Application of Euclidean Distance Power Graphs in Localization of Sensor Networks

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Abstract. Localization of sensor nodes in a wireless sensor network is needed for many practical uses. If the nodes are considered as vertices of a globally rigid graph then the nodes can be uniquely localized up to translation, rotation and reflection. We have given the construction of globally rigid graph through Euclidean distance powers of Unit Disk graph.

Keywords: UD Graph, powers of a graph, vertex connectivity and henneberg sequence.

1 Introduction

Unit Disk graph [4] is an important class of graphs, which finds application in modeling a wireless sensor network. The radio coverage range of sensors is based on Euclidean distance between the nodes and we utilize this concept of Euclidean distance in graph theory. This has given rise to a new branch termed as 'geometric graph theory'. Wireless sensor network can be modeled as unit disk graph. In this modeling sensors are denoted as vertices. The sensing coverage area of a sensor is represented by a unit disk centered at the corresponding vertex. The connectivity between two sensor nodes is determined if the first sensor is within the sensing coverage area of the second sensor. Thus there is an edge between u and v iff $d(u,v) \leq R$, where $d(u,v)$ is the Euclidean distance between u and v and R is the sensing range. Thus Unit Disk (UD) graph is a suitable model for a wireless sensor network.

Power of a graph [1] is an induced graphs which can be generated by making some additional edges to the original graphs. Square of a graph is a graph with the same vertex set in which vertices at distance 2 are connected through an edge. Cube of a graph is also the graph on the same set of vertices; however, additionally there is an edge between two vertices whenever they are at most distance 3. This concept can be applied to the UD graphs also. Powers of a UD graph as square and cube of a UD graph [1] represent the possible interfering nodes in network. Further Euclidean distance two graph [3] and Euclidean distance three graph of a UD graph also provide the information about interfering nodes in sensor network.

The nodes in a sensor network are in general deployed randomly and localization of nodes is an important issue in wireless sensor networks. Determining the geographical location of nodes in a sensor network is essential for many aspects of

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system operation, data stamping, tracking, signal processing, querying, topology control, clustering and routing. The localization problem in the sensor networks is to determine the location of all nodes. In space R^d (d = 2,3) the location of nodes can be easily determined if initial location of 2 and 3 nodes is known in the case of d = 2 and d = 3, respectively. These initially located sensor nodes are called anchor nodes [7].

The network is said to be uniquely localizable [5] if there is a unique set of locations consistent with the given data. Uniquely localizable network in two and three dimensions can be characterized by using the results of rigid graphs. A network N is uniquely localizable if and only if the *weighted grounded graph* [5] G_N' associated with N, is a globally rigid graph [8]. The terms: globally rigid graph and weighted grounded graph are defined hereafter.

In the weighted grounded graph G_N', the vertices are the corresponding nodes of the network N and the two vertices are connected by an edge if the distance between them is known initially. Since the location of anchor nodes is known, the distance between these nodes is known (can be directly calculated). Thus these anchor node are also connected to each other in G_N'.

An intuitive definition of globally rigid graph has been considered in [5] as follows: consider a configuration graph of points in plane in which edges between some of them represents distance constraints. If there is not another configuration consisting of different points on the plane that preserves all the distance constraints on the edges, then the configuration graph is said to be globally rigid graph in the plane. As shown in figure 1 and figure 2.

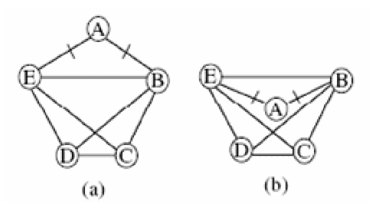


Fig. 1. Not globally rigid since distances between vertices are preserved in another configuration

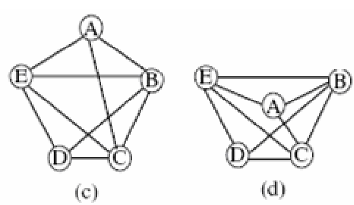


Fig. 2. Globally rigid graph since distances between vertices are not preserved in another configuration. Distance between A and C is not similar in both graphs (c) & (d).

In this paper we have provided an algorithm to construct systematically generically globally rigid graphs from those graphs which do not have these properties. This construction basically depends on adding of more edges in the graph. However, it is also important to know that how to add the extra edges in the underlying graph of a sensor network. The distance between sensor nodes and their neighbors (immediate, two hop, three hop and four hop) are involved in this construction. For a network whose graphical model is a UD graph, it corresponds to increasing the sensing radius (presumably by adjusting transmission powers) for each sensor. To determine the distance between two and three hop neighbors, the sensing radius has to be made double and triple respectively.

This paper is organized as follows: Some new terms and some other auxiliary definitions are described with examples in Section 2 for the completion of this paper. In Section 3 we have given the Condition for generically globally rigid graph and necessary definitions. In Section 4, we describe the construction of a globally rigid power graph of a UD graph. We also give the related Lemmas and Theorems for ED-2graph as well as ED-3 graph of a UD graph. The localization problem in sensor network is given in Section 4. In Section 5, we conclude the results.

2 Auxiliary Definition

2.1 Unit Disk Graph

A graph G is a Unit Disk graph if there is an assignment of unit disks centered at its vertices such two vertices are adjacent if and only if one vertex is within the unit disk centered at the other vertex. We denote a unit disk graph by G_{UD}.

2.2 Powers of a Unit Disk Graph

2.2.1 Square of a Unit Disk Graph (G_{UD}^2)

The Square G_{UD}^2 of a Unit Disk graph G_{UD} (V, E) is the graph whose vertex set is V and there is an edge between two vertices v_i and v_j if and only if their graph distance in G_{UD} is at most 2. [Figure 3 (a)].

2.2.2 Euclidean Distance Two Graph of a Unit Disk Graph (G_{UD}^{ED2})

Euclidean distance two graph of a unit disk graph G_{UD} (V, E) is the graph whose vertex set is V and there is an edge between two vertices v_i and v_j if and only if their Euclidean distance in G_{UD} is at most 2. [Figure 3 (b)].

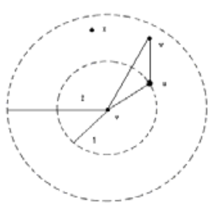


Fig. 3 (a). G_{UD}^2 Graph

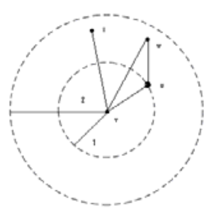


Fig. 3 (b). G_{UD}^{ED2} Graph

2.2.3 Cube of a Unit Disk Graph (G_{UD}^3)

The cube G_{UD}^3 of a Unit Disk graph G_{UD} (V, E) is the graph whose vertex set is V and there is an edge between two vertices v_i and v_j if and only if their graph distance in G_{UD} is at most 3. [Figure 4 (a)].

2.2.4 Euclidean Distance Three Graph of a Unit Disk Graph (G_{UD}^{ED3})

Euclidean distance three graph of a unit disk graph G_{UD} (V, E) is the graph whose vertex set is V and there is an edge between two vertices v_i and v_j if and only if their Euclidean distance in G_{UD} is at most 3. [Figure 4 (b)].

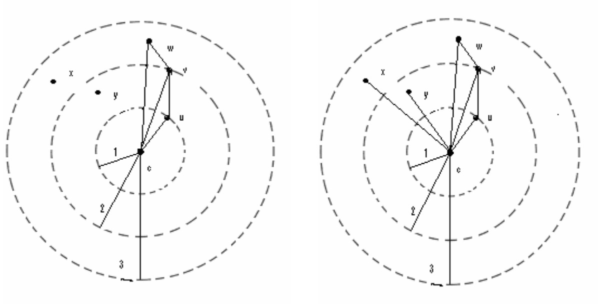


Fig. 4 (a). G_{UD}^3 Graph **Fig. 4 (b).** G_{UD}^{ED3} Graph

2.3 Rigidity in Unit Disk Graph

2.3.1 Realization UD Frame Work

A d- dimensional UD frame work (G_{UD},p) is a unit disk graph together with a map $p{:}V \longrightarrow \Re^d$, the framework is realization if it results in $\|p(i)-p(j)\| \leq R \ \forall i \ \& \ j \in V$ where $ij \in E$.

2.3.2 Equivalent UD Frameworks

Two UD frameworks (G_{UD},p) and (G_{UD},q) are equivalent if $\|p(i)-p(j)\| = \|q(i)-q(j)\|$ $\forall i \ \& \ j$ where $ij \in E$. i.e. two frameworks with the same graph G are equivalent if the lengths of their edges are the same. [Figure 5].

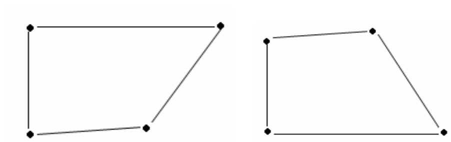


Fig. 5. Both are equivalent frameworks

2.3.3 Congruent UD Frameworks

Two UD frameworks (G_{UD},p) and (G_{UD},q) are congruent if $\|p(i)-p(j)\| = \|q(i)-q(j)\|$ $\forall i \ \& \ j \in V$. i.e. (G,q) can be obtained from (G,p) by applying a set of operations of translation, rotation and reflection.

2.3.4 Rigid UD Framework

A UD framework (G_{UD},p) is rigid if there exist a sufficiently small positive ϵ such that if (G_{UD}, q) is equivalent to (G_{UD},p) and $\|p(i)-q(i)\| \leq \epsilon \ \forall i \in V$ then (G_{UD}, q) is congruent to (G_{UD},p) or we can say A framework (or graph) is rigid iff continuous motion of the points of the configuration maintaining the bar constraints comes from a family of motions of all Euclidean space which are distance-preserving. A graph that is not rigid

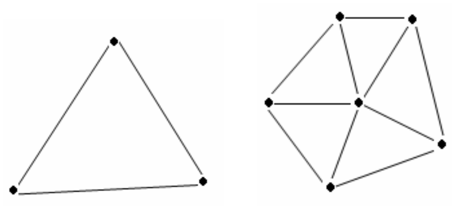


Fig. 6. Rigid Graphs

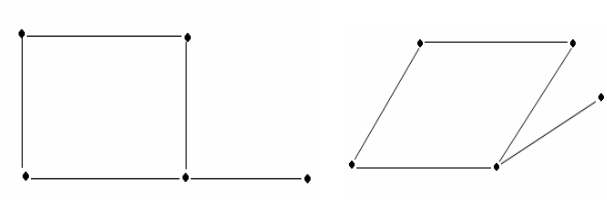


Fig. 7. Non rigid graph since it is not uniquely realizable

is said to be flexible. Figure 7 shows a non rigid graph since it is possible to rotate the pendent vertex around its neighbor and also it is possible to twist the square into parallelogram. It is causes of infinitely many configurations with respect to the edge length consistency.

2.3.5 Globally Rigid UD Framework

A UD framework (G_{UD}, p) is globally rigid if every framework which is equivalent to (G_{UD}, p) is congruent to (G_{UD}, p). i.e. it has unique realization up to rotation translation or reflection. [Figure 8].

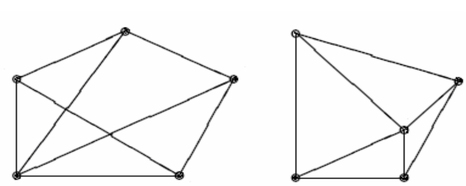


Fig. 8. Globally rigid graphs in $\Re^2$

2.3.6 Generic UD Framework

A UD framework (G_{UD}, p) is generic if the set containing the coordinates of all its point is algebraically independent over the rationals.

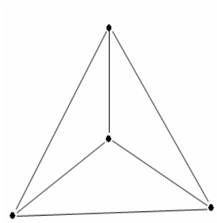
3 Condition for Generically Globally Rigid Graph

3.1 3-Connectivity

A graph G is said to be 3- connected if and only if it remains connected after removing any 2 vertices or we can say there are atleast three disjoint (vertex and edge disjoint) paths between every pair of vertices.

3.2 Redundantly Rigid

A graph is Redundantly Rigid if it remains rigid after removing any one of its edge. We have given a graph in figure (10). This graph is composed of two triangles connected by its vertices. It is rigid and it is 3-connected but figure 10(a) shows that there are two possible configurations for the vertices on the plane with respect to the edge length. Figure 10(b) shows that if the edge (a,f) is removed, the graph becomes non-rigid and it is possible to rotate the triangles until another position reached where the distance (a,f) is same as the previous one. In this way it is possible to find two configurations for the graph that are consistent with the edge length and therefore the graph is not uniquely realizable. So it is not a Redundantly Rigid graph.



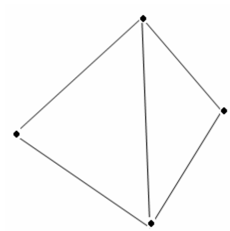
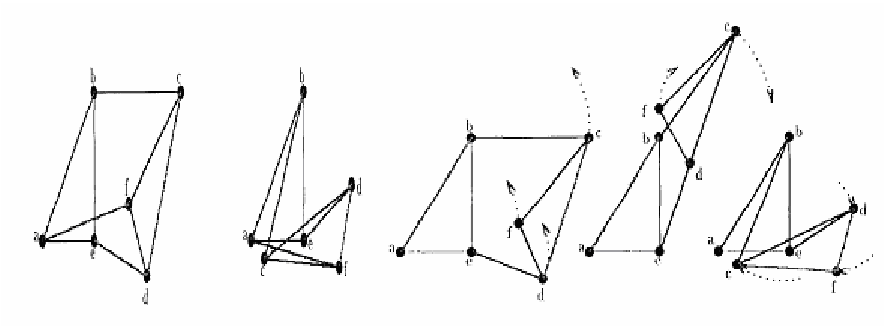


Fig. 9(a). 3-Connected Graph **Fig. 9(b).** Not 3-Connected Graph



(a) Two possible configurations (b) Deforming from one configuration to another

Fig. 10. Not a Redundantly Rigid Graph

3.3 Theorem: [9] Let (G,p) be a generic framework in $\Re^d$. if (G,p) is globally rigid then either G is a complete graph with at most d+1 vertices or G is (d+1)-connected and redundantly rigid in $\Re^d$.

3.4 Theorem: [8] Let (G,p) be a generic framework in $\Re^2$. If (G,p) is globally rigid if and only if either G is a complete graph with 2 or 3 vertices or G is 3-connected and redundantly rigid in $\Re^2$.

4 Generating Globally Rigid Power Graph of a UD Graph

4.1 Theorem: If G_{UD} be a UD graph and an edge 2- connected graph then G_{UD}^{ED2} is a generically globally rigid graph.

Proof: For proving this theorem we need the following Lemmas.

Lemma 1: If a UD graph corresponding to a wireless sensor network is in the form of a cycle C_{UD} then C_{UD}^{ED2} is generically globally rigid graph.

Proof: Let S be the set of all sensor nodes s_1, s_2, s_3,............s_{n-1}, s_n in a sensor network. If s_n and s_{n-1} ($n \geq 2$) are in the sensing range of each other and s_n and s_1 are also in the sensing range of each other, then the corresponding UD graph will be a cycle, say C_{UD}. In this cycle s_1, s_2, s_3,............s_{n-1}, s_n are vertices and s_1s_2, s_2s_3, s_3s_4, ...s_ns_1 are edges. If n=3, then it will be a complete graph on three vertices and result is trivial since complete graph is generically globally rigid graph. Now consider n>3. To prove C_{UD}^{ED2} is generically globally rigid graph we first show it is three connected and then it is generically redundantly rigid.

There exist three vertex disjoint paths between every pair of vertices in C_{UD}^2 as follows:

Consider two arbitrary vertices s_i and s_j C_{UD}.

Case I: If i and j both are even or odd then,

P_1: s_i s_{i-1} s_{i-2} ....s_1 s_n s_{n-1} s_{n-2} .... s_{j+2} s_{j+1} s_j, P_2: s_i s_{i+2} s_{i+4} .... s_{j-4} s_{j-2} s_j, P_3: s_i s_{i+1} s_{i+3} .... s_{j-3} s_{j-1} s_j are three different vertex disjoint paths between s_i and s_j.

Case II: If one of i or j is even and another is odd then,

P_1': s_i s_{i-1} s_{i-2} .... s_n s_{n-1} s_{n-2} .... s_{j+2} s_{j+1} s_j, P_2': s_i s_{i+2} s_{i+4} .... s_{j-3} s_{j-1} s_j, P_3': s_i s_{i+1} s_{i+3} .... s_{j-4} s_{j-2} s_j are again three vertex disjoint paths between s_i and s_j.

Therefore C_{UD}^2 is 3- connected. Also we know that C_{UD}^{ED2} is a supergraph of C_{UD}^2 with addition of more edges on same vertex set, so C_{UD}^{ED2} must be 3-connected.

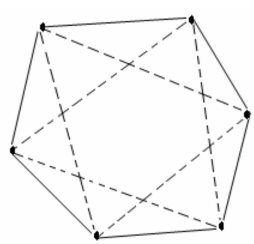


Fig. 11. C_{UD}^2

Now we prove generically redundantly rigidity of C_{UD}^{ED2}. We remove an edge from C_{UD}^{ED2} which is also an edge of C_{UD}. Let it be s_1s_n. Consider the sequence of triangles, whose all edges are in C_{UD}^{ED2}: $s_1s_2s_3$, $s_2s_3s_4$,..............$s_{n-2}s_{n-1}s_n$ and notice the corresponding subgraphs as each triangle is added.

We get a Henneberg sequence by vertex addition, in which each member of the sequence differing from the previous one by the addition of a two degree vertex. So it will be a generically globally rigid graph.

This subgraph C_{UD}^{ED2}- s_1s_n of C_{UD}^{ED2} retains all the vertices of C_{UD}^{ED2} but does not contain the edge s_1s_n. Hence C_{UD}^{ED2}- s_1s_n is a generically globally rigid graph. Thus C_{UD}^{ED2} is generically redundantly rigid.

Therefore C_{UD}^{ED2} is generically globally rigid graph.

Lemma 2: If H_0 be a generically globally rigid graph in $\Re^2$ with atleast three vertices and if a super graph H_1 be defined by adjoining one vertex to the vertex set of H_0 and three edges, each connecting the new vertex to three different vertices of H_0. Then H_1 is generically globally rigid.

Proof: The proof of this lemma is given in [2].

Lemma 3: if $H_1 = (V_1,E_1)$ and $H_2 = (V_2,E_2)$ be two generically globally rigid graphs in $\Re^2$ with atleast three vertices in common then $H_1 \bigcup H_2 = (V_1 \bigcup V_2, E_1 \bigcup E_2)$ is also generically globally rigid.

Proof: The proof of this Lemma is also given in [2].

Lemma 4: Every simple super graph of a generically globally rigid graph on same vertex set is generically globally rigid in $\Re^2$.

Now we provide the proof of main theorem:

Since G_{UD} is an edge 2- connected graph, it necessarily contains atleast one cycle. If it contains just one cycle then by above Lemma 1 the theorem is true. Now suppose G_{UD} contains more than one cycle. Let one of them be C_1: s_1s_2……………s_n, if the vertex set of C_1 is similar as of G_{UD} then the theorem is true by above Lemma 1.

Now suppose a vertex set of C_1 is not similar to G_{UD}. Since G_{UD} is connected, therefore every vertex in $G_{UD} \sim C_1$ is joined by a path to C_1 and hence there is a vertex of $G_{UD} \sim C_1$ joined by a single edge to a vertex of C_1. Call it s_L and let edge be s_1s_L.

Now consider $G_1 = (V_1,E_1)$ with vertex set of C_1 and edge set of C_1 together with the edge s_1s_L. Then G_1^2 has edge set with the edge set of C_1^2 and three more edges s_1s_L, s_ns_L, s_2s_L.

Using Lemma 2, if we consider C_1^2 as H_0 and G_1^2 as H_1 then G_1^2 must be generically globally rigid.

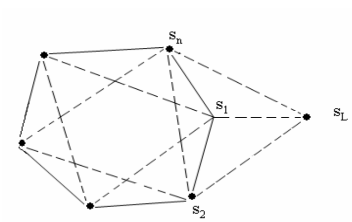


Fig. 12. G_{UD}^2

Since G_{UD} is edge 2- connected, thus there exist another path between s_1 and s_L except the single edge s_1s_L. Then there will be a cycle. Call it C_2, containing s_1 and s_L as adjacent vertices. Clearly this cycle can not contain both s_2 and s_n as adjacent vertices of s_1. Suppose it does not contain s_2 as an adjacent vertex of s_1. Now consider the graph $G_2 = (V_2,E_2)$ with vertex set of C_2 together with s_2 and with edge set of C_2 and also the edge s_1s_2. Then by previous argument we can say G_2^2 is generically globally rigid.

Using the Lemma 3, $G_1^2 \bigcup G_2^2$ must be generically globally rigid. It is a subgraph of $(G_1 \bigcup G_2)^2$ on same vertex set and hence generically globally rigid by Lemma 4. If the vertex set of this graph is that of G_{UD}, then the theorem is proved. If not then we must find another vertex joined by a single edge to $C_1 \bigcup C_2$ and by similar argument we proceed until the set of vertices of $G_1 \bigcup G_2 \bigcup G_3 \bigcup \ldots\ldots\ldots\ldots \bigcup G_r = G$ (for some r) is similar to G_{UD} and G^2 is generically globally rigid. G^2 is a subgraph of G_{UD}^2 retains all the vertices of G_{UD}. Thus G_{UD}^2 must be generically globally rigid. Furthermore G_{UD}^2 is a subgraph of G_{UD}^{ED2} on same vertex set. Therefore G_{UD}^{ED2} is a generically globally rigid graph.

4.2 Theorem 2: If G_{UD} be a UD graph and an edge 2- connected graph then G_{UD}^{ED3} is a generically globally rigid graph.

Proof: For proving this theorem we need the following Lemmas:

Lemma 5: If a UD graph corresponding to a wireless sensor network is in the form of a cycle C_{UD} then C_{UD}^{ED3} is generically globally rigid graph.

Proof: The proof is straight forward as lemma 1.

Lemma 6: If H_0 be a generically globally rigid graph in $\Re^3$ with atleast four vertices and if a super graph H_1 be defined by adjoining one vertex to the vertex set of H_0 and four edges, each connecting the new vertex to four different vertices of H_0. Then H_1 is generically globally rigid.

Lemma 7: If $H_1 = (V_1,E_1)$ and $H_2 = (V_2,E_2)$ be two generically globally rigid graphs in $\Re^3$ with atleast four vertices in common then $H_1 \bigcup H_2 = (V_1 \bigcup V_2, E_1 \bigcup E_2)$ is also generically globally rigid.

Now we provide the proof of main theorem:

If G contains just one cycle, we have done by Lemma [5]. Now suppose G contain more than one cycle and one of them is $C_1 = s_1s_2s_3\ldots\ldots\ldots\ldots s_n$. If the vertex set of C_1 is similar as of G_{UD} then also the theorem is true by Lemma [5].

Now suppose a vertex set of C_1 is not similar to G_{UD}. Since G_{UD} is connected, therefore every vertex in $G_{UD} \sim C_1$ is joined by a path to C_1 and hence there is a vertex of $G_{UD} \sim C_1$ by a single edge to a vertex of C_1. Call it s_L and let edge be s_1s_L. Since G_{UD} is edge 2- connected, thus there exist another path between s_1 and s_L except the single edge s_1s_L. Then there will be a cycle. Call it C_2, containing s_2 and s_n as adjacent vertices of s_1. Clearly this cycle can not contain both s_2 and s_n as an adjacent vertex of s_1. Suppose it does not contain s_2 as a successor of s_1. Consider a graph $G_1 = (V_1,E_1)$

with vertex set that of C_1 together with s_1s_L. Then G_1^3 has an its edge set the edge set of C_1^3 and five more edges s_1s_L, s_2s_L, s_3s_L, $s_{n-1}s_L$ and s_ns_L.

Lemma [6] Identifying C_1^3 as H_0, G_1^3 is generically globally rigid graph. Consider also the graph $G_2 = (V_2,E_2)$ with vertex set of C_2 together with s_2 and s_n if these are not in C_2 and with edge set of C_2 together with s_1s_2 and s_1s_n if s_n is not in C_2. Then arguing as in previous paragraph but applying twice to Lemma [6], we have that G_2^3 is generically globally rigid. The two graphs G_1^3 and G_2^3 are both generically globally rigid and have a common set of atleast four vertices (s_n, s_1, s_2 and s_L). Hence the graph formed from the union of the graph G_1^3 and G_2^3 is generically globally rigid by Lemma [7]. This graph is obviously a subgraph of $(G_1 \bigcup G_2)^3$ with the same vertex set. Thus $(G_1 \bigcup G_2)^3$ is generically globally rigid.

If there are any vertices of G_{UD} which are not vertices of $(G_1 \bigcup G_2)$, then above argument must br repeated by finding a vertex which is connected by a single edge to $(G_1 \bigcup G_2)$, then determining a cycle containing that edge and so on. The process must be repeated until the set of vertices of $G_1 \bigcup G_2 \bigcup G_3 \bigcup \ldots\ldots\ldots\ldots \bigcup G_r$=G for some r is identical with the vertex set of G_{UD} and G^3 is generically globally rigid. G^3 is a subgraph of G_{UD}^3 retains all the vertices of G_{UD}. Thus G_{UD}^3 must be generically globally rigid. Furthermore G_{UD}^3 is a subgraph of G_{UD}^{ED3} on same vertex set. Therefore G_{UD}^{ED3} is a generically globally rigid graph.

5 Localization Problem in Unit Disk Graph

Let S be the set of sensor nodes. d_{ij} be the given distance between the certain pair of nodes s_i and s_j. In unit disk graph s_i and s_j are connected if $d_{ij} \leq R$ were R is the radius of the sensing circle.

Suppose the coordinates p_i of certain nodes s_i (anchor nodes) are known. The localization problem is one of finding a map $p{:}S \longrightarrow \Re^d$ (d=2or3) which assign coordinate $p_j \in \Re^d$ to each node s_j such that $\|p(i)-p(j)\| \leq R$ holds for all pair i and j for which s_i and s_j are connected and assignment is consistent with any coordinate assignment provided in the problem statement.

The solvability of localization problem for sensor networks can be considered as follows: suppose a framework is constructed which is a realization i.e. the edge lengths corresponding to the collection of inter sensor distances. The framework may or may not be rigid and even if it is rigid there may be another and differently shaped framework which is a realization (constructible on same vertex set, edge set and length assignment). If there is a unique rigid realizing framework, up to congruence, consistent with the distances between nodes i.e. the framework is globally rigid then the sensor network can be consider as a rigid entity of known structure. Then we need only to know the Euclidian positions of several sensor nodes to locate the whole framework in two or three dimensions.

WSN localization is unique (up to rotation translation or reflection) if and only if its underlying graph is globally rigid. Thus the global rigidity of the graph ensures the unique localization of the node of a wireless sensor networks.

6 Conclusion

We have given an algorithm to construct systematically generically globally rigid graphs from those graphs which do not have these properties. This construction basically depends on adding of more edges in the graph. We have given the construction of globally rigid graph through Euclidean distance powers of Unit Disk graph. If the nodes are considered as vertices of a globally rigid graph then the nodes can be uniquely localized up to translation, rotation and reflection. In this way we can localize the nodes of a wireless sensor networks.

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