Characterizing Relevant Belief Revision Operators

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Abstract. This paper introduces a framework for relevant belief revision. We represent agent's beliefs in prime implicants and express agent's preference on beliefs as a pre-order over terms. We define a belief revision operator via minimising the change of the prime implicants of the existing beliefs and the incoming information with respect to agent's preferences. We show that such a belief revision operator satisfies Katsuno and Mendelzon's postulates for belief revision as well as Parikh's postulate for relevant revision. This paper demonstrates a natural way to identify relevance of beliefs and an implementation of Parikh's relevant belief revision.

1 Introduction

Belief revision is the process of incorporating new pieces of information into a set of existing beliefs. It is usually assumed that the operation follows the following two principles: (i) the resulting belief set is consistent and (ii) the change on the original belief set is minimal. Several formalisms of belief revision have been proposed in the literature (see [8] for more details). The most influential work is the AGM paradigm which characterises the belief revision operation by a set of plausible axioms, generally referred to as the AGM postulates [1].

Despite of the popularity of the AGM paradigm, the AGM postulates are not sufficient to capture the notion of minimal change. As stressed by Parikh in [19], the full meet revision operator (removing all statements of the original beliefs and keeping only the new piece of information) satisfies the AGM postulates, which is obviously not a minimal change. In order to avoid counter-intuitive change of beliefs, Parikh proposed an additional postulate to the AGM postulates, which characterises the notion of relevant revision. A revision is said to be *relevant* if it enables to keep all the initial beliefs of an initial belief set ψ that are not related to the new piece of information after the belief revision operation. Formally speaking, if a statement of ψ does not use any propositional symbols that are used in the new piece of information μ, then this statement should belong to the resulting belief set. In other words, Parikh's postulate is grounded in the symbols used in ψ and μ. However, Parikh did not provide an actual belief revision operator that satisfies his postulate and the AGM postulates. The main difficulty

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is that it is hard to find simple and intuitive criteria that separate relevant information from irrelevant information. Makinson in [16] proposed a formal approach that is able to split logical symbols that are used in a belief set. However, no construction of belief revision operator was provided based on this language splitting approach. Peppas *et al.* in [20] proposed a model based construction of belief revision operator (based on systems of spheres) that implements the AGM postulates and Parikh's postulate. However, relevance by its nature is a syntactical issue. A model based approach at most provides a peripheral solution.

This paper aims to offer a syntactical construction of belief revision operator based on prime implicants. In [3], Bittencourt *et al.* proposed a syntax-based belief revision operator that is constructed by using prime implicants and prime implicates. Such a construction provides a natural way to identify relevance of beliefs. However, the construction is based on Dalal's distance, which cannot capture the notion of minimal change based on general preference orderings. In this paper we redefine the belief revision operator based on minimal change on general preference orderings. We define a belief revision operator via minimising the change of the prime implicants of the existing beliefs and the incoming information. We show that such a belief revision operator satisfies Katsuno and Mendelzon's postulates for belief revision as well as Parikh's postulate for relevant revision. Our approach provides a clear and simple way to address the belief relevance issue and a natural implementation of Parikh's relevant belief revision.

The paper is organised as follows. Section 2 reviews the notions of implicant and prime implicant. Section 3 defines a class of revision operators based on the prime implicant representation of beliefs. Section 4 shows that Parikh postulate holds for this class of revision operators. Finally, we conclude the work and discuss the related work with a perspectives of possible future work.

2 Preliminaries

Let $P = \{p_0, \ldots, p_n\}$ be a finite set of propositional symbols and $LIT = \{L_0, \ldots, L_{2n}\}$ be the set of the associated literals: $L_i = p_j$ or $\neg p_j$. Let $\overline{L}$ be the complementary literal, s.t. $\overline{L} = p$ (respectively $\neg p$) iff $L = \neg p$ (respectively p). Let $\mathcal{L}(P)$ be the propositional language associated to P and $\psi \in \mathcal{L}(P)$ be an ordinary formula. Let Lang be the function that assigns to each formula the set of propositional symbols that are contained in the formula, i.e., $\mathsf{Lang} : \mathcal{L}(P) \mapsto 2^P$.

Let $\mathcal{W}(P)$ be the whole set of propositional interpretations associated to P (for the sake of conciseness, hereafter we skip parameter P) and $\models$ the satisfiability relation. Let $[\![\psi]\!]$ be the set of propositional interpretations that satisfy ψ (the models of ψ).

Any formula can be represented in a disjunctive normal form (DNF). Given a formula ψ, let DNF_ψ be a DNF of ψ. Assume that $DNF_\psi = D_0 \vee \cdots \vee D_w$ be the *disjunction of terms*, where each term D_i is a *conjunction of literals*: $D_i = L_0 \wedge \cdots \wedge L_k$. Let $\overline{D}$ be the mirror of term D s.t. $\overline{D} = L_0 \wedge \cdots \wedge L_{k_D}$ iff $D = \overline{L_0} \wedge \cdots \wedge \overline{L_{k_D}}$.

A term D is an *implicant* of ψ if $D \models \psi$. A term D is said to be a *prime implicant* [21] of ψ if D is an implicant of ψ and for any term D' such that $D' \subseteq D$, we have $D' \not\models \psi$, i.e., a prime implicant of a formula ψ is an implicant of ψ without any subsumed terms. In the following, terms can be seen as sets of literals. Hereafter, we frequently switch between the logical notation and the set notation. We write $D - D'$ to denote the subtraction operation over terms, that results from removing all literals that occur in D' from D, that is, $D - D' = \{L \in D : L \notin D'\}$. Although the definition of prime implicant includes contradictory terms, because a contradiction is an implicant of any formula, in the sequel we only consider terms that do not have any pair of contradictory literals. We also do not consider implicants with redundant literals (i.e. a literal can only appear at most once in an implicant). Let $\mathcal{D}_\psi$ be the set of all the non contradictory and non redundant implicants of ψ and $\mathcal{D}$ be the set of all non contradictory and non redundant terms. Notice that since P is finite, $\mathcal{D}$ is also finite.

We define PI_ψ as a disjunction of all non contradictory prime implicants of ψ such that $\psi \equiv PI_\psi$. Whenever it's clear, in the sequel we omit "non contradictory" and "non redundant" when we mention prime implicants.

2.1 Belief Revision Issue

Belief revision consists of inserting in a consistent way a new piece of information μ into a belief set ψ [6]. Revision operator is usually denoted by $\circ$ and the resulting belief set is denoted by $\psi \circ \mu$. The AGM postulates provide an axiomatic characterisation of belief revision operators [1,7]. In the context of finite propositional beliefs, AGM postulates can be rephrased as follows [12]:

(R1) $\psi \circ \mu$ implies μ.
(R2) If $\psi \wedge \mu$ is satisfiable then $\psi \circ \mu \equiv \psi \wedge \mu$.
(R3) If μ is satisfiable then $\psi \circ \mu$ is also satisfiable.
(R4) If $\psi_1 \equiv \psi_2$ and $\mu_1 \equiv \mu_2$ then $\psi_1 \circ \mu_1 \equiv \psi_2 \circ \mu_2$.
(R5) $(\psi \circ \mu) \wedge \phi$ implies $\psi \circ (\mu \wedge \phi)$.
(R6) If $(\psi \circ \mu) \wedge \phi$ is satisfiable then $\psi \circ (\mu \wedge \phi)$ implies $(\psi \circ \mu) \wedge \phi$.

As mentioned earlier, even if they have been widely accepted, these postulates are too weak to characterise minimal change with respect to relevant revision. For almost all belief revision operators, minimality is represented with the help of an extra logical criteria of distance between the initial belief set and the incoming information [10]. That is, revising ψ by μ consists of choosing the closest models of μ with respect to ψ [13,12]. Let $\preceq_\psi$ be a total pre-order representing preferences and defined over set $\mathcal{W}$ and representing the closeness criterion: $w \preceq_\psi w'$ states that w is at least as close as w' w.r.t. ψ. Faithful assignment represents preferences which are "centered" on ψ, i.e. the most preferred models are the models of ψ[1].

Theorem 1. *[13] Let $\mathcal{F}$ be a faithful assignment that maps each belief set ψ, to a total pre-order $\preceq_\psi$ over $\mathcal{W}$ such that the following three conditions hold:*

(C1) *if $w, w' \in [\![\psi]\!]$ then $w \not\prec_\psi w'$*
(C2) *if $w \in [\![\psi]\!]$ and $w' \notin [\![\psi]\!]$ then $w \prec_\psi w'$*
(C3) *if $\psi \equiv \varphi$ then $\preceq_\psi = \preceq_\varphi$*

A revision operator $\circ$ satisfies **(R1)–(R6)** *if and only if* $[\![\psi \circ \mu]\!] = \min([\![\mu]\!], \preceq_\psi)$

[1] $\prec_\psi$ is defined from $\preceq_\psi$ as usual, i.e., $w \prec_\psi w'$ iff $w \preceq_\psi w'$ but not $w' \preceq_\psi w$.

One of the simplest ways to set the preferences is to consider the propositional symbols that may change. This has been proposed by Dalal in [5]. It consists of characterising belief revision operator as a function which changes in each model of ψ the minimal number of propositional symbol truth values so that incoming information can be added without entailing inconsistency.

3 Prime Implicants Based Revision

Prime implicants enable us to define belief revision operators in a syntactic way while avoiding the issue of syntax dependency. This characteristic is due to the fact that each formula has only one set of prime implicants. The applicability of prime implicants and implicates in belief change area has been investigated and presented in several works, e.g [3,18,22,2,17], as well as the properties of prime implicants and prime implicates in [4]. We root our work in [3,17] which mainly focused on the notion of distance and the way to set preferences and extend these contributions to capture the notion of relevance.

3.1 Incorporating Prime Implicants

Given a belief set ψ and a new piece of information μ, let PI_ψ and PI_μ be the set of prime implicants of ψ and μ. In order to incorporate new information μ into the existing belief set ψ, we combine the prime implicants of ψ and μ in such a way that for every $D_\psi \in PI_\psi$ and $D_\mu \in PI_\mu$, a new term is obtained by adding to D_μ all the literals of D_ψ which are not conflicting with the literals to D_μ, as stressed by the following incorporating function Γ:

Definition 1. *Let* $\Gamma : \mathcal{L}(P) \times \mathcal{L}(P) \mapsto 2^{\mathcal{D}}$ *be a function defined as follows:*

$$\Gamma(\psi, \mu) = \{D_\mu \cup (D_\psi - \overline{D_\mu}) | D_\psi \in PI_\psi \text{ and } D_\mu \in PI_\mu\}$$

where PI_ψ *and* PI_μ *are the sets of prime implicants of* ψ *and* μ*.*

Intuitively, the set $\Gamma(\psi, \mu)$ contains all the terms that are obtained by extending each prime implicant of μ with the maximal consistent part of each prime implicant of ψ.

Example 1. Consider the following sets of prime implicants: $PI_\psi = (\neg p_2 \wedge \neg p_3) \vee (\neg p_2 \wedge p_4) \vee (\neg p_1 \wedge \neg p_3 \wedge p_4)$ and $PI_\mu = (p_3 \wedge \neg p_4) \vee (p_1 \wedge p_2)$. The following table presents the set of terms in $\Gamma(\psi, \mu)$:

We extend the previous definition with the set $\Gamma(\psi) \subseteq \mathcal{D}$ s.t. $\Gamma(\psi) = \bigcup_{D \in \mathcal{D}} \Gamma(\psi, D)$ which denotes the set of all terms that can be defined according to ψ (i.e. all possible consistent μ are considered).

3.2 Preference Ordering over Terms

To construct a belief revision operator using prime implicants, we need to set preferences over terms instead of worlds. Let $\leqslant_\psi$ be a preference relation defined over the set of possible terms $\mathcal{D}$: $D \leqslant_\psi D'$ states that D is at least as close as D' w.r.t. ψ. As for preferences set over worlds, we define the notion of faithful assignment.

Table 1. Incorporating prime implicants of two formulas

D_ψ	D_μ	$D_i \in \Gamma(\psi,\mu)$	
$\neg p_2 \wedge \neg p_3$	$p_3 \wedge \neg p_4$	$\neg p_2 \wedge p_3 \wedge \neg p_4$	(D_1)
$\neg p_2 \wedge \neg p_3$	$p_1 \wedge p_2$	$p_1 \wedge p_2 \wedge \neg p_3$	(D_2)
$\neg p_2 \wedge p_4$	$p_3 \wedge \neg p_4$	$\neg p_2 \wedge p_3 \wedge \neg p_4$	(D_1)
$\neg p_2 \wedge p_4$	$p_1 \wedge p_2$	$p_1 \wedge p_2 \wedge p_4$	(D_3)
$\neg p_1 \wedge \neg p_3 \wedge p_4$	$p_3 \wedge \neg p_4$	$\neg p_1 \wedge p_3 \wedge \neg p_4$	(D_4)
$\neg p_1 \wedge \neg p_3 \wedge p_4$	$p_1 \wedge p_2$	$p_1 \wedge p_2 \wedge \neg p_3 \wedge p_4$	(D_5)

Definition 2. *A* faithful assignment $\mathcal{F}$ *is a function which maps every formula* ψ *to a pre-order over* $\Gamma(\psi)$ *such that*[2]*:*

(C1-T) *if* $D_u, D_v \in \mathcal{D}_\psi$, *then* $D_u \not<_\psi D_v$.
(C2-T) *if* $D_u \in \mathcal{D}_\psi$ *and* $D_v \notin \mathcal{D}_\psi$, *then* $D_u <_\psi D_v$.
(C3-T) *if* $\psi \equiv \varphi$, *then* $\leqslant_\psi = \leqslant_\varphi$.
(CI-T) *For all* $D_u \notin D_\psi, D_v \notin D_\psi$, *if* $(D_u \subseteq D_v)$ *then* $D_u \sim_\psi D_v$.

The first key difference between the two notions of faithful assignment is the domain used for preferences: it is required that preferences have to be defined on a subset of terms rather than on the whole set of possible worlds. That is, the pre-order is only required to be set over the set of terms that can be built from ψ and function Γ. The three constraints **(C1-T)**–**(C3-T)** are similar to the constraints **(C1)**–**(C3)**. The second key difference is the constraint **(CI-T)** which states that preferences should not favour too specific terms.

Example 2. Suppose $\psi = p_1 \wedge \neg p_2$; suppose two terms $\neg p_1 \wedge \neg p_2$ and $\neg p_1 \wedge \neg p_2 \wedge p_3$ which belong to $\mathcal{D}_\psi$. Suppose that $\neg p_1 \wedge \neg p_2 \wedge p_3 <_\psi \neg p_1 \wedge \neg p_2$. It then means that if ψ is revised by $\neg p_1 \wedge \neg p_2$ then $\neg p_1 \wedge \neg p_2 \wedge p_3$ will be preferred to $\neg p_1 \wedge \neg p_2$ and thus the resulting belief set might contained extra and irrelevant information (p_3). Following the intuition of Parikh, relevance entails to focus changes on p_1 and p_2 and thus it cannot be the case that $\neg p_1 \wedge \neg p_2 \wedge p_3 <_\psi \neg p_1 \wedge \neg p_2$.

Hence, **(CI-T)** is a first step towards the enforcement of the notion of relevance.

Example 3. Let us pursue example 1. Suppose a faithful assignment such that $\leqslant_\psi$ is a preference ordering over the set of terms $\Gamma(\psi)$ based on Dalal's distance [5]. All terms $D \in \Gamma(\psi)$ are defined as follows: $D = D_\mu \cup (D_\psi - \overline{D_\mu})$ s.t. $D_\mu \in \mathcal{D}$ and $D_\psi \in PI_\psi$. $D \leqslant_\psi D'$ if and only if either (i) $D \in \mathcal{D}_\psi$ or (ii) the number of literals in the set $D_\psi \cap \overline{D_\mu}$ is less or equal to the number of literals that belongs to $D'_\psi \cap \overline{D'_\mu}$. As we can see, this pre-order is total over $\Gamma(\psi)$. It is straightforward to check that constraint **(C1-T)** and **(C2-T)** hold. Next, since the definition of prime implicants entails $\psi \equiv \varphi$ iff $PI_\psi = PI_\varphi$, constraint **(C3-T)** also holds. Finally, because Dalal's distance focuses on contradicting symbols, constraint **(CI-T)** also holds. Let us focus on the terms belonging to the set $\Gamma(\psi,\mu)$ shown in table 1. We get the following ordering over the terms in $\Gamma(\psi,\mu)$: $D_1 \sim_\psi D_2 \sim_\psi D_3 \sim_\psi D_5 <_\psi D_4$.

[2] $D \sim_\psi D'$ stands for $D \leqslant_\psi D'$ and $D' \leqslant_\psi D$.

3.3 Prime Implicant Based Revision

We are now able to define belief revision operators using prime implicants. A PI revision of ψ by μ is denoted by $\psi \circ_{PI} \mu$. Let us now characterise the PI revision operator:

Theorem 2. *Let $\mathcal{F}$ be a faithful assignment over $\mathcal{D}$ that maps each belief set ψ a total pre-order $\leqslant_\psi$. The PI revision operator $\circ_{PI}$ defined by $\mathcal{F}$ satisfies* **(R1)–(R6)** *if*

$$\psi \circ_{PI} \mu =_{def} \bigvee \min(\Gamma(\psi, \mu), \leqslant_\psi)$$

where $\min(\Gamma(\psi, \mu), \leqslant_\psi) = \{D \in \Gamma(\psi, \mu) \mid \forall D' \in \Gamma(\psi, \mu) \text{ and } D \leqslant_\psi D'\}$.

The proof is mainly based on [13], notice that constraint (**CI-T**) enable to enforce postulates (**R5**) and (**R6**)[3].

Example 4. According to the preferences detailed in the previous example, we get that terms D_1, D_2, D_3 and D_5 are minimal and compound the revised belief base:

$$\psi \circ_{PI} \mu = (\neg p_2 \wedge p_3 \wedge \neg p_4) \vee (p_1 \wedge p_2 \wedge \neg p_3) \vee \\ (p_1 \wedge p_2 \wedge p_4) \vee (p_1 \wedge p_2 \wedge \neg p_3 \wedge p_4)$$

Notice that the PI revision is more restricted than the AGM revision since AGM revision considers all possible worlds while *PI* revision only focuses on terms belonging to $\Gamma(\psi)$.

Example 5. Let $\psi = p_1 \wedge p_2$ and $\mu = \neg p_1$. It is easy to see that $PI_\psi = \{p_1 \wedge p_2\}$ and $PI_\mu = \{\neg p_1\}$. Therefore $\psi \circ_{PI} \mu = \neg p_1 \wedge p_2$, no matter what the preference over the set of terms is. However, there are more than one AGM revision outcomes. Let $\mathcal{W} = \{11, 10, 01, 00\}$ be the set of interpretations of the language $\mathcal{L} = \{p_1, p_2\}$. If the faithful order of ψ over $\mathcal{W}$ is $11 \prec_\psi 10 \sim_\psi 01 \sim_\psi 00$, the outcome of the revision will be $[\![\psi \circ \mu]\!] = \{01, 00\} = [\![\neg p_1]\!]$. If the faithful order is $11 \prec_\psi 10 \sim_\psi 01 \prec_\psi 00$, then we have $[\![\psi \circ \mu]\!] = \{01\} = [\![\neg p_1 \wedge p_2]\!]$. Notice that p_2 cannot be changed by the revision of ψ with $\neg p_1$ in the prime implicant based revision; while p_2 may change with the AGM revision.

Let us now relate $\circ$ and $\circ_{PI}$ by showing that if the preferences over terms are linked to the preferences overs worlds, then theorems 1 and 2 are similar. That is the revised belief sets are equivalent whether we use worlds or terms. The following constraint (**KP**) states that preferences over terms and worlds have to be closely connected; i.e. if a term D_u is preferred to a term D_v then we have the same preferences between the worlds in which these two terms are satisfied. Let $\leqslant_\psi$ be a faithful pre-order over $\mathcal{D}$ and $\preceq_\psi$ be a faithful pre-order over $\mathcal{W}$ associated with ψ such that:

$$D_u \leqslant_\psi D_v \iff \forall u \in [\![D_u]\!], \exists v \in [\![D_v]\!]\ u \preceq_\psi v \qquad \textbf{(KP)}$$

Let us now relate operators $\circ$ and $\circ_{PI}$ by stating that if constraint (**KP**) is satisfied then both operators give similar results.

[3] Due to space restrictions all proofs have been omitted. A longer version of the paper which includes all the proofs is downloadable at the URL
`http://www.irit.fr/~Laurent.Perrussel/ai10-long.pdf`.

Theorem 3. *Let $\preceq_\psi$ and $\leqslant_\psi$ be two faithful assignments over $\mathcal{W}$ and $\Gamma(\psi)$. Revision operators $\circ$ and $\circ_{PI}$ produce identical belief sets, that is $[\![\psi \circ_{PI} \mu]\!] = [\![\psi \circ \mu]\!]$ if and only if for all $D_u, D_v \in \Gamma(\psi)$ constraint* ***(KP)*** *is satisfied.*

The previous theorem confirms that operator $\circ_{PI}$ describes a specific family of AGM revision operators; that is, combined with additional constraints, postulates (**R1**)–(**R6**) also characterise PI revision. In the next section, we show that this specific aspect (or additional constraints) is in fact rooted in the notion of relevant revision.

4 Relevant Revision

The common shared opinion for setting the notion of relevant revision is to ground this notion into the languages used for describing belief bases [19,9,20,16]. If a statement φ in the belief base ψ does not share any propositional symbols with incoming information μ, then φ should belong to the resulting belief base. Parikh proposes the following postulate to capture the idea of relevant revision [19]:

(P) Let $\psi = \varphi \wedge \varphi'$ such that $\mathsf{Lang}(\varphi) \cap \mathsf{Lang}(\varphi') = \emptyset$. If $\mathsf{Lang}(\mu) = \mathsf{Lang}(\varphi)$, then $\psi \circ \mu \equiv (\varphi \circ' \mu) \wedge \varphi'$, where $\circ'$ is the revision operator restricted to language $\mathsf{Lang}(\varphi)$.

In general it is not easy to split the irrelevant statements from the belief base because the syntactical representation of the belief base could "falsify" us [16]. However, if we represent the belief set in prime implicants, this splitting becomes much more visible. This motivates us to rephrase Parikh's postulate in terms of the prime implicant representation of formulas:

(P-T) Let $\psi = \varphi \wedge \varphi'$. If $\mathsf{Lang}(PI_\varphi) \cap \mathsf{Lang}(PI_{\varphi'}) = \emptyset$ and $\mathsf{Lang}(PI_\mu) = \mathsf{Lang}(PI_\varphi)$, then $\psi \circ_{PI} \mu \equiv (\varphi \circ'_{PI} \mu) \wedge \varphi'$, where $\circ'_{PI}$ is the revision operator restricted to the language $\mathsf{Lang}(PI_\varphi)$.

However, the construction of $\circ_{PI}$ is not sufficient for enforcing postulate **(P-T)**. As stressed in [20], the local revision operator mentioned in postulate **(P)** has to be context-independent. Suppose that there are two belief sets ψ and ψ' such that $\psi \equiv \varphi \wedge \varphi'$, $\psi' \equiv \varphi \wedge \varphi''$, $\mathsf{Lang}(PI_\varphi) \cap \mathsf{Lang}(PI_{\varphi'}) = \emptyset$ and $\mathsf{Lang}(PI_\varphi) \cap \mathsf{Lang}(PI_{\varphi''}) = \emptyset$. Then there should exist only one single version of the local revision operator $\circ'$ such that $\psi \circ \mu \equiv (\varphi \circ' \mu) \wedge \varphi'$ and $\psi' \circ \mu \equiv (\varphi \circ' \mu) \wedge \varphi''$ for any μ s.t. $\mathsf{Lang}(PI_\mu) \subseteq \mathsf{Lang}(PI_\varphi)$. We also agree for this reading of postulate **(P)** qualified by [20] as the *strong* version of postulate **(P)**. Let us represent this notion in our framework. Assume that $\psi \equiv \varphi \wedge \varphi'$ s.t. $\mathsf{Lang}(PI_\varphi) \cap \mathsf{Lang}(PI_{\varphi'}) = \emptyset$. Having one local revision operator means that we have only one pre-order $\leqslant_\varphi$ associated to φ. Now, let us suppose two terms D and D' such that $D \leqslant_\varphi D'$. Pre-order $\leqslant_\psi$ should also reflect these preferences; that is extending terms D and D' with any prime implicants belonging to $PI_{\varphi'}$ will not change the preferences. The following constraint states this by saying how we can switch from one pre-order to a second one.

(PS-T) Let $\psi \equiv \varphi \wedge \varphi'$ such that $\mathsf{Lang}(PI_\varphi) \cap \mathsf{Lang}(PI_{\varphi'}) = \emptyset$. For any $D, D' \in \Gamma(\varphi)$: $D \leqslant_\varphi D'$ iff $D \cup D_{\varphi'} \leqslant_\psi D' \cup D'_{\varphi'}$ such that $D_{\varphi'}, D'_{\varphi'} \in PI_{\varphi'}$ and $D \cup D_{\varphi'}, D' \cup D'_{\varphi'} \in \Gamma(\psi)$.

In other words, this constraint expresses the strong notion of relevance by considering multiple faithful assignments. Now, we conclude that operator $\circ_{PI}$ characterizes relevant belief revision by satisfying postulate **(P)**. We first show that satisfying constraint **(PS-T)** entails that operator $\circ_{PI}$ satisfies the relevance postulate.

Theorem 4. *If faithful assignment $\leqslant_\psi$ satisfies **(PS-T)** then $\circ_{PI}$ postulate* **(P-T)**.

The theorem shows that the relevance is rooted in two key aspects: the definition of the revision operator $\circ_{PI}$ and the commitment to the strong version of relevance postulate.

Now, let us look at the opposite way. The question is: is operator $\circ_{PI}$ too restrictive or not? That is, if a revision operator $\circ$ satisfies postulate **(P)**, then can we exhibit an operator $\circ_{PI}$ which produces the same result? If the answer is positive then it means that in fact operator $\circ_{PI}$ characterizes the family of belief revision operators that satisfy postulate **(P)**. The following theorem shows that it is in fact the case under the condition that we focus on strong meaning of relevance.

Theorem 5. *Suppose a revision operator $\circ$ such that (**R1**)–(**R6**) and (**P**) hold. There always exists an operator $\circ_{PI}$ such that (**PS-T**) holds for $\circ_{PI}$ and $\psi \circ \mu \equiv \psi \circ_{PI} \mu$.*

The theorem tells us that $\circ_{PI}$ can represent operators for relevant belief change. Indeed, in [19,20] it has been shown that we can always define an operator $\circ$ which satisfies postulates (**R1**)–**(R6)** and **(P)**. In other words, theorems 2, 4 and 5 show that prime implicant based revision operator exactly characterizes the notion of relevant belief revision: (**R1**)–**(R6)** and **(P)** $\iff \circ_{PI}$.

Example 6. It can be easily shown that condition **(PS-T)** holds for Dalal-based preferences introduced in the previous examples. The consequence of the previous theorems is that Dalal's revision operator is relevant. A second consequence is that preferences underlying a relevant revision can always be connected through constraint (**KP**).

5 Conclusion

In this paper we have proposed a general characterisation of relevant revision that states the family of operators for relevant belief change. We rephrased the relevance postulate in terms of prime implicants, which leads to the characterisation of relevant revision.The use of prime implicant representation not only provides a natural way to identify relevance in beliefs but also has advantages in computation [11]. Also our approach is syntax based. It avoids the syntax dependency, therefore possess the advantages of syntactical and semantical approaches.

Most of the work about the notion of relevant belief revision is based on language splitting. A splitting of a language with respect a belief set ψ is a partition of the language such that ψ can be resembled by a set of belief sets described in each sub-language. Makinson in [14,15] shown that any AGM compliant contraction operator which performs contraction on the belief set in each sub-language satisfies Parikh's postulate. That is, only partitions concerned by incoming information should be contracted. Makinson argues that there should exist a language-based dependence relation between the impacted partition and incoming information. He also argues this dependence relation should be rooted on the notion of canonical language. We actually follow a similar idea: the prime implicant set Γ focuses on the dependence relation and the $\circ_{PI}$ operation focuses on the splitting language. That is, prime implicants (and also prime implicates) ensure that we focus on the smallest language for describing ψ.

Peppas *et al.* proposed an other approach to relevant revision [20]. Their contribution is two fold. Firstly they proposed two conditions that are imposed on an AGM revision operator and guarantee the satisfaction of Parikh's postulate. Secondly, they proposes a set of semantic conditions based on the system of spheres model so that a belief revision operator that satisfies the semantic conditions is a relevant revision operator. As we have seen, our approach is purely syntax-based. Moreover, our framework embeds a significant part of the notion of relevance in the definition of the operator itself while existing approaches [15,20] consider the relevance with the help of additional constraints.

As future work, we want to pursue this characterization of relevance. That is, at this stage, relevance only describes what should not change. It does give a positive perspective by stating what should change. For instance, consider $PI_\psi = p_1 \wedge p_2 \wedge p_3$ and $PI_\mu = \neg p_1 \wedge p_2$. According to postulate **(P)**, we conclude that p_3 should remain unchanged. At the same time, only p_1 represents a disagreement point: at least p_1 should be changed. Prime implicants help us to to focus on the literals which represent disagreements, that is the set $\cup\{D_\psi \cap \overline{D_\mu} | D_\psi \in PI_\psi$ and $D_\mu \in PI_\mu\}$. This set represents in fact a lower bound for revision: "at least, what should change?" while postulate **(P)** describes the upper bound of relevant revision: "at most, what should change?". Our aim is to investigate the interactions between these bounds in order to characterize relevance from a positive perspective.

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