990) gave an empirical relationship for the estimation of stability of particles in an armor layer as

$$\frac{\Theta_{\mathrm{ca}}}{\Theta_{\mathrm{c}}} = \left[0.4\left(\frac{d_{50}}{d_{50\mathrm{a}}|_{\max}}\right)^{0.5} + 0.6\right]^2 \quad \wedge \quad \Theta_{\mathrm{ca}} = \frac{\tau_{0\mathrm{ca}}}{\Delta\rho g d_{50\mathrm{a}}|_{\max}} \quad \vee \quad \frac{d_{50\mathrm{a}}|_{\max}}{d_{100}} \leq 0.55 \tag{5.149}$$

where Θ_{ca} is the threshold Shields parameter for the armoring particles and $d_{50\mathrm{a}}|_{\max}$ is the maximum size of the armoring particles, being determined by extrapolating the particle size distribution curve on the basis of last two or three points. Importantly, no armoring takes place for uniform sediments.

In case of a nonuniform sediment sample with a mixture of fine and large particles (for example, sand and gravel), Chin et al. (1994) observed that the stability of individual large particles and their number in the bed govern the process of formation of a stable armor layer. The removal of finer particles from the bed surface exposes individual large particles. As a large particle is exposed considerably to the flow, it leads to the formation of an erosion pit in the front and a deposition of finer particles at the rear. The large particle may then slide into the erosion pit, reducing its exposure to the flow and becoming more stable. Medium and relatively coarse particles may also be accumulated within the scour pit and finer particles may hide behind and in between the larger ones. Gradually, this rearrangement of the surface particles leads to the formation of clusters of particles of various sizes. A cluster may slowly collapse with an erosion of the bed at its periphery. The anchor large particle may then be moved downstream to another stable position; and the process of cluster formation may be repeated. Thus, the formation of an armor layer in a nonuniform sediment mixture with fine and large particles is a continuous process involving formation and collapse of clusters. In Sect. 8.6, formation of cluster is further discussed.

5.14 Sediment Entrainment Probability to Bed Load

Determination of sediment entrainment probability to bed load is an essential prerequisite in developing a probabilistic theory of bed-load transport. Einstein (1942) laid the foundation of the probabilistic concept to study the bed-load transport, in which the probability of sediment removal was introduced. The most innovative development was due to Einstein (1950) to introduce a formula for the probability of sediment transport [see Eq. (5.70)]. It was based on the probability of hydrodynamic lift induced by the fluctuating velocity to exceed submerged particle weight, using the Gaussian distribution, as already discussed. The probability function p that is given by Eq. (5.69) can be written in a simplified form as

$$p = 1 - \frac{1}{\pi^{0.5}} \int_{-0.143\Theta^{-1}-2}^{0.143\Theta^{-1}-2} \exp(-t^2)\mathrm{d}t \tag{5.150}$$

Subsequent investigations by other researchers viewed the probability of sediment removal in different ways and put forward formulation for probability in terms of entrainment or pickup probability function. The *entrainment probability function* is a function of Shields parameter Θ. Engelund and Fredsøe (1976) gave an empirical formula for the entrainment probability function [see Eq. (5.87)] by using experimental data of Guy et al. (1966) and Fernandez Luque (1974). The formula was subsequently modified by Fredsøe and Deigaard (1992) in the form

$$p = \left[1 + \left(\frac{\mu_\mathrm{d}\pi/6}{\Theta - \Theta_\mathrm{c}}\right)^4\right]^{-0.25} \tag{5.151}$$

However, following Einstein's concept of bed-load transport, Cheng and Chiew (1998) obtained an expression for the entrainment probability function, based on the assumption of the Gaussian distribution for the streamwise velocity fluctuations. They expressed the sediment entrainment probability in hydraulically rough flow regime as

$$p = p(F_\mathrm{L} > F_\mathrm{G}) = p(u_\mathrm{d}^2 > B^2) = p(u_\mathrm{d} > B) + p(u_\mathrm{d} < -B) \tag{5.152}$$

where u_d is the instantaneous near-bed velocity and $B = [4\Delta gd/(3C_\mathrm{L})]^{0.5}$. They estimated the time-averaged near-bed velocity $\bar{u}_\mathrm{d}$, using the logarithmic law and fixing the zero-plane displacement level at $0.25d$ and the zero-velocity level z_0 at $k_\mathrm{s}/30$ below the top of the closely packed bed particles. Here, k_s was considered as $2d$. They assumed that a particle placed in an interstice between two bed particles is about to move. In this way, they estimated $\bar{u}_\mathrm{d} = 5.52u_*$ acting on the particle in an initial position at $z = 0.6d$. Quoting Kironoto and Graf (1994), Cheng and Chiew (1998) assumed $\sqrt{\overline{u'^2}} = 2u_*$ and finally obtained the entrainment probability as

$$\begin{aligned} p = 1 - 0.5\frac{0.21 - \sqrt{\Theta C_\mathrm{L}}}{|0.21 - \sqrt{\Theta C_\mathrm{L}}|}\sqrt{1 - \exp\left[-\left(\frac{0.46}{\sqrt{\Theta C_\mathrm{L}}} - 2.2\right)^2\right]} \\ - 0.5\sqrt{1 - \exp\left[-\left(\frac{0.46}{\sqrt{\Theta C_\mathrm{L}}} + 2.2\right)^2\right]} \end{aligned} \tag{5.153}$$

Cheng and Chiew (1998) selected a value of $C_\mathrm{L} = 0.25$ to fit Eq. (5.153) with the previous experimental data, as shown in Fig. 5.17.

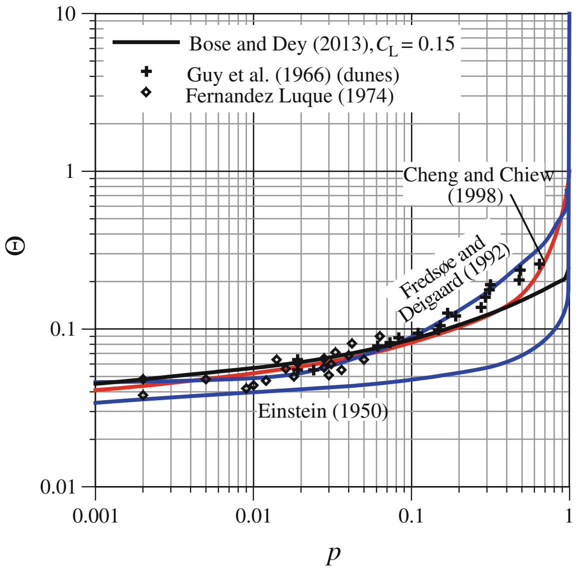

Fig. 5.17 Variation of Shields parameter Θ with probability p of sediment entrainment (Bose and Dey 2013). The Θ(p) curves obtained from the approaches given by Einstein (1950), Fredsøe and Deigaard (1992) and Cheng and Chiew (1998), and the experimental data of Guy et al. (1966) and Fernandez Luque (1974) are shown for the comparison

Later, Wu and Lin (2002) noted that since only positive fluctuations in the streamwise velocity could cause entrainment of bed particles, a log-normal distribution could be better suited to derive an expression for the entrainment probability. They therefore modified the concept of entrainment probability as

$$p = p(u_\mathrm{d} > B) = p(\ln u_\mathrm{d} > \ln B) = 1 - p(-\infty < \ln u_\mathrm{d} < \ln B) \qquad (5.154)$$

Wu and Lin (2002) finally expressed the entrainment probability as

$$p = 0.5 - 0.5\frac{\ln(0.044\Theta^{-1}C_\mathrm{L}^{-1})}{|\ln(0.044\Theta^{-1}C_\mathrm{L}^{-1})|}\sqrt{1 - \exp\left\{-\frac{2}{\pi}\left[\frac{\ln(0.044\Theta^{-1}C_\mathrm{L}^{-1})}{0.724}\right]^2\right\}} \qquad (5.155)$$

Bose and Dey (2013) argued that the Gaussian and the log-normal distributions primarily occur when there is additive and multiplicative accumulation of errors. This is, however, not the case of turbulent velocity fluctuations in open channel flow. *Bose–Dey universal probability theory* (see Sect. 3.17.1), on the other hand, gave the Gram–Charlier series expansion of the probability densities based on the two-sided exponential or Laplace distribution. They explained that the probability density function (henceforth pdf) $p_{\hat{u}}(\hat{u})$ for the nondimensional streamwise velocity fluctuations $\hat{u}$ can be given by

$$p_{\hat{u}}(\hat{u}) = \frac{1}{2}\left[1 + \frac{1}{2}C_{10}\hat{u} - \frac{1}{8}C_{20}(1 + |\hat{u}| - \hat{u}^2) - \frac{1}{48}C_{30}\hat{u}(3 + 3|\hat{u}| - \hat{u}^2) + \frac{1}{384}C_{40}(9 + 9|\hat{u}| - 3\hat{u}^2 - 6|\hat{u}^3| + \hat{u}^4) + \cdots\right]\exp(-|\hat{u}|) \quad \wedge \quad \hat{u} = \frac{u'}{\sqrt{\overline{u'^2}}} \tag{5.156}$$

Dey et al. (2012) obtained that the coefficients C_{10} and C_{30} are of the order of 0.001; while $C_{20} \approx -0.5$ and $C_{40} \approx 0.6$. Thus, it was assumed that $C_{20} \approx -0.5$ and the rest of the coefficients are effectively negligible due to their smallness or division by a large number, such as 384. Then, Eq. (5.156) reduces to

$$p_{\hat{u}}(\hat{u}) = \frac{1}{32}(17 + |\hat{u}| - \hat{u}^2)\exp(-|\hat{u}|) \tag{5.157}$$

The instantaneous near-bed streamwise velocity u_d, which can be decomposed as $u_d = \bar{u}_d + u'$, is the cause of an entrainment of particles lying on the bed. Wu and Lin (2002), following Nelson et al. (1995), argued that the entrainment is only possible when the velocity fluctuations $u' > 0$, for which the pdf according to Eq. (5.157) becomes the one-sided exponential based Gram–Charlier series. Therefore,

$$\left.\begin{aligned} p_{u'}(u' \geq 0) &= \frac{1}{16\sqrt{\overline{u'^2}}}(17 + \hat{u} - \hat{u}^2)\exp(-\hat{u}) \\ p_{u'}(u' < 0) &= 0 \end{aligned}\right\} \tag{5.158}$$

where $p_{u'}(u')$ is the pdf for u'. It satisfies the condition

$$\int_{-\infty}^{\infty} p_{u'}(u')\mathrm{d}u' = \int_{0}^{\infty} p_{u'}(u')\mathrm{d}u' = 1$$

Following Einstein (1950), a particle placed on the bed is likely to be lifted by the flowing fluid provided $F_L > F_G$. Importantly, the instantaneous lift force F_L acting on a particle fluctuates in accordance with the velocity fluctuations u' of the near-bed velocity u_d; while the submerged weight F_G of a particle is a constant for a given particle size. Therefore, $F_L > F_G$ implies that $u_d > B$ or $u' > B - \bar{u}_d$, where $B = [4\Delta gd/(3C_L)]^{0.5}$. Thus, using Eq. (5.158), the total entrainment probability p is

$$p = \int_{B-\bar{u}_d}^{\infty} p_{u'}(u')\mathrm{d}u' = \frac{1}{16}(16 - a - a^2)\exp(-a) \quad \wedge \quad a = \frac{B - \bar{u}_d}{\sqrt{\overline{u'^2}}} \tag{5.159}$$

It is pertinent to mention that Dey et al. (2012) found that when the bed particles move, the von Kármán constant κ diminishes from its universal value 0.41, and the zero-plane displacement level and the zero-velocity level move up as compared to their values in immobile beds [also available in Dey and Raikar (2007), Gaudio et al. (2010), Dey et al. (2011), and Gaudio and Dey (2012)]. These modify the estimation of near-bed velocity from the logarithmic law as $\bar{u}_d = 6.4u_*$, which was used by Bose and Dey. Quoting Kironoto and Graf (1994), Cheng and Chiew (1998) estimated $\sqrt{\overline{u'^2}} = 2u_*$. Using these results, the a can be expressed as

$$a = \frac{B - \bar{u}_d}{\sqrt{\overline{u'^2}}} = \frac{1}{2\sqrt{\Delta g d \Theta}} \left(\sqrt{\frac{4\Delta g d}{3C_L}} - 6.4\sqrt{\Delta g d \Theta} \right) = \frac{1}{\sqrt{3C_L \Theta}} - 3.2 \quad (5.160)$$

Figure 5.17 depicts the theoretical $\Theta(p)$ curve for $C_L = 0.15$ obtained by solving Eq. (5.159) using Eq. (5.160). The theoretical curve matches well with the experimental data of Guy et al. (1966) and Fernandez Luque (1974). The data of Guy et al. (1966) that correspond to dunes have less agreement, because the analysis by Bose and Dey (2013) did not include the flow resistance due to bed-forms. However, the curve obtained by Bose and Dey (2013) corresponds closely with the curves of Fredsøe and Deigaard (1992) and Cheng and Chiew (1998) for $p < 0.2$. The Shields parameter Θ for rough flow regime ($R_* > 70$, where R_* is the shear Reynolds number, $u_* k_s/\upsilon$) according to Yalin and Karahan's (1979) diagram is 0.046, for which the probability of entrainment is 0.1 % as obtained from Fig. 5.17. It implies that 0.1 % of all the particles on a given bed area are in motion under the threshold of sediment entrainment.

5.15 Effects of Bed Load on Velocity Distribution

Dey et al. (2012) conducted experiments to measure the velocity distributions and turbulence parameters in mobile-bed flow with bed-load transport and to compare them with those in a clear-water (immobile bed) flow. The experimental data for clear-water flow were used as a reference. For each sediment sample, an experimental set comprised of two different runs, such as clear-water and mobile-bed flow conditions. Fixed-bed roughness for a clear-water flow was prepared by gluing sediment on the flume bottom. The mobile-bed experiments were conducted to simulate the bed-load transport at a certain rate corresponding to the same flow condition as that of the clear-water flow. In mobile-bed experiments, the same sediment that was glued on the flume bottom was fed in the flow at a uniform rate to achieve a dynamic equilibrium condition of the mobile bed. A continuous weak sediment transport (as bed load) was established by the flow in the form of a thin sediment layer disallowing any bedforms to develop.

Figure 5.18 shows the vertical distributions of nondimensional time-averaged streamwise velocity u^+ for clear-water and mobile-bed flows. In order to fit the data points in the inner layer ($z \leq 0.2h$) to the universal logarithmic law of wall, the time-averaged streamwise velocity $\bar{u}$ and the vertical distance z are scaled by u_* and d_{50}, such that $u^+ = \bar{u}/u_*$ and $z^+ = z/d_{50}$. For the convenience, the origin of z-axis is set at the top of the bed particles (that is the bed surface). As the flow regime was the rough-turbulent flow, it is customary to use d_{50} to scale z. Dey et al. used the values of u_* that were obtained from the Reynolds shear stress plots by extrapolating a linear curve fitting onto the bed surface. To plot the experimental data, they consider the logarithmic law expressed in nondimensional form. It is

$$u^+ = \frac{1}{\kappa}\ln\left(\frac{z^+ + \Delta z^+}{\zeta^+}\right) \tag{5.161}$$

where $\Delta z^+ = \Delta z/d_{50}$, Δz is the depth of the virtual bed level from the bed surface, $\zeta^+ = z_0/d_{50}$, and z_0 is the zero-velocity level. Figure 5.18 describes the logarithmic law showing the variations of u^+ with $z^+ + \Delta z^+$ for the experimental datasets. It is clear that a prior estimation of Δz^+ was an essential prerequisite to plot the data, and subsequent determination of κ and ζ^+ was required to fit the data to the logarithmic law given by Eq. (5.161). The determination of these parameters was done independently, as described below:

Step 1: Having obtained u_* from the Reynolds shear stress plots by projecting straight line on the bed surface [see Eq. (3.20) and Fig. 3.11], such that $u_* = (-\overline{u'w'})^{0.5}\big|_{z=0}$, the dataset $u^+(z^+)$ for the range $z \leq 0.2h$ were prepared for the data analysis.

Step 2: As an initial trial, considering $\Delta z^+ = 0$, the values of κ and ζ^+ were determined from Eq. (5.161) by the regression analysis, and then, the regression coefficient *RC* was evaluated.

Step 3: The values Δz^+ were increased at a regular interval by a small magnitude (say 0.001), and the values of κ and ζ^+ were determined in the same way as in step 2. The values of *RC* for each value of Δz^+ were checked, till *RC* became the maximum. Then, the corresponding values of Δz^+, κ and ζ^+ were considered as parameters for Eq. (5.161).

The average values of $\Delta z^+ = 0.39$, $\kappa = 0.413$, and $\zeta^+ = 0.034$ obtained for clear-water flow are in agreement with those for the traditional logarithmic law over rough boundary. Typically, the customary values of these parameters for the rough beds are $\Delta z^+ = 0.25$, $\kappa = 0.41$, and $\zeta^+ = 0.033$ (van Rijn 1984a). Thus, for clear-water flow, the data collapse well on the average logarithmic law curve shown by a solid line in Fig. 5.18. On the other hand, the average values of $\Delta z^+ = 0.21$, $\kappa = 0.37$, and $\zeta^+ = 0.04$ obtained for mobile-bed flow suggest the modified values of the parameters for the logarithmic law over a rough mobile bed. It is obvious that for mobile-bed flow, the data exhibit some degree of scatter about the average logarithmic law curve. A comparison of the values of Δz^+ and ζ^+ for

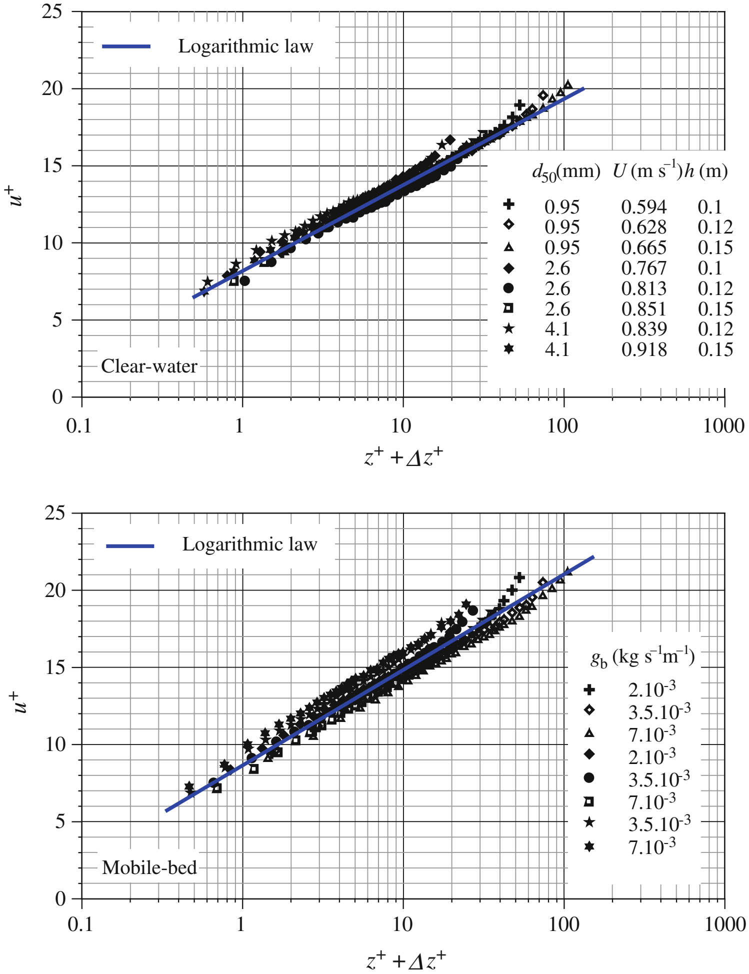

Fig. 5.18 Vertical distributions of nondimensional time-averaged streamwise velocity u^+ for clear-water and mobile-bed flows (Dey et al. 2012)

clear-water and mobile-bed flows reveals that the virtual bed and zero-velocity levels move up in the presence of bed-load transport. Although the data analysis related to the logarithmic law was done considering the data range $z \leq 0.2h$, Fig. 5.18 displays all the data plots for $z \leq 0.2h$ and $z > 0.2h$. Thus, the data plots depart from the logarithmic law in the outer layer to some extent. Additionally, the values of Nikuradse's equivalent sand roughness k_s can be determined from the relationship of zero-velocity level as $k_s = 30\zeta^+ d_{50}$. Finally, it can be concluded that for mobile-bed flow, (1) the von Kármán constant decreases and (2) the virtual bed and the zero-velocity levels move up.

5.16 Effects of Bed Load on Length Scales of Turbulence

According to Prandtl, the mixing length l, which defines a distance that a fluid parcel (eddy) keeps its original characteristics before dispersing into the surrounding fluid, is given by

$$l = \frac{(-\overline{u'w'})^{0.5}}{\mathrm{d}\bar{u}/\mathrm{d}z} \tag{5.162}$$

To calculate the mixing length l from Eq. (5.162), Dey et al. (2012) used the measured velocity profiles to determine the velocity gradients $\mathrm{d}\bar{u}/\mathrm{d}z$ by a smooth curve fitting to the data plots. They obtained the values of $-\overline{u'w'}$ directly from the measured Reynolds shear stress distributions. The variations of nondimensional mixing length $\tilde{l}(= l/h)$ with $\tilde{z}(= z/h)$ for clear-water and mobile-bed flows as obtained by Dey et al. are shown in Fig. 5.19. Within the wall shear layer ($z \leq 0.2h$), $\tilde{l}$ varies linearly with $\tilde{z}$. All the experimental data points for clear-water and mobile-bed flows collapse reasonably on a single band, which is in conformity with Prandtl's mixing length hypothesis. Also, the data points collapse satisfactorily on the curves obtained from the theoretical equation of $\tilde{l} = \kappa\tilde{z}(1-\tilde{z})^{0.5}$ given by Nezu and Nakagawa (1993). The slope of the linear portion defining von Kármán constant $\kappa(= \tilde{l}/\tilde{z} = l/z)$ for mobile-bed flow is smaller than that for clear-water flow. It suggests that the traversing length of an eddy decreases with bed-load transport and increases more rapidly with z in a clear-water flow. A detailed discussion on nonuniversality of von Kármán constant κ is given in next section.

Studies by Gore and Crowe (1991), Hetsroni (1993), Crowe (1993), Best et al. (1997) argued that in flow with transported particles, the ratio of the size of transported particles to the length scale of turbulence is involved in influencing the enhancement or attenuation of the streamwise turbulence intensity. Taylor microscale λ_T, which defines the eddy size in the inertial subrange, is the relevant length scale of turbulence and is given by

$$\lambda_\mathrm{T} = \left(\frac{15\upsilon\overline{u'^2}}{\varepsilon}\right)^{0.5} \tag{5.163}$$

where ε is the turbulent kinetic energy dissipation rate. The estimation of ε is done by using Kolmogorov second hypothesis that predicts the following equality describing the true inertial subrange (Pope 2001):

$$k_\mathrm{w}^{5/3} S_{uu} = C\varepsilon^{2/3} \tag{5.164}$$

where k_w is the wave number, S_{uu} is the spectral density function for u', and C is the constant approximately equaling 0.5 (Monin and Yaglom 2007).

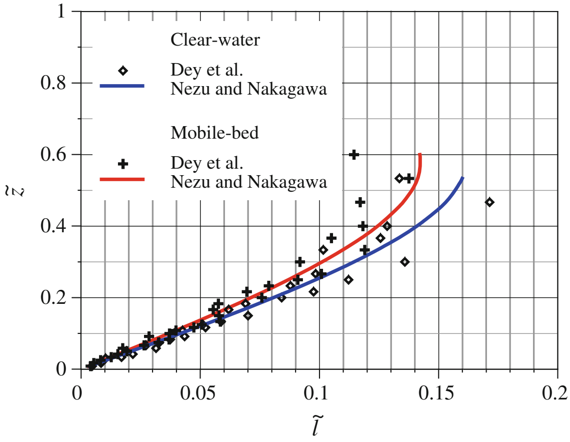

Fig. 5.19 Nondimensional mixing length $\tilde{l}$ as a function of nondimensional vertical distance $\tilde{z}$ for clear-water and mobile-bed flows (Dey et al. 2012)

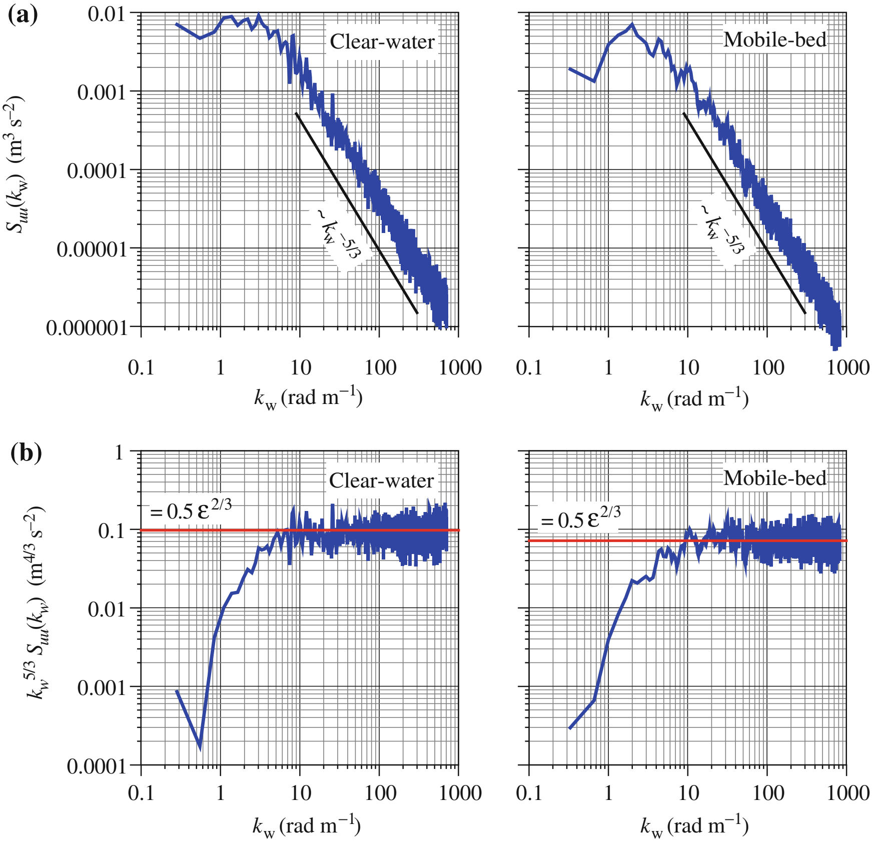

Fig. 5.20 **a** Velocity power spectra $S_{uu}(k_w)$. **b** Estimation of turbulent dissipation rate ε for clear-water and mobile-bed flows (Dey et al. 2012)

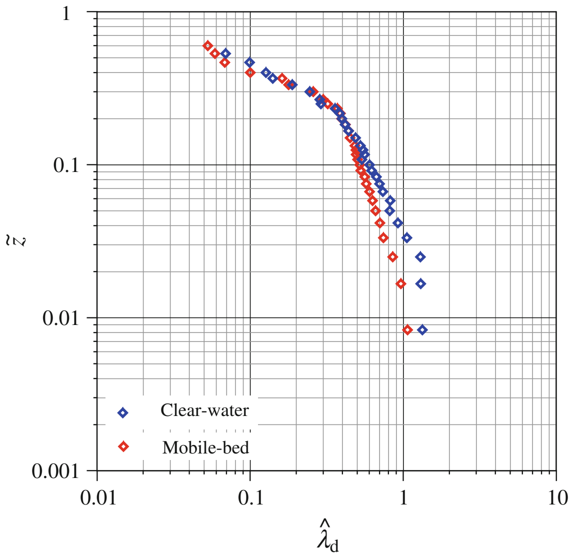

Fig. 5.21 Ratio of particle size to Taylor microscale $\hat{\lambda}_d$ as a function of nondimensional vertical distance $\tilde{z}$ for clear-water and mobile-bed flows (Dey et al. 2012)

In Fig. 5.20a, the spectra $S_{uu}[= (0.5\bar{u}/\pi)F_{uu}(f)]$ as a function of $k_w[= (2\pi/\bar{u})f]$ are drawn using the despiked instantaneous velocity data at $z = 2$ mm (near-bed point) from the bed surface having $d_{50} = 2.6$ mm for clear-water and mobile-bed flows. For both flow conditions, the depth-averaged flow velocity was 0.851 m s^{-1} and the flow depth 0.15 m. The bed-load transport rate in mobile-bed experiment was 7×10^{-3} kg s^{-1} m^{-1}. The inertial subranges in clear-water and mobile-bed flows are characterized by Kolmogorov's *–5/3-th power law*. It corresponds to a subrange of k_w, where the average value of $k_w^{5/3} S_{uu}$ is relatively constant (that is independent of k_w), as shown in Fig. 5.20b. Then, the ε was estimated from Eq. (5.164) and λ_T from Eq. (5.163).

Figure 5.21 shows the variations of the ratio of sediment size to Taylor microscale, that is $\hat{\lambda}_d = d_{50}/\lambda_T$, with $\tilde{z}$ obtained by Dey et al. (2012) for the same flow condition mentioned above (clear-water and mobile-bed cases). Near the bed ($z \leq 0.1\ h$), $\hat{\lambda}_d$ for mobile-bed flow is smaller than that for a clear-water flow. In the outer layer, $\hat{\lambda}_d$ for both the cases, the variation being almost same decreases away from the bed. The values of λ_T near the bed are 2 and 2.44 mm in clear-water and mobile-bed flows, respectively. Hence, the eddy size close to the bed increases in the presence of bed-load transport. Other studies on two-phase flows reported that the range $\hat{\lambda}_d \approx 0.2-1.2$ corresponds to the turbulence enhancement; while the range $\hat{\lambda}_d \approx 0.2-0.065$ corresponds to the turbulence attenuation (Gore and Crowe 1991; Hetsroni 1993; Best et al. 1997).

Figure 5.22 presents the data plots of the ratio of particle size to Taylor microscale, $\hat{\lambda}_d$, for mobile-bed flow as a function of relative difference of streamwise turbulence intensities $\Delta\sigma_{uu}[= (\overline{u'^2})^{0.5}|_{mb}/(\overline{u'^2})^{0.5}|_{cw} - 1]$. Here, subscripts "cw" and

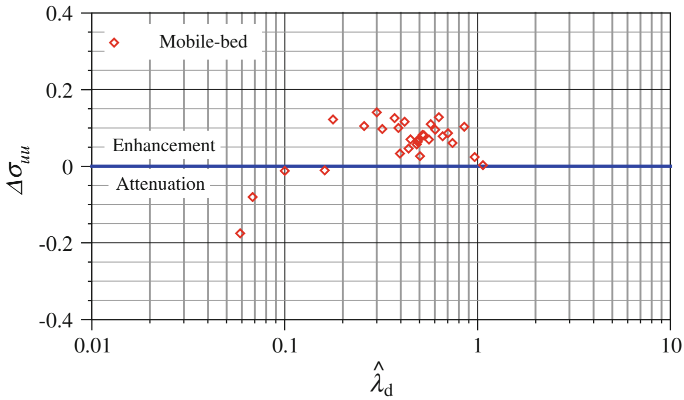

Fig. 5.22 Ratio of particle size to Taylor microscale $\hat{\lambda}_d$ for mobile-bed flow as a function of relative difference of streamwise turbulence intensities $\Delta\sigma_{uu}$ between clear-water and mobile-bed flows (Dey et al. 2012)

"mb" refer to clear-water and mobile-bed flows, respectively. The positive values of $\Delta\sigma_{uu}(\tilde{z} < 0.2)$ indicate that the streamwise turbulence intensity in mobile bed is greater than that in clear-water flow. This is in conformity to the findings of Sumer et al. (2003), who studied the role of externally induced turbulence fields on bed-load transport and argued that the sediment transport rate increases considerably with an increase in streamwise turbulence intensity $(\overline{u'^2})^{0.5}$.

5.17 Effects of Bed Load on von Kármán Constant κ

During bed-load transport, the sediment motion (by rolling, sliding, and saltation) produces an expansion of the roughness layer. Recking et al. (2008) reported that the Nikuradse's equivalent sand roughness k_s increases from the particle size d_{50} for immobile-bed condition to $2.6d_{50}$ for intense bed-load transport condition. The expansion of the roughness layer modifies the logarithmic wall shear layer, resulting in the variation of von Kármán constant κ from its universal value 0.41. Gaudio et al. (2010) and Gaudio and Dey (2012) reviewed the studies on the effects of sediment transport on κ, which is discussed below:

Gust and Southard (1983) analyzed the velocity data in the wall shear layer ($z/h \leq 0.2$) measured by a hot-wire anemometer. They observed a decrease in κ from its universal value with an increase in bed-load transport rate. After a transitional regime corresponding to the entrainment threshold of sediment, κ adjusted to a constant value of 0.32 ± 0.04 for all the experiments with bed-load transport, in which the transport rate varied by a factor 10. Best et al. (1997) used a phase Doppler anemometer to differentiate the characteristics of the fluid from those of the sediment particles and to quantify the influence of the sediments on the carrier fluid turbulence. They observed that the average value of κ was 0.385 in the presence of bed-load transport. Nikora and Goring (2000) reported a study on the characteristics of turbulent structure of high Reynolds number in quasi-two-dimensional flow with fixed and weakly mobile gravel-beds. The flow measurements by an acoustic Doppler velocimeter in an irrigation field canal were carried out for two bed conditions: fixed and weakly mobile beds. Measurements were first taken with a weakly mobile-bed flow (WMBF) and then repeated for a fixed-bed flow (FBF). They obtained $\kappa \approx 0.29$ for the WMBF being significantly smaller than $\kappa \approx 0.4$ for the FBF. They argued that the value $\kappa \approx 0.4$ with the WMBF would have been achieved with an adjustment of the virtual bed level if the bed level was shifted by 30 mm upwards. Since such a shift is physically unjustifiable, it corroborates that the difference of κ values between the WMBF and the FBF is possible due to the effects of bed-load transport. Bennett and Bridge (1995), Nikora and Goring (1999), and Gallagher et al. (1999) also revealed an appreciable decrease in κ under bed-load transport. Nikora and Goring (1999) anticipated that the reduction in κ might reflect the special turbulence characteristics within a narrow range of the Shields parameter when the bed shear stress is approximately equal to the threshold bed shear stress. In Nikora and Goring (2000), the drag reduction effects were expressed as decreased values of κ. The general concept is that the drag reduction prevails when the spacing between turbulent bursting events increases in comparison to the spacing in flow with no sediment (Tiederman et al. 1985). However, it is revealed that the κ reduces when spanwise (lateral) spacing between bursting events increases; while streamwise spacing remains unchanged (Hetsroni et al. 1997). Nikora and Goring (2000) found that the streamwise spacing between bursting events was approximately the same for both the WMBF and the FBF, referring to an increase in spanwise spacing for the WMBF. Dey and Raikar (2007) reported the laboratory experimental results on the turbulent flow characteristics measured by an acoustic Doppler velocimeter. The primary endeavor was to investigate the response of the turbulent flow field, having zero-pressure gradient, to the uniform gravel-beds at the near-threshold of sediment of motion. They observed that the variation of mixing length is considerably linear with the distance from the bed within the wall shear layer, whose thickness was obtained as 0.23 times the boundary layer thickness; and von Kármán κ was estimated as 0.35.

Gaudio et al. (2011) performed laboratory tests in a narrow flume with sediment feeding to simulate bed load on a rough bed and measured the velocity within the wall shear layer ($z/h \leq 0.2$) by using a Pitot-Prandtl tube. They obtained a decrease in κ, that κ varied from 0.3 with bed load ($0.0334 \leq g_b \leq 0.0649$ kg s^{-1} m^{-1}) to 0.4 with clear-water flow condition. Further, Dey et al. (2012) fitted a logarithmic law for mobile-bed flow to obtain $\kappa = 0.37$ for bed-load transport rates ($2 \times 10^{-3} \leq g_b \leq 7 \times 10^{-3}$ kg s^{-1} m^{-1}), as already discussed. Table 5.3 furnishes a summary of the results on κ in flow with bed-load transport. The available experimental data are so limited that the variation of κ with bed-load transport rate (q_b or g_b) is not so specific, although it has been well-recognized that the κ values with bed-load transport are less than its universal value 0.41.

Table 5.3 Experimental results on the effects of bed-load transport on κ

References	d_{50} (mm)	R_*	g_b (kg s^{-1} m^{-1})	κ
Gust and Southard (1983)	0.16	–	$0.15 - 1.5 \times 10^{-5}$	0.32 ± 12.5 %
Best et al. (1997)	0.22	8.9	$9 - 22 \times 10^{-3}$	0.385
Nikora and Goring (2000)	6.4	429	0.0138	0.29 ± 10.3 %
Dey and Raikar (2007)	4.1–14.25	210–1,573	$1.23 - 0.09$	0.35 ± 0.86 %
Gaudio et al. (2011)	1	101–120	$3.34 - 0.0649$	$0.3 - 0.39 \pm 10.7$ %
Dey et al. (2012)	0.95, 2.6, 4.1	63–508	$2–7 \times 10^{-3}$	$0.35 - 0.42$

5.18 Examples

Example 5.1 The flow velocity in a wide river is 1.65 m s^{-1}, flow depth 3.2 m, and energy slope 5×10^{-4}. The flow is uniform within the measuring reach. The bed sediment has a median size $d_{50} = 1.5$ mm, $d_{65} = 1.8$ mm, and $d_{90} = 3$ mm, a static angle of repose of 32°, a dynamic angle of repose of 20°, and a relative density of 2.65. Consider coefficient of kinematic viscosity of water $\upsilon = 10^{-6}$ m^2 s^{-1} and mass density of water $\rho = 10^3$ kg m^{-3}.

Compute the bed-load transport rate (in volume per unit time and width) by using formulas/methodologies proposed by du Boys, Shields, Schoklitsch, Meyer-Peter and Müller, Einstein, Brown/Julien (empirical form of Einstein's method), Bagnold, Engelund and Fredsøe, Yalin, and van Rijn.

Also, compute the saltation characteristics of the particle.

Solution

Given data are as follows:
Flow velocity, $U = 1.65$ m s^{-1}; flow depth, $h = 3.2$ m; energy slope, $S_f = 5 \times 10^{-4}$; sediment size, $d_{50} = 1.5$ mm, $d_{65} = 1.8$ mm, and $d_{90} = 3$ mm; static angle of repose, $\phi = 32°$; dynamic angle of repose, $\phi_d = 20°$; relative density, $s = 2.65$; kinematic viscosity of water, $\upsilon = 10^{-6}$ m^2 s^{-1}; and mass density of water, $\rho = 10^3$ kg m^{-3}
For uniform flow, the energy slope equals the streamwise bed slope. Thus, $S_f = S_0 = 5 \times 10^{-4}$
Applied bed shear stress, $\tau_0 = \rho g h S_0 = 10^3 \times 9.81 \times 3.2 \times 5 \times 10^{-4} = 15.7$ Pa
Shear velocity, $u_* = (\tau_0/\rho)^{0.5} = (15.7/10^3)^{0.5} = 0.125$ m s^{-1}
Shields parameter, $\Theta = \tau_0/(\Delta \rho g d_{50}) = 15.7/(1.65 \times 10^3 \times 9.81 \times 1.5 \times 10^{-3}) = 0.647$
Use van Rijn's empirical formula for the determination of threshold bed shear stress and threshold shear velocity (see Table 4.1):

Particle parameter, $D_* = d_{50}(\Delta g/\upsilon^2)^{1/3} = 1.5 \times 10^{-3}[1.65 \times 9.81/(10^{-6})^2]^{1/3} = 37.94$
Threshold Shields parameter, $\Theta_c(20 < D_* \leq 150) = 0.013 D_*^{0.29} = 0.013 \times 37.94^{0.29} = 0.037$
Threshold bed shear stress, $\tau_{0c} = \Theta_c \Delta \rho g d_{50} = 0.037 \times 1.65 \times 10^3 \times 9.81 \times 1.5 \times 10^{-3} = 0.898$ Pa
Threshold shear velocity, $u_{*c} = (\tau_{0c}/\rho)^{0.5} = (0.898/10^3)^{0.5} = 0.03$ m s^{-1}

Computation of bed load by du Boys formula

$$\chi = 6.89 \times 10^{-6}/1.5^{0.75} = 5.083 \times 10^{-6}\,\text{kg}^{-2}\,\text{m}^4\,\text{s}^3 \Leftarrow \text{Eq. (5.13) (Note: } d_{50} \text{ is in mm)}$$

$$q_b = 5.083 \times 10^{-6} \times 15.7(15.7 - 0.898) = 1.181 \times 10^{-3}\,\text{m}^2\,\text{s}^{-1} \Leftarrow \text{Eq. (5.11)}$$

Computation of bed load by Shields formula

$$q = Uh = 1.65 \times 3.2 = 5.28\ \text{m}^2\ \text{s}^{-1}$$

$$q_b = \frac{10 \times 5.28 \times 5 \times 10^{-4}}{2.65 \times 1.65^2 \times 10^3 \times 9.81 \times 1.5 \times 10^{-3}}(15.7 - 0.898) = 3.681 \times 10^{-3}\ \text{m}^2\ \text{s}^{-1} \Leftarrow \text{Eq. (5.14)}$$

Computation of bed load by Schoklitsch formula

$$q_c = 1.944 \times 10^{-5}/S_0^{4/3} = 1.944 \times 10^{-5}/(5 \times 10^{-4})^{4/3} = 0.49\ \text{m}^2\ \text{s}^{-1}$$

$$g_b = \frac{7000}{(1.5 \times 10^{-3})^{0.5}}(5 \times 10^{-4})^{1.5}(5.28 - 0.49) = 9.679\ \text{N s}^{-1}\text{m}^{-1} \Leftarrow \text{Eq. (5.43)}$$

$$q_b = g_b/(\rho_s g) = 9.679/(2.65 \times 10^3 \times 9.81) = 3.723 \times 10^{-4}\ \text{m}^2\ \text{s}^{-1}$$

Computation of bed load by Meyer-Peter and Müller formula

$$C_R = U/(hS_0)^{0.5} = 1.65/(3.2 \times 5 \times 10^{-4})^{0.5} = 41.25\ \text{m}^{0.5}\ \text{s}^{-1}$$

$$C'_R = 18\log(12h/d_{90}) = 18\log[12 \times 3.2/(3 \times 10^{-3})] = 73.93\ \text{m}^{0.5}\ \text{s}^{-1}$$

$$\eta_C = (C_R/C'_R)^{1.5} = (41.25/73.93)^{1.5} = 0.417$$

Meyer-Peter and Müller recommended $\Theta_c = 0.047$ and corresponding $\tau_{0c} = \Theta_c \Delta \rho g d_{50} = 0.047 \times 1.65 \times 10^3 \times 9.81 \times 1.5 \times 10^{-3} = 1.14$ Pa

$$q_b = \frac{8}{1.65(10^3)^{1.5} \times 9.81}(0.417 \times 15.7 - 1.14)^{1.5} = 1.965 \times 10^{-4}\ \text{m}^2\ \text{s}^{-1} \Leftarrow \text{Eq. (5.15)}$$

Computation of bed load by Einstein's method
Assume $R'_b = R_b = h = 3.2$ m (for the wide channel)

$$\Psi_b = \Delta d_{65}/(R'_b S_0) = 1.65 \times 1.8 \times 10^{-3}/(3.2 \times 5 \times 10^{-4}) = 1.86$$

From Fig. 5.10, Φ_b ($\Psi_b = 1.86$) = 4

$$q_b = \Phi_b(\Delta g d_{50}^3)^{0.5} = 4[1.65 \times 9.81(1.5 \times 10^{-3})^3]^{0.5} = 0.349 \times 10^{-4} \text{ m}^2 \text{ s}^{-1}$$

Computation of bed load by empirical form of Einstein's method
$\Psi_b = 1.86 \leq 1.92$; thus, Julien formula, given by Eq. (5.72), is applicable.

$$K_f = \left[\frac{2}{3} + \frac{36(10^{-6})^2}{1.65 \times 9.81(1.5 \times 10^{-3})^3}\right]^{0.5} - \left[\frac{36(10^{-6})^2}{1.65 \times 9.81(1.5 \times 10^{-3})^3}\right]^{0.5} = 0.791 \Leftarrow \text{Eq. (5.71a)}$$

$$\Phi_b(\Psi_b \leq 1.92) = 15 \times 0.791 \times \frac{1}{1.86^{1.5}} = 4.677 \Leftarrow \text{Eq. (5.72)}$$

$$q_b = \Phi_b(\Delta g d_{50}^3)^{0.5} = 4.677[1.65 \times 9.81(1.5 \times 10^{-3})^3]^{0.5} = 1.093 \times 10^{-3} \text{ m}^2 \text{ s}^{-1}$$

Computation of bed load by Bagnold formula
Assume $e_b = 0.1$

$$g_{bs} = 15.7 \times 1.65 \times 0.1/\tan 20^\circ = 7.12 \text{ N s}^{-1}\text{m}^{-1} \Leftarrow \text{Eq. (5.104)}$$

$$g_b = (s/\Delta)g_{bs} = (2.65/1.65)7.12 = 11.44 \text{ N s}^{-1}\text{m}^{-1}$$

$$q_b = g_b/(\rho_s g) = 11.44/(2.65 \times 10^3 \times 9.81) = 4.4 \times 10^{-4} \text{ m}^2 \text{ s}^{-1}$$

Computation of bed load by Engelund and Fredsøe formula
Dynamic coefficient of friction, $\mu_d = \tan 20°$

$$\Phi_b = \frac{9.3}{\tan 20^\circ}(0.647 - 0.037)(0.647^{0.5} - 0.7 \times 0.037^{0.5}) = 10.44 \Leftarrow \text{Eq. (5.88)}$$

$$q_b = \Phi_b(\Delta g d_{50}^3)^{0.5} = 10.44[1.65 \times 9.81(1.5 \times 10^{-3})^3]^{0.5} = 2.44 \times 10^{-3} \text{ m}^2 \text{ s}^{-1}$$

The formula of Engelund and Fredsøe seems to produce a higher estimation.

Computation of bed load by Yalin formula

$$a_1 = 2.45\Theta_c^{0.5}/s^{0.4} = 2.45 \times 0.037^{0.5}/2.65^{0.4} = 0.319$$

$$(\Theta/\Theta_c) - 1 = (0.647/0.037) - 1 = 16.49$$

$$\Phi_b = 0.635 \times 0.647^{0.5} \times 16.49\left[1 - \frac{1}{0.319 \times 16.49}\ln(1 + 0.319 \times 16.49)\right]$$

$$= 5.486 \Leftarrow \text{Eq. (5.115)}$$

$$q_b = \Phi_b(\Delta g d_{50}^3)^{0.5} = 5.486[1.65 \times 9.81(1.5 \times 10^{-3})^3]^{0.5} = 1.282 \times 10^{-3} \text{ m}^2 \text{ s}^{-1}$$

Table 5.4 Saltation length λ_b, height h_s and particle velocity $\bar{u}_b$ obtained from different formulas

References	λ_b (m)	h_s (m)	$\bar{u}_b$ (m s^{-1})	Remark
Fernandez Luque and van Beek (1976)	0.024	–	1.196	
Engelund and Fredsøe (1976)	–	–	1.229	
Abbott and Francis (1977)	–	–	1.33	$a = 14$
Sekine and Kikkawa (1992)	2.55	–	0.971	From Cheng formula, $w_s = 0.15$ m s^{-1} (Table 1.3)
Niño et al. (1994)	0.06	–	0.71	$a = 7.5$
Lee and Hsu (1994)	0.2	0.016	1.322	
Hu and Hui (1996)	0.07	0.005	1.33	
Lajeunesse et al. (2010)	0.066	–	–	

Computation of bed load by van Rijn formula

$$\Phi_b = (5.3 \times 10^{-2}/37.94^{0.3})16.49^{2.1} = 6.41 \Leftarrow \text{Eq. (5.26)}$$

$$q_b = \Phi_b(\Delta g d_{50}^3)^{0.5} = 6.41[1.65 \times 9.81(1.5 \times 10^{-3})^3]^{0.5} = 1.498 \times 10^{-3} \text{ m}^2 \text{ s}^{-1}$$

Computation of saltation characteristics

By van Rijn formulas:

$$\text{Saltation length}, \lambda_b/d_{50} = 3 \times 37.94^{0.6} \times 16.49^{0.9} \Rightarrow \lambda_b = 0.497 \text{ m} \Leftarrow \text{Eq. (5.135a)}$$

$$\text{Saltation height}, h_s/d_{50} = 0.3 \times 37.94^{0.7} \times 16.49^{0.5} \Rightarrow h_s = 0.023 \text{ m} \Leftarrow \text{Eq. (5.135b)}$$

$$\text{Particle velocity}, \bar{u}_b/(\Delta g d_{50})^{0.5} = 1.5 \times 16.49^{0.6} \Rightarrow \bar{u}_b = 1.256 \text{ m s}^{-1} \Leftarrow \text{Eq. (5.137)}$$

Further, estimates of saltation length λ_b, height h_s, and particle velocity $\bar{u}_b$ by using the formulas (see Tables 5.1 and 5.2) proposed by different investigators are given in Table 5.4.

Example 5.2 Water flows with a depth-averaged velocity of 1.5 m s^{-1} through a wide channel having a uniform flow depth of 3 m. The channel has a streamwise bed slope of 8×10^{-4}. The size classes of nonuniform sediment obtained from the sieve analysis are 35 % between 0.1 and 0.5 mm, 30 % between 0.5 and 1 mm, 20 % between 1 and 2 mm, 10 % between 2 and 3 mm, and 5 % between 3 and 4 mm. Relative density of sediment is 2.65; and sediment size, $d_{50} = 0.75$ mm and $d_{90} = 3$ mm.

Find the bed-load transport rate by using the methods of (1) Meyer-Peter and Müller and (2) Ashida and Michiue.

Table 5.5 Calculation by Meyer-Peter and Müller's method

Size class (mm)	d_i (mm)	p_i	$p_i d_i$ (m)	ξ_i	$\eta_C \Theta_i$	$\xi_i \Theta_c$	q_{bi} ($m^2\ s^{-1}$)
0.1–0.5	0.3	0.35	1.05×10^{-4}	3.064	1.304	0.104	7.695×10^{-5}
0.5–1	0.75	0.3	2.25×10^{-4}	1.284	0.522	0.0437	6.555×10^{-5}
1–2	1.5	0.2	3×10^{-4}	0.8	0.261	0.0272	4.223×10^{-5}
2–3	2.5	0.1	2.5×10^{-4}	0.6	0.157	0.0204	2.021×10^{-5}
3–4	3.5	0.05	1.75×10^{-4}	0.506	0.112	0.0172	9.693×10^{-6}
			$\sum 1.06 \times 10^{-3}$				$\sum 2.146 \times 10^{-4}$

Solution

Given data are as follows:
Flow velocity, $U = 1.5$ m s^{-1}; flow depth, $h = 3$ m; bed slope, $S_0 = 8 \times 10^{-4}$; bed sediment size, $d_{50} = 0.75$ mm and $d_{90} = 3$ mm; and relative density, $s = 2.65$

1. *Meyer-Peter and Müller's method*
   Weighted mean size, $d_m = \sum p_i d_i = 1.06$ mm (see Table 5.5)

$$C_R = U/(hS_0)^{0.5} = 1.5/(3 \times 8 \times 10^{-4})^{0.5} = 30.62\ \text{m}^{0.5}\ \text{s}^{-1}$$
$$C'_R = 18 \log(12h/d_{90}) = 18 \log[12 \times 3/(3 \times 10^{-3})] = 73.43\ \text{m}^{0.5}\ \text{s}^{-1}$$
$$\eta_C = (C_R/C'_R)^{1.5} = 0.269$$

   Applied bed shear stress, $\tau_0 = \rho g h S_0 = 10^3 \times 9.81 \times 3 \times 8 \times 10^{-4} = 23.54$ Pa

$$\Theta_i = \tau_0/(\Delta \rho g d_i) = 23.54/(1.65 \times 10^3 \times 9.81 \times d_i) = 1.454 \times 10^{-3}/d_i$$

   Threshold Shields parameter, $\Theta_c = 0.034$ that is obtained from van Rijn's empirical formula (Table 4.1) for the sediment size $d_m = 1.06$ mm. Bed-load transport rate for fractional size d_i is $q_{bi} = 8(\Delta g)^{0.5} p_i d_i^{1.5} (\eta_C \Theta_i - \xi_i \Theta_c)^{1.5}$[see Eq. (5.146)]
   Therefore, the total bed-load transport rate for all size fractions, $q_b = 2.146 \times 10^{-4}$ m^2 s^{-1} (Table 5.5)
   However, one can check the difference in estimation of bed-load transport rate obtained using the weighted mean size d_m.

$$\Theta = \tau_0/(\Delta \rho g d_m) = 23.54/(1.65 \times 10^3 \times 9.81 \times 1.06 \times 10^{-3}) = 1.372$$
$$q_b = 8(\Delta g d_m^3)^{0.5} (\eta_C \Theta - \Theta_c)^{1.5} = 8[1.65 \times 9.81 (1.06 \times 10^{-3})^3]^{0.5} (0.269 \times 1.372 - 0.034)^{1.5}$$
$$= 2.154 \times 10^{-4}\ \text{m}^2\ \text{s}^{-1}$$

   which is almost equaling the estimate of total bed-load transport rate for all size fractions.

Table 5.6 Calculation by Ashida and Michiue's method

Size class (mm)	d_i (mm)	p_i	Θ_i	Θ_{ci}	$\Theta_i - \Theta_{ci}$	$\Theta_i^{0.5} - \Theta_{ci}^{0.5}$	q_{bi} ($m^2\ s^{-1}$)
0.1–0.5	0.3	0.35	4.85	0.039	4.811	2.005	1.2×10^{-3}
0.5–1	0.75	0.3	1.94	0.03	1.91	1.22	9.82×10^{-4}
1–2	1.5	0.2	0.97	0.037	0.933	0.793	5.88×10^{-4}
2–3	2.5	0.1	0.582	0.043	0.539	0.556	2.56×10^{-4}
3–4	3.5	0.05	0.416	0.048	0.368	0.426	1.11×10^{-4}
							$\sum 3.14 \times 10^{-3}$

2. *Ashida and Michiue's method*

Bed-load transport rate for fractional size d_i is $q_{bi} = 17(\Delta g)^{0.5} p_i d_i^{1.5}(\Theta_i - \Theta_{ci}) \times (\Theta_i^{0.5} - \Theta_{ci}^{0.5})$ [see Eq. (5.140)]

Threshold Shields parameter Θ_{ci} for the fractional size d_i can be obtained by using van Rijn's empirical formula (Table 4.1) for the size d_i (Table 5.6).

Therefore, the total bed-load transport rate for all size fractions, $q_b = 3.14 \times 10^{-3}\ m^2\ s^{-1}$ (Table 5.6)

However, one can check the difference in estimation of bed-load transport rate obtained using the weighted mean size d_m.

$$q_b = 17(\Delta g d_m^3)^{0.5}(\Theta - \Theta_c)(\Theta^{0.5} - \Theta_c^{0.5}) = 17[1.65 \times 9.81(1.06 \times 10^{-3})^3]^{0.5} \times (1.372 - 0.034)\left(1.372^{0.5} - 0.034^{0.5}\right) = 3.12 \times 10^{-3}\ m^2\ s^{-1}$$

which is very close to the estimate of total bed-load transport rate for all size fractions.

References

Abbott JE, Francis JRD (1977) Saltation and suspension trajectories of solid grains in a water stream. Proc R Soc London A 284(1321):225–254

Ashida K, Michiue M (1972) Study on hydraulic resistance and bed-load transport rate in alluvial streams. Trans Japan Soc Civ Eng 206:59–69

Bagnold RA (1954) Experiments on a gravity-free dispersion of large solid spheres in a Newtonian fluid under shear. Proc R Soc London A 255(1160):49–63

Bagnold RA (1956) The flow of cohesionless grains in fluids. Philos Trans R Soc London A 249(964):315–319

Bagnold RA (1966) An approach to the sediment transport problem from general physics. Geological survey professional paper 422-I, Washington, DC

Bagnold RA (1977) Bed load transport by natural rivers. Water Resour Res 13(2):303–312

Barekyan AS (1962) Discharge of channel forming sediments and elements of sand waves. Trans Am Geophys Union 2:128–130

Bennett SJ, Bridge JS (1995) The geometry and dynamics of low-relief bed forms in heterogeneous sediment in a laboratory channel, and their relationship to water flow and sediment transport. J Sediment Res 65A(1):29–39

Best J, Bennett S, Bridge J, Leeder M (1997) Turbulence modulation and particle velocities over flat sand beds at low transport rates. J Hydraul Eng 123(12):1118–1129
Borah DK (1989) Scour-depth prediction under armoring conditions. J Hydraul Eng 115(10): 1421–1425
Borah DK, Alonso CV, Prasad SN (1982) Routing graded sediment in streams: formation. J Hydraul Div 108(12):1486–1503
Bose SK (2009) Numeric computing in Fortran. Narosa, New Delhi
Bose SK, Dey S (2013) Sediment entrainment probability and threshold of sediment suspension: Exponential-based approach. J Hydraul Eng 139(10):1099–1106
Brown CB (1950) Sediment transportation. In: Rouse H (ed) Engineering hydraulics. Wiley, New York, pp 769–857
Chang FM, Simons DB, Richardson EV (1967) Total bed-material discharge in alluvial channels. In: Proceedings of the twelfth congress of International Association for Hydraulic Research, Fort Collins, Colorado, pp 132–140
Cheng N-S (2002) Exponential formula for bedload transport. J Hydraul Eng 128(10):942–946
Cheng N-S, Chiew Y-M (1998) Pickup probability for sediment entrainment. J Hydraul Eng 124(2):232–235
Chien N (1954) Meyer-Peter formula for bed load transport and Einstein bed load function. Missouri River division sediment series number 7, University of California-Berkeley and Missouri River Division, United States Army Corps of Engineers, Berkeley, California
Chien N, Wan Z (1999) Mechanics of sediment transport. ASCE Press, Reston
Chin CO, Melville BW, Raudkivi AJ (1994) Streambed armoring. J Hydraul Eng 120(8):899–918
Correia L, Graf WH (1988) Grain-size distribution and armoring in gravel-bed rivers. Activities report of the hydraulics research laboratory, École Polytechnique Fédérale de Lausanne, Lausanne
Crowe CT (1993) Modelling turbulence in multiphase flows. In: Rodi W, Martelli F (eds) Engineering turbulence modelling and experiments, vol 2., Elsevier, Amsterdam, pp 899–913
Damgaard JS, Whitehouse RJS, Soulsby RL (1997) Bedload sediment transport on steep longitudinal slopes. J Hydraul Eng 123(12):1130–1138
de Ruiter JCC (1982) The mechanism of sediment transport on bed forms. In: Proceedings of the Euromech conference, vol 156, Technical University of Istanbul, Istanbul
de Ruiter JCC (1983) Incipient motion and pick-up of sediment as function of local variables. Report R 657-XI, Delft Hydraulics Laboratory, Delft
Dey S, Das R, Gaudio R, Bose SK (2012) Turbulence in mobile-bed streams. Acta Geophys 60(6):1547–1588
Dey S, Debnath K (2001) Sediment pick-up on stream-wise sloping beds. J Irrig Drainage Eng 127(1):39–43
Dey S, Raikar RV (2007) Characteristics of loose rough boundary streams at near-threshold. J Hydraul Eng 133(3):288–304
Dey S, Sarkar S, Solari L (2011) Near-bed turbulence characteristics at the entrainment threshold of sediment beds. J Hydraul Eng 137(9):945–958
Donat J (1929) Über sohlangriff und geschiebetrieb. Wasserwirtschaft 26:27–36
Dou GR (1964) Bed-load transport. Report, Nanjing Hydraulic Research Institute, Nanjing
du Boys MP (1879) Le rhone et les rivieres a lit affouillable. Annales des Ponts et Chaussés 18(5):141–195
Egiazaroff JV (1965) Calculation of non-uniform sediment concentrations. J Hydraul Div 91(4):225–247
Einstein HA (1942) Formulas for the transportation of bed load. Trans Am Soc Civ Eng 107:561–577
Einstein HA (1950) The bed-load function for sediment transportation in open channel flows. Technical bulletin number 1026, United States Department of Agriculture, Soil Conservation Service, Washington, DC
Einstein HA, El-Samni EA (1949) Hydrodynamic forces on rough wall. Rev Modern Phys 21(3):520–524

Engelund F, Fredsøe J (1976) A sediment transport model for straight alluvial channels. Nord Hydrol 7(5):293–306
Engelund F, Hansen E (1967) A monograph on sediment transport in alluvial streams. Technical Press (Teknish Forlag), Copenhagen
Exner FM (1925) Über die wechselwirkung zwischen wasser und geschiebe in flüssen. Sitzungsberichte der Akademie der Wissenschaften 134(2a):165–203
Fernandez Luque R (1974) Erosion and transport of bed-load sediment. PhD thesis, Delft University of Technology, Meppel
Fernandez Luque R, van Beek R (1976) Erosion and transport of bed-load sediment. J Hydraul Res 14(2):127–144
Francis JRD (1973) Experiments on the motion of solitary grains along the bed of a water stream. Proc R Soc London A 332(1591):443–471
Fredsøe J, Deigaard R (1992) Mechanics of coastal sediment transport. World Scientific, Singapore
Frijlink HC (1952) Discussion of bed load transport formulas. Report number X2344/LV, Delft Hydraulics, Delft
Froehlich DC (1995) Armor-limited clear-water contraction scour at bridges. J Hydraul Eng 121(6):490–493
Gallagher M, McEwan I, Nikora V (1999) The changing structure of turbulence over a self-stabilising sediment bed. Internal report number 21, Department of Engineering, University of Aberdeen, Aberdeen
Gaudio R, Dey S (2012) Evidence of non-universality of von Kármán's κ. In: Rowinski P (ed) Experimental and computational solutions of hydraulic problems. Springer, Heidelberg, pp 71–83
Gaudio R, Miglio A, Calomino F (2011) Friction factor and von Kármán's κ in open channels with bed-load. J Hydraul Res 49(2):239–247
Gaudio R, Miglio A, Dey S (2010) Non-universality of von Kármán's κ in fluvial streams. J Hydraul Res 48(5):658–663
Gilbert GK (1914) Transportation of debris by running water. Professional paper number 86, United States Geological Survey, Washington DC
Gore RA, Crowe CT (1991) Modulation of turbulence by a dispersed phase. J Fluids Eng 113(6):304–307
Graf WH (1971) Hydraulics of sediment transport. McGraw-Hill Book Company, New York
Graf WH, Suszka L (1987) Sediment transport in steep channels. J Hydrosci Hydraul Eng 5(1):11–26
Gust G, Southard JB (1983) Effects of weak bed load on the universal law of the wall. J Geophys Res 88(C10):5939–5952
Guy HP, Simons DB, Richardson EV (1966) Summary of alluvial channel data from flume experiments, 1956–1961. United States Geological survey professional paper, 462-1, Washington, DC
Hanes DM (1986) Grain flows and bed-load sediment transport: review and extension. Acta Mech 63(1–4):131–142
Hetsroni G (1993) The effect of particles on the turbulence in a boundary layer. In: Raco MC (ed) Particulate two-phase flow. Butterworth-Heinemann, pp 244–264
Hetsroni G, Zakin JL, Mosyak A (1997) Low-speed streaks in drag-reduced turbulent flow. Phys Fluids 9(8):2397–2404
Hsu SM, Holly FM (1992) Conceptual bed-load transport model and verification for sediment mixtures. J Hydraul Eng 118(8):1135–1152
Hu CH, Hui YJ (1996) Bed-load transport. I: mechanical characteristics. J Hydraul Eng 122(5):245–254
Julien PY (1998) Erosion and sedimentation, 1st edn. Cambridge University Press, Cambridge
Kalinske AA (1947) Movement of sediment as bed-load in rivers. Trans Am Geophys Union 28(4):615–620

Kironoto BA, Graf WH (1994) Turbulence characteristics in rough uniform open-channel flow. Water Marit Energ Proc Inst Civ Eng (London) 106(4): 333–344

Lajeunesse E, Malverti L, Charru F (2010) Bed load transport in turbulent flow at the grain scale: Experiments and modeling. J Geophys Res 115(F04001). doi:10.1029/2009JF001628

Lee HY, Hsu I-S (1994) Investigation saltating particle motions. J Hydraul Eng 120(7):831–845

Madsen OS (1991) Mechanics of cohesionless sediment transport in coastal waters. In: Kraus NC, Gingerich KJ, Kriedel DL (eds) Coastal sediment '91. American Society of Civil Engineers, New York, pp 15–27

Meyer-Peter E, Favre H, Einstein HA (1934) Neuere versuchsresultate über den geschiebetrieb. Schweizerische Bauzeitung 103(13):147–150

Meyer-Peter E, Müller R (1948) Formulas for bed-load transport. In: Proceedings of the second meeting of International Association for Hydraulic Research, vol 3. Stockholm, pp 39–64

Misri RL, Garde RJ, Ranga Raju KG (1984) Bed load transport of coarse nonuniform sediment. J Hydraul Eng 110(3):312–328

Monin AS, Yaglom AM (2007) Statistical fluid mechanics, volume II: mechanics of turbulence. Dover Publications, New York

Morsi SA, Alexander AJ (1972) An investigation of particle trajectories in two-phase flow systems. J Fluid Mech 55:193–208

Nakagawa H, Tsujimoto T (1980) Sand bed instability due to bed load motion. J Hydraul Div 106(12):2029–2051

Nelson J, Shreve RL, McLean SR, Drake TG (1995) Role of near-bed turbulence structure in bed load transport and bed form mechanics. Water Resour Res 31(8):2071–2086

Nezu I, Nakagawa H (1993) Turbulence in open-channel flows. Balkema, Rotterdam

Nielsen P (1992) Coastal bottom boundary layers and sediment transport. World Scientific, Singapore

Nikora VI, Goring DG (1999) Effects of bed mobility on turbulence structure. NIWA internal report number 48, National Institute of Water & Atmospheric Research (NIWA), Christchurch

Nikora V, Goring D (2000) Flow turbulence over fixed and weakly mobile gravel beds. J Hydraul Eng 126(9):679–690

Niño Y, García M, Ayala L (1994) Gravel saltation: 1. experiments. Water Resour Res 30(6):1907–1914

Niño Y, García MH (1998) Using Lagrangian particle saltation observations for bedload sediment transport modelling. Hydrol Process 12(8):1197–1218

Paintal A (1971) Concept of critical shear stress in loose boundary open channels. J Hydraul Res 9(1):91–113

Parker G (1979) Hydraulic geometry of active gravel rivers. J Hydraul Div 105(9):1185–1201

Parker G, Kilingeman PC, McLean DG (1982) Bed load and size distribution in paved gravel-bed streams. J Hydraul Div 108(4):544–571

Patel PL, Ranga Raju KG (1996) Fractionwise calculation of bed load transport. J Hydraul Res 34(3):363–379

Pope SB (2001) Turbulent flows. Cambridge University Press

Poreh M, Sagiv A, Seginer J (1970) Sediment sampling efficiency of slots. J Hydraul Div 96(10):2065–2078

Qin YY (1980) Incipient motion of nonuniform sediment. J Sediment Res 1:83–91

Raudkivi AJ (1990) Loose boundary hydraulics. Pergamon, New York

Recking A, Frey P, Paquier A, Belleudy P, Champagne JY (2008) Feedback between bed load transport and flow resistance in gravel and cobble bed rivers. Water Resour Res 44(W05412). doi:10.1029/2007WR006219

Rickenmann D (1991) Bed load transport and hyperconcentrated flow at steep slopes. Fluvial hydraulics of mountain regions, Lecture notes in earth sciences, vol 37, pp 429–441

Rubey W (1933) Settling velocities of gravel, sand and silt particles. Am J Sci 225(148):325–338

Rubinow SI, Keller JB (1961) The transverse force on a spinning sphere moving in a viscous fluid. J Fluid Mech 11:447–459

Saffman PG (1968) Corrigendum, the lift on a small sphere in a slow shear flow. J Fluid Mech 31:624
Schoklitsch A (1914) Über schleppkraft und geschiebebewegung. Engelmann, Leipzig
Schoklitsch A (1934) Geschiebetrieb und die geschiebefracht. Wasserkraft und Wasserwirtschaft 39(4):1–7
Sekine M, Kikkawa H (1992) Mechanics of saltating grains.II. J Hydraul Eng 118(4):536–558
Shields AF (1936) Application of similarity principles and turbulence research to bed-load movement. Mitteilungen der Preussischen Versuchsanstalt für Wasserbau und Schiffbau 26:5–24
Smart GM (1984) Sediment transport formula for steep channels. J Hydraul Eng 110(3):267–276
Straub LG (1935) Missouri river report. In-House document 238, Seventy-third Congress, Second Session, United States Government Printing office, Washington, DC
Sumer BM, Chua LHC, Cheng N-S, Fredsøe J (2003) Influence of turbulence on bed load sediment transport. J Hydraul Eng 129(8):585–596
Tiederman WG, Luchik TS, Bogard DG (1985) Wall layer structure and drag reduction. J Fluid Mech 156:419–437
van Rijn LC (1981) Computation of bed-load concentration and bed-load transport. Research report S 487-1, Delft Hydraulics Laboratory, Delft
van Rijn LC (1984a) Sediment transport, part I: bed load transport. J Hydraul Eng 110(10):1431–1456
van Rijn LC (1984b) Sediment pick-up function. J Hydraul Eng 110(10):1494–1502
van Rijn LC (1993) Principles of sediment transport in rivers, estuaries and coastal seas. Aqua Publications, The Netherlands
Wang X, Zheng J, Danxun L, Qu Z (2008) Modification of the Einstein bed-load formula. J Hydraul Eng 134(9):1363–1369
White BR, Schultz JC (1977) Magnus effect in saltation. J Fluid Mech 81:497–512
Williams PG (1970) Flume width and water depth effects in sediment transport experiments. Geological survey professional paper 562-H, Washington, DC
Wilson KC (1966) Bedload transport at high shear stresses. J Hydraul Div 92(6):49–59
Wong M, Parker G (2006) Re-analysis and correction of bedload relation of Meyer-Peter and Muller using their own database. J Hydraul Eng 132(11):1159–1168
Wu F-C, Lin Y-C (2002) Pickup probability of sediment under log-normal velocity distribution. J Hydraul Eng 128(4):438–442
Wu W, Wang SSY, Jia Y (2000) Nonuniform sediment transport in alluvial rivers. J Hydraul Res 38(6):427–434
Yalin MS (1977) Mechanics of sediment transport. Pergamon, Oxford
Yalin MS, Karahan E (1979) Inception of sediment transport. J Hydraul Div 105(11):1433–1443
Yen CL, Lee HY, Chang SY, Hsu SH (1988) A study on aggradation/degradation of coarse sediment in alluvial stream. Research report number 88, Hydraulic Research Laboratory, National Taiwan University, Taipei (in Chinese)
Zhang RJ, Xie JH, Wang MF, Huang JT (1989) Dynamics of river sedimentation. Water and Power Press, Beijing