Constrained Surface-Level Gateway Placement for Underwater Acoustic Wireless Sensor Networks

Deying Li, Zheng Li, Wenkai Ma, and Hong Chen

Key Laboratory of Data Engineering and Knowledge Engineering, MOE,
School of Information, Renmin University of China, China

Abstract. One approach to guarantee the performance of underwater acoustic sensor networks is to deploy multiple Surface-level Gateways (SGs) at the surface. This paper addresses the connected (or survivable) Constrained Surface-level Gateway Placement (C-SGP) problem for 3-D underwater acoustic sensor networks. Given a set of candidate locations where SGs can be placed, our objective is to place minimum number of SGs at a subset of candidate locations such that it is connected (or 2-connected) from any USN to the base station. We propose a polynomial time approximation algorithm for the connected C-SGP problem and survivable C-SGP problem, respectively. Simulations are conducted to verify our algorithms' efficiency.

Keywords: Underwater sensor networks, Surface-level gateway placement, Connectivity, Survivability, Approximation algorithm.

1 Introduction and Motivations

Underwater Acoustic Wireless Sensor Networks (UA-WSNs) consist of underwater sensors that are deployed to perform collaborative monitoring tasks over a given region [1]. Underwater sensors are prone to failures because of fouling and corrosion. It is important that the deployed network is highly reliable, so as to avoid failure of monitoring missions due to failure of single or multiple sensors. Additionally, the network topology is in general a crucial factor in determining the energy consumption, the capacity and the communication delay of a network [2]. Hence, the network topology should be carefully engineered, and post-deployment topology optimization should be performed, when possible.

There is an architecture for 3-D UA-WSNs, consisting of resource-constrained Underwater Sensor Nodes (USNs) floating at different depths in order to observe a given phenomenon, some resource-rich SGs which are placed at the surface, and BSs (onshore sink or satellite etc.). The SG is equipped with an acoustic transceiver that is able to handle multiple parallel communications with the USNs. It is also endowed with a long range Radio Frequency (RF) transmitter to communicate with other SGs and the Base Stations (BSs). This network architecture provides better QoS and is used to quickly forward sensing data packets

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to the user [1,3,4]. In practice, however, there are some physical constraints on the placement of the SGs (or relay nodes). For example, there should be a minimum distance between two SGs in the network to avoid interference. Also, there may be some regions where SGs cannot be placed. In practice, there may be a forbidden regions where SGs cannot be placed [8].

In this paper, we study the Constrained Surface-level Gateway Placement (C-SGP) problem for 3-D underwater acoustic sensor networks, in which the optimization objective is to place the minimum number of SGs at a subset of candidate locations to meet 1-connectivity and Survivability (2-connectivity) requirements. We propose an approximate algorithm for the two problems respectively, and corroborate the algorithms' performance through theoretical analysis and simulations.

The rest of this paper is organized as follows. In Section II we present related works. Section III describes the network model and basic notations. The 1-connected and survivable C-SGP problems are studied in section IV. Section V presents the simulation results, and Section VI concludes this paper.

2 Related Works

The benefits of using SGs have been presented in previous research[1,3,4]. The work in [1] introduces a type of 3-D UA-WSNs architecture, consisting of USNs, SGs, and BSs (onshore sink or satellite etc.). The role of SGs is to communicate USNs with BSs. The work in [3] mainly focuses on the surface gateway placement. And the tradeoff between the number of surface gateways and the expected delay and energy consumption was analyzed. In [4], the authors propose a novel virtual sink architecture for UA-WSNs that aims to achieve robustness and energy efficiency in harsh under water channel conditions.

The majority of the existing work in relay node deployment problem is based on the 2-D network model derived from the terrestrial wireless sensor networks [5–8]. In addition, almost all of the above works study unconstrained version problem, in the sense that the relay nodes can be placed anywhere. However, in reality there are some physical constraints on the placement of the SGs (or relay nodes). Only works in [3,8] address the constrained surface-level gateway (relay node) placement problem. In this paper, we focus on the constrained surface-level gateway placement problem in 3-D networks to meet 1-connectivity and 2-connectivity, which is different from the problems in [3,8]. The authors in [3] only formulated the problem as Integer Linear Programming, but did not give any algorithm. In [8], the authors studied the constrained relay node placement problem in 2-D WSNs to meet 1-connectivity and survivability requirements. However, approaches proposed for 2-D networks can not be directly applied in 3-D networks. Therefore, some new research challenges are posed.

3 Notations and Basic Concepts

Let us consider a 3-D heterogeneous UA-WSN consisting of USNs, SGs and a BS. The USNs are pre-deployed in the sensing area and floated at different depths, each of them is equipped with an acoustic communicator which has communication range R_A. On the other hand, SGs only can be deployed on the surface, and are equipped with acoustic communicators and RF transceivers which have communication ranges R_A and R_{RF}, respectively. Compared with wireless RF links among ground-based or surface-level gateways, underwater acoustic wireless links have higher attenuation and path loss [1]. We assume that the wireless RF transceiver has longer effective distance than the acoustic modem. R_A and R_{RF} are given positive constants and $R_{RF} > R_A > 0$. We also assume that the BS is powerful enough so that its communication range is much greater than R_{RF} and R_A.

In this paper, $d_{Euc}(u, v)$ represents the Euclidean distance between u and v. Let b be the base station, S be a set of USNs, and L be a set of candidate locations where SGs can be placed. We use an undirected graph $G(V, E)$ to model the network architecture of a 3-D UA-WSN, where $V(G)=\{b\} \cup S \cup L$. The edge set $E(G)$ can be defined as follows:

- For any SG $u \in L$, and any node $v \in \{b\} \cup L$ which could be either a SG or the BS, $(u, v) \in E$ if and only if $d_{Euc}(u, v) \leq R_{RF}$.
- For any USN $w \in S$ and any node $z \in S \cup L \cup \{b\}$ which could be either a USN, a SG or the BS, $(w, z) \in E$ if and only if $d_{Euc}(w, z) \leq R_A$.

Definition 1. Suppose $G(V, E)$ is a 3-D graph to model a 3-D UA-WSN. Let H be a subgraph of G and u be a SG in H. The *USN degree* of u in H, denoted by $\delta_s(u, H)$, is the number of USNs that are neighbors of u in H. The *maximum USN degree* of H is defined as $\Delta_s(H)=\max\{\delta_s(u, H) | u \in V(H) \cap L\}$.

Definition 2. Suppose $G(V, E)$ is a graph. For $V' \subseteq V$, we denote $G[V'] = G(V', E')$ as an induced subgraph of $G(V, E)$ by V', in which, for any two nodes u and v in V', $(u, v) \in E'$ if and only if $(u, v) \in E$. For $E' \subseteq E$, we denote $G[E'] = G(V', E')$ as an induced subgraph of $G(V, E)$ by E', where V' is a set of endpoints of all edges in E'.

In this paper, we focus on the connected (or survivable) C-SGP Problem, which are formally represented as follows:

The connected (or survivable) C-SGP Problem: Given an UA-WSN (R_{RF}, R_A, $\{b\}$, S, L), the connected (or survivable) C-SGP problem is to place SGs at a subset L' of candidate locations in L such that there exists 1 routing path (or 2 node disjoint routing paths) connecting any USN in S to the BS and $|L'|$ is minimized.

The connected and survivable C-SGP problems are NP-hard since they have been proved to be NP-hard even for the scenario of 2-D network model[8]. In addition, the authors [9] proved that the 1-connected node cover placement problem (which is the special case of the connected C-SGP problem) is NP-hard where all the nodes are on regular triangular grid points.

4 Algorithms for the Constrained Surface-Level Gateway Placement Problems

In order to design approximation algorithms for the C-SGP problems, we construct a weighted graph $G(V, E, w)$. We give a weight for each edge in $G(V, E)$ as follows:

- For any edge $(u, v) \in E(G)$, we define its *weight* as $w(u, v) = |\{u, v\} \cap L|$. Let H be a subgraph of G, the weight of H is define as: $w(H) = \sum_{e \in E(H)} w(e)$.

From above definition, we know that the weight of any edge in E connecting two nodes u and v in L is assigned to 2. Similarly, any edge in E connecting a node in $\{b\} \cup S$ with a node in L is assigned weight of 1. Any edge connecting two USNs is assigned weight of 0. We have the following lemma.

Lemma 1. Let H be a subgraph of $G(V, E, w)$ such that each node's degree in $V(H) \cap L$ is at least 2 (within H). Then $w(H) \geq 2 \cdot |V(H) \cap L|$.

Proof. Initially, each node's weight in H is initialized to 0. Let (u, v) be an edge of H which is incident with two SGs. According to our definition, the weight of this edge is 2. In this case, we divide the edge weight into two equal pieces, add weight 1 to node u, add another 1 weight to node v. Let (u, v) be an edge of G where u is a SG and v is not. According to our definition, the weight of this edge is 1. In this case, we add weight 1 to node u, add weight 0 to node v. Let (u, v) be an edge of H where neither u nor v is a SG. According our definition, the weight of this edge is 0. In this case, we do not add any weight to node u and v. When all edges are executed over, we have shifted the edge weights of H to the SGs in H. Note that any SG u is getting a weight of 1 from every edge of H which is incident with u, resulting in that the weight of u is equal to the degree of u. Since each SG in H is incident to at least two edges in H, it receives at least weight 2. Therefore, $w(H) \geq 2 \cdot |V(H) \cap L|$. ■

4.1 An Algorithm for the Connected C-SGP Problem

In this subsection, we propose a polynomial time approximation algorithm for the connected C-SGP problem.

The algorithm includes two steps: (1) construct an edge-weighted undirected graph $G(V, E, w)$; (2) using the existing algorithm for the minimum Steiner tree problem on weight graph $G(V, E, w)$ to get a feasible solution for the connected C-SGP problem.

The algorithm is presented as Algorithm 1.

Lemma 2. Suppose Y_{opt} is an optimal solution to the connected C-SGP problem. Let T_{opt} be a Minimum Spanning Tree (MST) of $G[\{b\} \cup Y_{opt} \cup S]$ which is a induced subgraph of $G(V, E, w)$ by $\{b\} \cup Y_{opt} \cup S$. Then $\Delta_s(T_{opt}) \leq 12$.

Proof. We prove this lemma by contradiction. Assume that there is a SG u which can be connected to more than 12 USNs in T_{opt}. Without of the generality, we

Algorithm 1. An approximation algorithm for the connected C-SGP Problem

Input: An UA-WSN (R_{RF}, R_A, $\{b\}$, S, L).
Output: A feasible solution Y_A for the connected C-SGP problem.

Begin:
Construct an edge-weighted undirected graph $G(V, E, w)$ based on this UA-WSN, where $V=\{b\} \cup S \cup L$.
if The nodes in $\{b\} \cup S$ are not in a single connected component H of $G(V, E)$ **then**
  The connected C-SGP problem does not have a feasible solution. Stop.
end if
Apply the existing algorithm A for the Steiner Minimum Tree problem to compute a low weight Steiner Tree T_A of $G(V, E, w)$ for $\{b\} \cup S$.
Output $Y_A = V(T_A) \cap L$.
End.

assume that u is connected to 13 USNs $v_1, v_2, ..., v_{13}$. We will prove that these 13 USNs can not communicate with each other. Otherwise, we assume v_1 and v_2 can communicate with each other, i. e., (v_1, v_2) is an edge in $G(V, E, w)$. Since T_{opt} is a tree, it does not contain edge (v_1, v_2), otherwise there would be a cycle (u, v_1, v_2, u) in T_{opt}. Replacing edge (u, v_1) in T_{opt} by edge (v_1, v_2), we obtain another tree T_1 spanning all nodes in $\{b\} \cup Y_{opt} \cup S$. Since $w(u, v_1) = 1$ and $w(v_1, v_2) = 0$, we have $w(T_1) < w(T_{opt})$, contracting to the assumption that T_{opt} is a MST.

Since the acoustic communication range of any SG u is at most R_A, this assumption that SG u is connected to at least 13 USNs (which can not communicate with each other) implies that the maximum cardinality of the *MIS* (Maximal Independent Set) in u's neighbors in 3-D space is at least 13. Note that SG u and its USN neighbors all have the communication range R_A, i.e., when the Euclidean distance between u and one of its USN neighbors is less than R_A, there is a edge in G. Thus SG u and its neighbors can construct a local UBG (Unit Ball Graph). However, the authors in [11,12] had proved that the maximum cardinality of the *MIS* in a node's neighbors in 3-D space is at most 12. Therefore, this contradiction proves that SG u can not be connected to more than 12 USNs in T_{opt}, i.e., $\Delta_s(T_{opt}) \leq 12$. This lemma holds. ■

Theorem 1. Algorithm 1 can guarantee getting a feasible solution which uses no more than $(7.5 \cdot \alpha \cdot |Y_{opt}|)$ SGs, where α is an approximation ratio of algorithm A for the Steiner minimum tree problem.

Proof. Let T_{min} be the minimum weight tree of $G(V, E, w)$ which connects all nodes in $\{b\} \cup S$, and T_{opt} be a minimum weight spanning tree of $G[\{b\} \cup Y_{opt} \cup S]$ which is an induced subgraph of $G(V, E, w)$ by $\{b\} \cup Y_{opt} \cup S$. Y_A and T_A be a feasible solution and a subgraph corresponding Y_A got by Algorithm 1, respectively.

We denote T_{opt}^1 as an induced subgraph of T_{opt} by all 1-weight edges, and T_{opt}^2 as an induced subgraph of T_{opt} by all 2-weight edges. Then $w(T_{opt}) = w(T_{opt}^1) + w(T_{opt}^2)$.

From the definition of $\Delta_s(T_{opt})$ and the structure of T_{opt}^1, since each edge in T_{opt}^1 has weight 1 and can only contain a SG and a USN (or BS), and there is only one BS, we know each SG in (T_{opt}^1) is incident with at most $\Delta_s(T_{opt}) + 1$ edges. Therefore we have

$$w(T_{opt}^1) \le (\Delta_s(T_{opt}) + 1) \cdot |Y_{opt}|. \tag{1}$$

Since T_{opt} is a tree of $G(V, E)$, it has at most $(|Y_{opt}| - 1)$ 2-weight edges. Then,

$$w(T_{opt}^2) \le 2 \cdot (|Y_{opt}| - 1). \tag{2}$$

Therefore

$$w(T_{opt}) \le (2 + \Delta_s(T_{opt}) + 1) \cdot |Y_{opt}| - 2. \tag{3}$$

Since T_{min} is a minimum weight tree for $\{b\} \cup S$, we have

$$w(T_{min}) \le w(T_{opt}). \tag{4}$$

Since algorithm A's approximation ratio is α, we have

$$w(T_A) \le \alpha \cdot w(T_{min}) \le \alpha \cdot w(T_{opt}) \tag{5}$$

Note that T_A must satisfy the condition of Lemma 1, this is because, if there exists a node u in $V(T_A) \cap L$ such that $d_{T_A}(u) = 1$, where $d_{T_A}(u)$ represents u's degree in T_A, then we delete u from T_A and still get a feasible solution. Therefore, $|Y_A| \le \frac{1}{2} w(T_A)$.

Combining above inequations, we have

$$|Y_A| \le \frac{\alpha}{2} \cdot (2 + \Delta_s(T_{opt}) + 1) \cdot |Y_{opt}|. \tag{6}$$

From Lemma 2, we have $\Delta_s(T_{opt}) \le 12$, therefore,

$$|Y_A| \le 7.5 \cdot \alpha \cdot |Y_{opt}|. \tag{7}$$

This theorem holds. ■

We can use $(1+\frac{\ln 3}{2})$-approximation algorithm in [16] as algorithm A in Algorithm 1. From theorem 1, we have the following corollary:

Corollary 1. The connected C-SGP problem has a polynomial time 11.625-approximation algorithm.

4.2 An Algorithm for Survivable C-SGP Problem

In the UA-WSNs, USNs are prone to failures because of fouling and corrosion. Thus, survivability is an important requirement for topology construction or data routing. The network connectivity should be preserved even when some USNs fail or deplete their power. One way to preserve survivability is to construct 2-connected paths from any USN to base station. In this section, we present a polynomial time approximation algorithm for the survivable C-SGP problem. Our algorithm is based on polynomial time approximation algorithms for minimum weight 2-connected many-to-one routing problem. The algorithm for the survivable C-SGP problem is presented as Algorithm 2.

Algorithm 2. A approximation algorithm for the Survivable C-SGP problem.

Input: An UA-WSN (R_{RF}, R_A, $\{b\}$, S, L).
Output: A feasible solution $Y_A \subseteq L$.
Begin:

1: Construct an edge-weighted undirected graph $G(V, E, w)$ based on this UA-WSN, where $V = \{b\} \cup S \cup L$.
2: **if** The nodes in $\{b\} \cup S$ are not in a single 2-connected component H of $G(V, E, w)$ **then**
3: The survivable C-SGP problem does not have a feasible solution. Stop.
4: **end if**
5: Apply the existing algorithm A for the 2-connected many-to-one routing problem to compute a low weight subgraph H_A of $G(V, E, w)$ from S to b.
6: Output $Y_A = V(H_A) \cap L$.

End.

Lemma 3[10]. Let $G(V, E)$ be an undirected k-connected graph where $|V| \geq 3k - 2$ and $H(V, E')$ be a k-connected subgraph of H with minimum number of edges. Then $|E'| \leq k \cdot (|V| - k)$.

Lemma 4. Let $G(V, E)$ be an undirected graph and $H(V, E')$ be a many-to-one 2-connected subgraph from D to b with minimum number of edges, where D and b are a source set and a destination node, respectively. Then $|E'| \leq 2 \cdot (|V| - 1)$.

Proof. Since each node in D has two node disjoint paths to b which can construct a cycle containing b, a many-to-one 2-connected subgraph with minimum number of edges consists of some 2-node-connected components of H. Let H_1, H_2, ..., H_m be these 2-connected components, where H_i has $|V_i| \geq 3$ vertices, $i = 1, 2, ..., m$. Note that these 2-connected components must contain source node b and any two 2-connected components can not share a common node in $V \setminus \{b\}$, otherwise, the two components can merge into one 2-connected component. Furthermore, each 2-connected component $H_i(V_i, E_i)$ in $H(V, E')$ is a 2-connected spanning subgraph for V_i in $H(V, E')$ with minimum number of edges. If not, we can construct another many-to-one 2-connected subgraph from D to b in G with less number of edges than H, which contradicts with the assumption that H is a many-to-one 2-connected subgraph with minimum number of edges. We apply Lemma 3 for each 2-connected component H_i with $|V_i| \geq 4$ ($i = 1, 2, ..., m$), and since those 2-connected components with 3 nodes must be a cycle with 3 edges, therefore,

$$|E'| = \sum_{i=1}^{m} |E(H_i)| \le \sum_{i=1}^{m} 2(|V_i| - 2) = 2(\sum_{i=1}^{m} |V_i| - 2m)$$
$$= 2(|V| + m - 1 - 2m) = 2(|V| - 1 - m) \le 2(|V| - 1) \quad (8)$$

Note that $|V| = \sum_{i=1}^{m} |V_i| - m + 1$. ■

Lemma 5. Y_{opt} is an optimal solution for the Survivable C-SGP problem. Let H_{opt} be a minimum weight spanning subgraph of $G[\{b\} \cup Y_{opt} \cup S]$ which meets the 2-connected requirement from all USNs to b. Then $\Delta_s(H_{opt}) \le 12$.

Proof. We first prove that for any SG u, if it connects to more than 3 USNs in H_{opt}, then these USNs can not communicate with each other in $G(V, E)$. We prove it by contradiction. Without loss of generality, we assume that a SG u is to connect to m USNs $v_1, v_2, ..., v_m$ in H_{opt}($m \ge 3$), and v_1 and v_2 can communicate with each other, i.e., (v_1, v_2) is an edge in $G(V, E, w)$. Since H_{opt} is 2-connected from all USNs to b, and $m \ge 3$, there are two node disjoint paths from v_1 to b and v_3 to b, respectively. Therefore, there must be a path P from v_1 to v_3 which does not go through u. If P does not go through the USN v_2, H_{opt} contains a cycle $C_1 = \{(u, v_2), (v_2, v_1), P, (v_3, u)\}$. Then we have following obversion: For a USN node x, there are two node disjoint paths from x to b in H_{opt} which can construct a cycle C_2 containing b and x. If C_2 does not contain the edge (u, v_1), replacing (u, v_1) by (v_1, v_2) in H_{opt} does not destroy the 2-connectivity from x to b. If C_2 contains edge (u, v_1), then there are at least two intersect points (u and v_1) for C_1 and C_2. Then we can find at least three node disjoint paths between u and v_1. If delete the edge (u, v_1), there also exists a cycle containing b and x, i.e., there exist two node disjoint paths from x to b. For the arbitrary choice of x, we know that replacing edge (u, v_1) by (v_1, v_2) in H_{opt} does not destroy the many-to-one 2-connectivity from S to b.

From above discussion, we know that the subgraph H_1 got by replacing edge (u, v_1) in H_{opt} by edge (v_1, v_2) also can span all nodes in $\{b\} \cup Y_{opt} \cup S$ while meeting the many-to-one 2-connected requirement. Since $w(u, v_1) = 1$ and $w(v_1, v_2) = 0$, we have $w(H_1) < w(H_{opt})$, contradicting to the assumption that H_{opt} is a minimum weight subgraph. If path P goes through the USN v_2, H_{opt} has to contain a cycle $\{(u, v_1), P, (v_3, u)\}$. Similarly, deleting the edge (u, v_2) from H_{opt} will reduce its weight and H_{opt} is also a subgraph which can meet the many-to-one 2-connected requirement. This again contradicts to the minimum weight property of H_{opt}. So, we proved that for any SG u, if it connects to more than 3 USNs in H_{opt}, then these USNs can not communicate with each other in H_{opt}.

We prove this lemma by contradiction. Assume that in H_{opt}, a SG u can connect to more than 12 USNs. From above result, these USNs with at least 13 can not communicate with each other. Therefore, this also contradicts with the conclusions in [11,12]. Similar with the proof of Lemma 2, we also can prove that a SG u cannot be connected to more than 12 USNs in H_{opt}, i.e., $\Delta_s(H_{opt}) \le 12$. This proves this lemma. ■

Theorem 2. Algorithm 2 can guarantee getting a feasible solution which uses no more than $(8.5 \cdot \alpha \cdot |Y_{opt}|)$ SGs, where α is an approximation ratio of algorithm A for the 2-connected Steiner Minimum Subgraph problem.

Proof. Let H_{min} be the minimum weight many-to-one 2-node connected subgraph of $G(V, E, w)$ from S to b and H_{opt} be a minimum weight many-to-one 2-node connected subgraph of $G[Y_{opt} \cup S \cup \{b\}]$. Suppose Y_A is a feasible solution got by Algorithm 2, and H_A is a subgraph corresponding to Y_A.

We denote H^1_{opt} as an induced subgraph of H_{opt} by all 1-weight edges, and H^2_{opt} as an induced subgraph of H_{opt} by all 2-weight edges. Then $w(H_{opt}) = w(H^1_{opt}) + w(H^2_{opt})$.

From the definition of $\Delta_s(H_{opt})$ and the structure of H^1_{opt}, since each edge in H^1_{opt} has weight 1 and can only contain a SG and a USN (or BS), and there is only one BS, we know each SG in (H^1_{opt}) is incident with at most $\Delta_s(H_{opt}) + 1$ edges. Therefore we have

$$w(H^1_{opt}) \le (\Delta_s(H_{opt}) + 1) \cdot |Y_{opt}|. \tag{9}$$

$$w(H^2_{opt}) \le 2|E(H_{opt})| \le 2 \cdot (2|Y_{opt}| - 2). \tag{10}$$

Note that we use Lemma 4 to get the second inequation in (10) since H_{opt} satisfies the condition of Lemma 4. Therefore,

$$w(H_{opt}) \le (4 + \Delta_s(H_{opt}) + 1) \cdot |Y_{opt}| - 4. \tag{11}$$

Since H_{min} is the minimum weight many-to-one 2-node connected subgraph of $G(V, E, w)$ from S to b, we have $w(H_{min}) \le w(H_{opt})$. Because the approximation ratio of algorithm A is α, therefore

$$w(H_A) \le \alpha \cdot w(H_{min}) \le \alpha \cdot w(H_{opt}) \tag{12}$$

Note that H_A must satisfy the condition of Lemma 1, this is because, if there exists a node $u \in V(H_A) \cap L$ such that $d_{H_A}(u) = 1$, then we delete u from H_A and still get a feasible subgraph. So, $|Y_A| \le \frac{1}{2} w(H_A)$. Combining above inequations, we have

$$|Y_A| \le \frac{\alpha}{2} \cdot (5 + \Delta_s(H_{opt})) \cdot |H_{opt}|. \tag{13}$$

From Lemma 5, we have $\Delta_s(H_{opt}) \le 12$, therefore

$$|Y_A| \le 8.5 \cdot \alpha \cdot |Y_{opt}|. \tag{14}$$

This proves this theorem. ■

Corollary 2. The survivable C-SGP problem has a polynomial time 17-approximation algorithm.

Proof. According to the algorithm in [15], there is a polynomial time 2-approximation algorithm for the many-to-one 2-connected problem. The conclusion of this corollary can be achieved by choosing the algorithm in [15] as A in Algorithm 2. ■

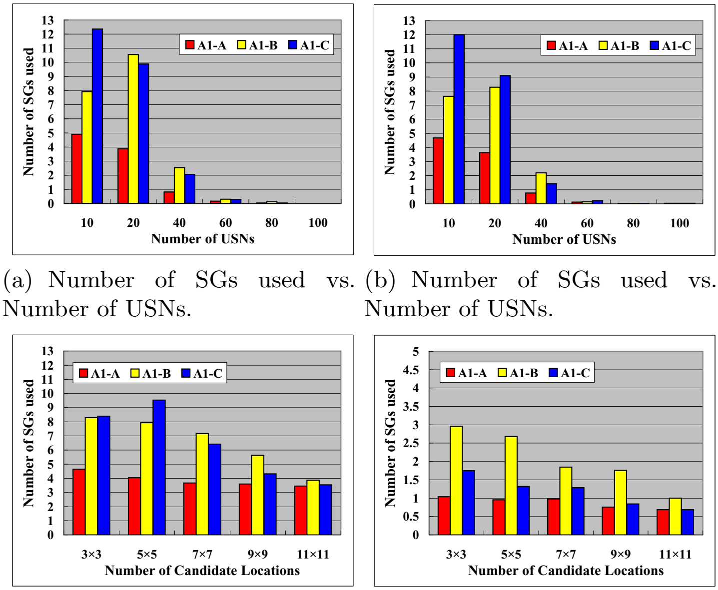
5 Performance Evaluations

In this section, we evaluate the performance of our algorithms by simulations. We implemented approximation Algorithm 1 with A being the MST based 2-approximation SMT algorithm in [13] (denoted by A1-A and simpler than the algorithm in [16]) and another algorithms in [14] (denoted by A1-B and A1-C) for Steiner Minimum Tree problem. In the simulations, we focus on comparing the approximation algorithms A1-A and heuristic algorithms A1-B and A1-C. We study how the required number of SGs is affected by two parameters varying over a wide range: the number of USNs in the space and the number of the candidate locations in the upper plane.

The simulation is conducted in a $100 \times 100 \times 30$ 3-D space. We used both regular grid points and randomly generated points as the candidate locations for the SGs, and obtained similar results. For convenience of presentation, we used regular grid points as the surface-level candidate locations for the SGs in upper plane. In this setting, the playing field consists of $K \times K$ small squares contained by the upper plane of the space, with the $(K+1)^2$ grid points as L. We present averages of 50 separate runs for each result.

In Fig. 1 (a) and (b), we compare the number of SGs required with number of USNs varying. In both cases, the number of candidate locations was 100 (10×10). The number of USNs was varied from 10 to 100. The R_A is set to 25 for both cases and R_{RF} is set to 25 and 50 for case (a) and (b), respectively. In both cases, we can see that the required number of SGs decreases with the increment of USNs. With the increment of USNs, the USNs trend to self-connected and only few SGs are required to connectethe USNs and BSs. The algorithm A1-A always performs better than the algorithms A1-B and A1-C.

In Fig. 1 (c) and (d), we also study the relationship between the number of SGs required and number of the candidate locations. We addressed two cases for 20 and 40 USNs respectively. We set $R_{RF} = 40$ and $R_A = 25$. There is no obvious variety of the number of SGs used for A1-A in Fig. 1 (c) and (d). Since, the SGs' main function is to connect USNs and BSs, some isolated USNs have to send data to BS by the SGs nearby them. Thus the variety of number of candidate locations may change the number of required SGs a little when the total number of used SGs trend to the optimal solution. This indicates that our approximation algorithm performs well. However, for the results delivered by A1-B and A1-C, these results are worse than A1-A's, and the more choice of candidate locations, the less redundant SGs will be used. Therefore, with the increment of candidate locations, the numbers of used SGs produced by A1-B and A1-C decrease gradually.



(a) Number of SGs used vs. Number of USNs.

(b) Number of SGs used vs. Number of USNs.

(c) Number of SGs used vs. Number of candidate locations.

(d) Number of SGs used vs. Number of candidate locations.

Fig. 1. The simulation results for the 1-connected C-SGP problem

6 Conclusions

In this paper, we studied the C-SGP problems in UA-WSNs. We mainly addressed the connected and survivable C-SGP problems, which can ensure to meet the connectivity and survivability requirements for some application environments of UA-WSNs. We discussed the computational complexity and presented an approximate algorithm for the two problems respectively, and corroborate the algorithms' performance through theoretical analysis and simulations.

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