Output Regulation of Arneodo-Coullet Chaotic System

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Abstract. This paper investigates the problem of output regulation of the Arneodo-Coullet chaotic system, which is one of the paradigms of the chaotic systems proposed by A. Arneodo, P. Coullet and C. Tresser (1981). Explicitly, state feedback control laws to regulate the output of the Arneodo-Coullet chaotic system have been derived so as to track the constant reference signals as well as to track periodic reference signals. The control laws are derived using the regulator equations of C.I. Byrnes and A. Isidori (1990), who solved the problem of output regulation of nonlinear systems involving neutrally stable exosystem dynamics. The output regulation of the Coullet chaotic system has important applications in Electrical and Communication Engineering. Numerical simulations are shown to verify the results.

Keywords: Arneodo-Coullet system; output regulation; nonlinear control systems; feedback stabilization.

1 Introduction

Output regulation of control systems is one of the very important problems in control systems theory. Basically, the output regulation problem is to control a fixed linear or nonlinear plant in order to have its output tracking reference signals produced by some external generator (the exosystem). For linear control systems, the output regulation problem has been solved by Francis and Wonham (1975, [1]). For nonlinear control systems, the output regulation problem has been solved by Byrnes and Isidori (1990, [2]) generalizing the internal model principle obtained by Francis and Wonham [1]. Byrnes and Isidori [2] have made an important assumption in their work which demands that the exosystem dynamics generating reference and/or disturbance signals is a neutrally stable system (Lyapunov stable in both forward and backward time). The class of exosystem signals includes the important particular cases of constant reference signals as well as sinusoidal reference signals. Using Centre Manifold Theory [3], Byrnes and Isidori have derived regulator equations, which completely characterize the solution of the output regulation problem of nonlinear control systems.

The output regulation problem for linear and nonlinear control systems has been the focus of many studies in recent years ([4]-[14]). In [4], Mahmoud and Khalil obtained results on the asymptotic regulation of minimum phase nonlinear systems using output feedback. In [5], Fridman solved the output regulation problem for nonlinear control

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systems with delay, using Centre Manifold Theory [3]. In [6]-[7], Chen and Huang obtained results on the robust output regulation for output feedback systems with nonlinear exosystems. In [8], Liu and Huang obtained results on the global robust output regulation problem for lower triangular nonlinear systems with unknown control direction. In [9], Immonen obtained results on the practical output regulation for bounded linear infinite-dimensional state space systems. In [10], Pavlov, Van de Wouw and Nijmeijer obtained results on the global nonlinear output regulation using convergence-based controller design. In [11], Xi and Ding obtained results on the global adaptive output regulation of a class of nonlinear systems with nonlinear exosystems. In [12]-[14], Serrani, Marconi and Isidori obtained results on the semi-global and global output regulation problem for minimum-phase nonlinear systems.

In this paper, the output regulation problem for the Arneodo-Coullet chaotic system [15] has been solved using the Byrnes-Isidori regulator equations [2] to derive the state feedback control laws for regulating the output of the Coullet chaotic system for the important cases of constant reference signals (set-point signals) and periodic reference signals. The Arneodo-Coullet chaotic system is one of the simplest three-dimensional chaotic systems studied by A. Arneodo, P. Coullet and C. Tresser (1981, [15]). It has important applications in Electrical and Communication Engineering.

This paper is organized as follows. In Section 2, a review of the solution of the output regulation for nonlinear control systems and Byrnes-Isidori regulator equations has been presented. In Section 3, the main results of this paper, namely, the feedback control laws solving the output regulation problem for the Coullet chaotic system for the cases of constant reference signals and periodic reference signals have been detailed. In Section 4, the numerical results illustrating the main results of the paper have been described. Section 5 summarizes the main results obtained in this paper.

2 Review of the Output Regulation for Nonlinear Control Systems

In this section, we consider a multi-variable nonlinear control system modelled by equations of the form

$$\dot{x} = f(x) + g(x)u + p(x)\omega \tag{1}$$

$$\dot{\omega} = s(\omega) \tag{2}$$

$$e = h(x) - q(\omega) \tag{3}$$

Here, the differential equation (1) describes the *plant dynamics* with state x defined in a neighbourhood X of the origin of $\mathbb{R}^n$ and the input u takes values in $\mathbb{R}^m$ subject to the effect of a disturbance represented by the vector field $p(x)\omega$. The differential equation (2) describes an autonomous system, known as the *exosystem*, defined in a neighbourhood W of the origin of $\mathbb{R}^k$, which models the class of disturbance and reference signals taken into consideration. The equation (3) defines the error between the actual plant output $h(x) \in \mathbb{R}^p$ and a reference signal $q(\omega)$, which models the class of disturbance and reference signals taken into consideration.

We also assume that all the constituent mappings of the system (1)-(2) and the error equation (3), namely, f, g, p, s, h and q are $\mathcal{C}^1$ mappings vanishing at the origin, *i.e.*

$$f(0) = 0,\ g(0) = 0,\ p(0) = 0,\ h(0) = 0 \ \text{ and } \ q(0) = 0.$$

Thus, for $u = 0$, the composite system (1)-(2) has an equilibrium state $(x, \omega) = (0, 0)$ with zero error (3).

A *state feedback controller* for the composite system (1)-(2) has the form

$$u = \alpha(x, \omega) \tag{4}$$

where α is a $\mathcal{C}^1$ mapping defined on $X \times W$ such that $\alpha(0, 0) = 0$. Upon substitution of the feedback law (4) in the composite system (1)-(2), we get the closed-loop system given by

$$\begin{aligned} \dot{x} &= f(x) + g(x)\alpha(x, \omega) + p(x)\omega \\ \dot{\omega} &= s(\omega) \end{aligned} \tag{5}$$

The purpose of designing the state feedback controller (4) is to achieve both *internal stability* and *output regulation*. Internal stability means that when the input is disconnected from (5) (*i.e.* when $\omega = 0$), the closed-loop system (5) has an exponentially stable equilibrium at $x = 0$. Output regulation means that for the closed-loop system (5), for all initial states $(x(0), \omega(0))$ sufficiently close to the origin, $e(t) \to 0$ asymptotically as $t \to \infty$. Formally, we can summarize the requirements as follows.

State Feedback Regulator Problem [2]:

Find, if possible, a state feedback control law $u = \alpha(x, \omega)$ such that

(OR1) [*Internal Stability*] The equilibrium $x = 0$ of the dynamics

$$\dot{x} = f(x) + g(x)\alpha(x, 0)$$

is locally asymptotically stable.

(OR2) [*Output Regulation*] There exists a neighbourhood $U \subset X \times W$ of $(x, \omega) = (0, 0)$ such that for each initial condition $(x(0), \omega(0)) \in U$, the solution $(x(t), \omega(t))$ of the closed-loop system (5) satisfies

$$\lim_{t \to \infty} [h(x(t)) - q(\omega(t))] = 0.$$ ■

Byrnes and Isidori [2] have solved this problem under the following assumptions.

(H1) The exosystem dynamics $\dot{\omega} = s(\omega)$ is neutrally stable at $\omega = 0$, i.e. the system is Lyapunov stable in both forward and backward time at $\omega = 0$.

(H2) The pair $(f(x), g(x))$ has a stabilizable linear approximation at $x = 0$, i.e. if

$$A = \left[\frac{\partial f}{\partial x}\right]_{x=0} \quad \text{and} \quad B = \left[\frac{\partial g}{\partial x}\right]_{x=0},$$

then (A, B) is stabilizable, which means that we can find a gain matrix K so that $A + BK$ is Hurwitz. ■

Next, we recall the solution of the output regulation problem derived by Byrnes and Isidori [2].

Theorem 1. *[2] Under the hypotheses (H1) and (H2), the state feedback regulator problem is solvable if, and only if, there exist* $\mathcal{C}^1$ *mappings* $x = \pi(\omega)$ *with* $\pi(0) = 0$

and $u = \phi(\omega)$ *with* $\phi(0) = 0$*, both defined in a neighbourhod of* $W^0 \subset W$ *of* $\omega = 0$ *such that the following equations (called the* Byrnes-Isidori regulator equations*) are satisfied:*

$$\text{(1) } \frac{\partial \pi}{\partial \omega} s(\omega) = f(\pi(\omega)) + g(\pi(\omega))\phi(\omega) + p(\pi(\omega))\omega$$

$$\text{(2) } h(\pi(\omega)) - q(\omega) = 0$$

When the Byrnes-Isidori regulator equations (1) and (2) are satisfied, a control law solving the state feedback regulator problem is given by

$$u = \phi(\omega) + K[x - \pi(\omega)] \tag{6}$$

where K *is any gain matrix such that* $A + BK$ *is Hurwitz.* ■

3 Output Regulation of Arneodo-Coullet Chaotic System

The Arneodo-Coullet chaotic system [15] is one of the paradigms of the three-dimensional chaotic models described by the dynamics

$$\begin{aligned} \dot{x}_1 &= x_2 \\ \dot{x}_2 &= x_3 \\ \dot{x}_3 &= ax_1 - bx_2 - cx_3 - x_1^3 + u \end{aligned} \tag{7}$$

where a, b and c are positive constants.

A. Arneodo, P. Coullet and C. Tresser [15] studied the chaotic system (7) with $a = 5.5, b = 3.5, c = 1.0$ and $u = 0$. The chaotic portrait of the unforced Arneodo-Coullet chaotic system is illustrated in Figure 1.

In this paper, we consider two important cases of output regulation for the Arneodo-Coullet chaotic system [15]:

(I) Tracking of Constant Reference Signals
(II) Tracking of Periodic Reference Signals

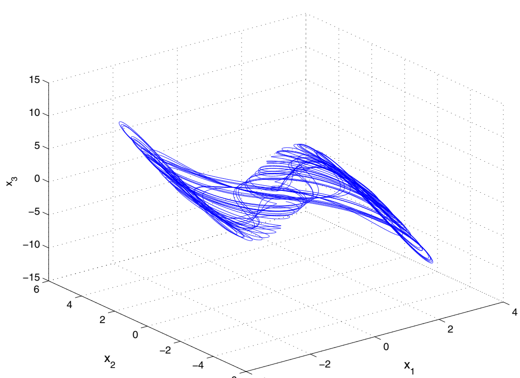

Fig. 1. Chaotic Portrait of the Arneodo-Coullet System

Case I: Tracking of Constant Reference Signals
In this case, the exosystem is given by the scalar dynamics

$$\dot{\omega} = 0 \tag{8}$$

It is important to observe that the exosystem (8) is neutrally stable because the solutions of (8) are only constant trajectories, *i.e.*

$$\omega(t) \equiv \omega(0) = \omega_0 \quad \text{for all} \;\; t$$

Thus, the assumption (H1) of Theorem 1 (Section 2) holds trivially.

Linearizing the dynamics of the Arneodo-Coullet system (7) yields the system matrices

$$A = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ a & -b & -c \end{bmatrix} \quad \text{and} \quad B = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \tag{9}$$

which is in Bush companion form.

Using Kalman's rank test for controllability ([16], p.738), it can be easily seen that the pair (A, B) is completely controllable. Since (A, B) is in Bush companion form, the characteristic equation of $A + BK$ is given by

$$\lambda^3 + (c - k_3)\lambda^2 + (b - k_2)\lambda - (a + k_1) = 0 \tag{10}$$

where $K = [\, k_1 \quad k_2 \quad k_3 \,]$.

By the Routh's stability criterion ([16], p.234), it can be easily shown that the closed-loop system matrix $A + BK$ is Hurwitz if and only if

$$k_1 < -a, \;\; k_2 < b, \;\; k_3 < c, \quad (c - k_3)(b - k_2) + (a + k_1) > 0 \tag{11}$$

Thus, the assumption (H2) of Theorem 1 (Section 2) also holds. Hence, Theorem 1 can be applied to solve the output regulation problem for the Arneodo-Coullet chaotic system (7) for the tracking of constant reference signals (*set-point signals*).

Case I (a): The error equation is $e = x_1 - \omega$
Solving the Byrnes-Isidori regulator equations (Theorem 1), we get the solution

$$\pi_1(\omega) = \omega, \; \pi_2(\omega) = 0, \; \pi_3(\omega) = 0, \; \phi(\omega) = \omega\left(\omega^2 - a\right) \tag{12}$$

By Theorem 1 (Section 2), a state feedback control law solving the output regulation problem is given by

$$u = \phi(\omega) + K[x - \pi(\omega)] = \omega\left(\omega^2 - a\right) + k_1(x_1 - \omega) + k_2 x_2 + k_3 x_3 \tag{13}$$

where k_1, k_2 and k_3 satisfy the inequalities (11).

Case I (b): The error equation is $e = x_2 - \omega$
A direct calculation shows that the Byrnes-Isidori regulator equations are not solvable in this case. Hence, by Theorem 1 (Section 2), we conclude that the output regulation problem is not solvable for this case.

Case I (c): The error equation is $e = x_3 - \omega$
A direct calculation shows that the Byrnes-Isidori regulator equations are not solvable in this case. Hence, by Theorem 1 (Section 2), we conclude that the output regulation problem is not solvable for this case.

Case II: Tracking of Periodic Reference Signals
In this case, the exosystem is given by the planar dynamics

$$\begin{aligned} \dot{\omega}_1 &= \nu\,\omega_2 \\ \dot{\omega}_2 &= -\nu\,\omega_1 \end{aligned} \tag{14}$$

where $\nu > 0$ is any fixed constant.

Clearly, the assumption (H1) (Theorem 1) holds. Also, as established in Case I, the assumption (H2) (Theorem 1) also holds and that the closed-loop system matrix $A + BK$ will be Hurwitz if the constants k_1, k_2 and k_3 of the gain matrix K satisfy the inequalities (11).

Case II (a): The error equation is $e = x_1 - \omega_1$
Solving the Byrnes-Isidori regulator equations (Theorem 1) for this case, we get the solution

$$\pi_1(\omega) = \omega_1, \pi_2(\omega) = \nu\omega_2, \pi_3(\omega) = -\nu^2\omega_1 \tag{15}$$

$$\phi(\omega) = \omega_1^3 - (a + c\nu^2)\omega_1 + (b\nu - \nu^3)\omega_2 \tag{16}$$

By Theorem 1 (Section 2), a state feedback control law solving the output regulation problem is given by

$$u = \phi(\omega) + K[x - \pi(\omega)] \tag{17}$$

where $\pi(\omega)$ is given by (15), $\phi(\omega)$ is given by (16) and k_1, k_2 and k_3 satisfy the inequalities (11).

Case II (b): The error equation is $e = x_2 - \omega_2$
Solving the Byrnes-Isidori regulator equations (Theorem 1), we get the solution

$$\pi_1(\omega) = -\nu^{-1}\,\omega_2,\ \pi_2(\omega) = \omega_1,\ \pi_3(\omega) = \nu\omega_2 \tag{18}$$

$$\phi(\omega) = (b - \nu^2)\omega_1 + (a\nu^{-1} + c\nu)\omega_2 - \nu^{-3}\,\omega_2^3 \tag{19}$$

By Theorem 1 (Section 2), a state feedback control law solving the output regulation problem is given by

$$u = \phi(\omega) + K[x - \pi(\omega)] \tag{20}$$

where $\pi(\omega)$ is given by (18), $\phi(\omega)$ is given by (19) and k_1, k_2 and k_3 satisfy the inequalities (11).

Case II (c): The error equation is $e = x_3 - \omega_3$
Solving the Byrnes-Isidori regulator equations (Theorem 1), we get the solution

$$\pi_1(\omega) = -\nu^{-2}\,\omega_1,\ \pi_2(\omega) = -\nu^{-1}\,\omega_1,\ \pi_3(\omega) = \omega_1 \tag{21}$$

$$\phi(\omega) = \left(c + a\nu^{-2}\right)\omega_1 + \left(\nu + b\nu^{-1}\right)\,\omega_2 - \nu^{-6}\,\omega_1^3 \tag{22}$$

By Theorem 1 (Section 2), a state feedback control law solving the output regulation problem is given by

$$u = \phi(\omega) + K[x - \pi(\omega)] \tag{23}$$

where $\pi(\omega)$ is given by (21), $\phi(\omega)$ is given by (22) and k_1, k_2 and k_3 satisfy the inequalities (11).

4 Numerical Simulations

For classical case, we consider the classical chaotic case of Arneodo-Coullet chaotic system [15] with parameter values $a = 5.5, b = 3.5$ and $c = 1.0$. For achieving the internal stability of the state feedback regulator problem, a gain matrix K which satisfies the inequalities (11) must be used.

With the choice

$$K = [\,k_1 \quad k_2 \quad k_3\,] = [\,-130.5 \quad -71.5 \quad -14\,],$$

the matrix $A + BK$ is Hurwitz with the eigenvalues $-5, -5, -5$.

In the periodic tracking output regulation problem, the value $\nu = 1$ is taken in the exosystem dynamics given by (14).

Case I (a): Constant Tracking Problem with Error Equation $e = x_1 - \omega$

Here, the initial conditions are taken as

$$x_1(0) = 7, x_2(0) = 5, x_3(0) = 6 \;\text{ and }\; \omega(0) = 2$$

The simulation graph is depicted in Figure 2 from which it is clear that the state trajectory $x_1(t)$ tracks the constant reference signal $\omega(t) \equiv 2$ in 2.5 seconds.

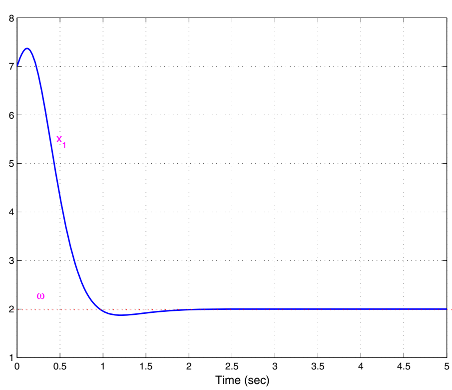

Fig. 2. Constant Tracking Problem - Case I (a)

Case I (b): Constant Tracking Problem with Error Equation $e = x_2 - \omega$
As pointed out in Section 3, the output regulation problem is not solvable for this case because the Byrnes-Isidori regulator equations do not admit any solution.

Case I (c): Constant Tracking Problem with Error Equation $e = x_3 - \omega$
As pointed out in Section 3, the output regulation problem is not solvable for this case because the Byrnes-Isidori regulator equations do not admit any solution.

Case II (a): Periodic Tracking Problem with Error Equation $e = x_1 - \omega_1$
Here, the initial conditions are taken as

$$x_1(0) = 4, \;\; x_2(0) = 3, \;\; x_3(0) = -2 \;\text{ and }\; \omega_1(0) = 1, \omega_2(0) = 0$$

Also, it is assumed that $\nu = 1$. The simulation graph is depicted in Figure 3 from which it is clear that the state trajectory $x_1(t)$ tracks the periodic reference signal $\omega_1(t) = \sin t$ in 1.5 seconds.

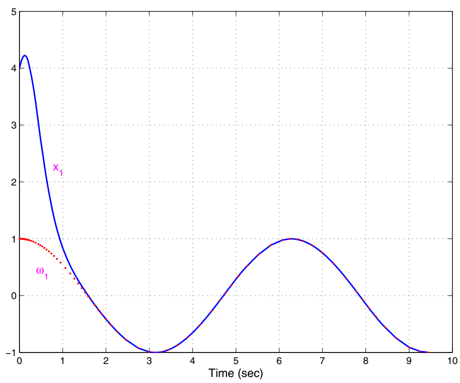

Fig. 3. Periodic Tracking Problem - Case II (a)

Case II (b): Periodic Tracking Problem with Error Equation $e = x_2 - \omega_1$
Here, the initial conditions are taken as

$$x_1(0) = 8, \;\; x_2(0) = 3, \;\; x_3(0) = -2 \;\text{ and }\; \omega_1(0) = 1, \omega_2(0) = 0$$

Also, it is assumed that $\nu = 1$. The simulation graph is depicted in Figure 4 from which it is clear that the state trajectory $x_2(t)$ tracks the periodic reference signal $\omega_1(t) = \sin t$ in 2 seconds.

Case II (c): Periodic Tracking Problem with Error Equation $e = x_3 - \omega_1$
Here, the initial conditions are taken as

$$x_1(0) = 4, \;\; x_2(0) = -5, \;\; x_3(0) = 7 \;\text{ and }\; \omega_1(0) = 1, \omega_2(0) = 0$$

Also, it is assumed that $\nu = 1$. The simulation graph is depicted in Figure 5 from which it is clear that the state trajectory $x_2(t)$ tracks the periodic reference signal $\omega_1(t) = \sin t$ in 2 seconds.

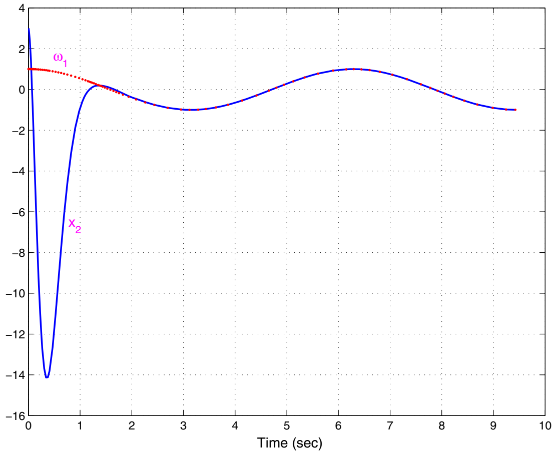

Fig. 4. Periodic Tracking Problem - Case II (b)

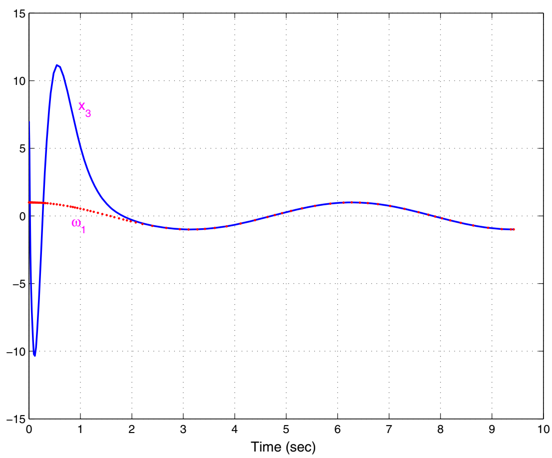

Fig. 5. Periodic Tracking Problem - Case II (c)

5 Conclusions

In this paper, the output regulation problem for the Arneodo-Coullet chaotic system (1981) has been studied in detail and a complete solution for the output regulation problem for the Arneodo-Coullet chaotic system has been presented as well. Explicitly, using the Byrnes-Isidori regulator equations (1990), state feedback control laws for regulating the output of the Arneodo-Coullet chaotic system have been derived. As tracking reference signals, constant and periodic reference signals have been considered and in each case, feedback control laws regulating the output of the Arneodo-Coullet chaotic system have been derived. Numerical simulations are shown to verify the results.

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