From Inconsistency to Consistency: Knowledge Base Revision by Tableaux Opening

Camilla Schwind

Laboratoire d'Informatique Fondamentale, CNRS
Université de Marseille
163 Avenue de Luminy - F-13288 Marseille Cedex09, case 901
camilla.schwind@lif.univ-mrs.fr

Abstract. We present a formal framework for theory revision that is based on a tableaux algorithm. We use a semantic tableaux system for generating the tableau of a belief base and of a revision formula. Specific revision operators can then be defined in terms of tableaux branches. Tableaux branches of a formula correspond to implicants of the formula. We define a new set of revision postulates for implicant sets and we show that they verify the AGM revision postulates. Then we consider specific distance based revision operators and in particular those based on the symmetrical difference between interpretations. We show that the minimal symmetric difference between the model sets of two implicants can be obtained by a simple binary operation on these implicants. Finally, we define three specific distance based revision operators. The representation of a formula by implicants is not unique (our approach is not based on prime implicants). We show that our implicant revision approach obtains the same revision result operated on different equivalent implicant sets for the three revision operators we propose.

1 Introduction

Knowledge base revision is an important subject in artificial intelligence and in data base systems. Considerations about the evolution of knowledge and belief are very important in many other domains such as philosophy and philosophy of science, history, law and politics. Theory revision as we will consider it in this article has been introduced in the framework of logic by Alchourron, Gärdenfors and Makinson in [2]. They considered a belief set as a deductively closed set of sentences (of a propositional logic). Revision is then an operator between a belief set and a formula that will be added to the belief set. The authors do not define specific revision operators but formulate postulates, called AGM postulates, that every reasonable revision operator should satisfy. Although the postulates have frequently been criticized they are widely used as a very usefull framework to study belief revision. The AGM postulates are very general, they specify sort of "minimal requirements" for revision operators. Katsuno and Mendelson have reformulated the postulates for belief sets which are represented by a finite set of formulas [14]. In this framework a revision is an operator between two formulas, one representing the belief base and the other representing the revision formula.

A. Kuri-Morales and G.R. Simari (Eds.): IBERAMIA 2010, LNAI 6433, pp. 120–132, 2010.
© Springer-Verlag Berlin Heidelberg 2010

In this paper, we propose an approach where revision is defined as an operator on implicant sets. Intuitively, our approach is due to the following observation. Revision is the process of integrating new information into a belief base. In our framework, a belief base is represented by a set of formulas Γ of some logic. If we add new information represented by a formula μ to Γ, we consider the set $\Gamma \cup \{\mu\}$. There are two situations, either $\neg\mu \notin \Gamma$ or $\neg\mu \in \Gamma$. In the first case the AGM postulates tell us that $\Gamma * \mu$ is $\Gamma \wedge \mu$, in the second case $\Gamma \cup \{\mu\}$ is inconsistent and we must find a new set Γ' that contains μ and most of the information from Γ, that is consistent with μ. If $\Gamma \cup \{\mu\}$ is inconsistent, a tableaux theorem prover will produce a closed tableau for $\Gamma \cup \{\mu\}$, that is a tree, where every branch contains a formula and its negation. The idea underlying our approach is to "open" tableaux branches in order to obtain a consistent knowledge set. Opening a closed branch consists in suppressing the literals "responsible" of the contradiction. Since we want to keep the revision information μ we do not suppress literals coming from μ. Since we want to conserve as much information from Γ as possible, we will suppress a minimum of literals in branches of Γ just enough to obtain a consistent belief base. This minimum may be defined in different ways, yielding different revision operators.

The tableau for a formula can be seen as a set of implicants of the formula, an implicant being a finite set of literals. We define binary *revision functions* on implicant sets and we formulate implicant revision postulates that specify properties every revision function should have. We show that every implicant revision function satisfying the implicant revision postulates defines an AGM revision operator. Our new implicant revision operators verify one additional property that does not follow from the AGM postulates.

Specific revision operators are frequently defined in terms of models and most time they take into account the set of atoms in which two models differ. This means that the symmetrical difference between models must be calculated and we show that it is easily obtained by a simple operation on implicants.

We illustrate our approach by defining three specific implicant revision operators all based on the symmetric difference between models. We minimize this difference in two ways, first by set inclusion of the differences and secondly by minimizing the number of elements where the two models differ. The third operator is based on literal weights where a weight is a natural number that indicates the importance of the literal. The idea is that the higher the weight the more important is the literal in the belief base and the less a user wants to give it up. This approach offers the possibility to express preferences on literal level. It seems to us that the preference of literals is more concrete for a user than preference of models. We use the weights for revision by minimizing the sum of the weights of the symmetric difference thus giving up globally the less important information units.

This paper is organized as follows. In the next section, we recall elements and results of theory revision. In the third part, we introduce our tableaux calculus based approach and we introduce the new implicant revision postulates for which we proof a representation theorem. In the fourth part we study specific distance based implicant revision operators. The fifth section describes some related work and section six concludes. Due to space limit, proofs could not be included in the paper.

It can happen that we use the terms knowledge set or belief base or belief set to speak of a belief base. We know that there might be subtle differences between these concepts. Nevertheless, we think that they all can be subject to implicant based revision.

2 Background

Preliminaries and Notation. We consider a propositional language over a set of propositional variables $\mathcal{P}$ (also called *atoms*). We note $\mathcal{M}$the set of all interpretations, $\mathcal{F}$ the set of all formulas, LIT the set of all literals, i.e. $LIT = \mathcal{P} \cup \{\neg a : a \in \mathcal{P}\}$. We call $\neg l$ also the opposite of l. We will identify a literal $l \in LIT$ with $\neg\neg l$, etc. as well as $\neg l$ with $\neg\neg\neg l, \ldots$, etc. For a formula $\phi \in \mathcal{F}$, $[\phi]$ is the set of models of ϕ (the set of interpretations that satisfy ϕ). We identify an interpretation with the set of atoms evaluated to true. Then $\mathcal{M} = 2^{\mathcal{P}}$[1]. Given a set of interpretations $M \subseteq \mathcal{M}$, we define $FOR(M) = \{\phi \in \mathcal{F} : M \subset [\phi]\}$ the set of formulas satisfied by all models in M. A set of interpretations M is called $\mathcal{F}$-representable if there is a formula $\psi \in \mathcal{F}$ such that $[\psi] = M$. $FOR(\mathcal{F})$ is the set of all $\mathcal{F}$-representable model sets. If $\mathcal{P}$ is finite then every set of models is $\mathcal{F}$-representable, i.e. $FOR(\mathcal{F}) = 2^{\mathcal{P}}$. In this paper we will be only concerned with a language over a finite set of atoms $\mathcal{P}$. The consequence relation is noted "$\models$", i.e. $\phi \models \psi$ iff $[\phi] \subseteq [\psi]$ and we denote $Cn(\phi) = \{\psi : \phi \models \psi\}$ the set of consequences of formula ϕ. A belief base is a deductively closed set of formulas. It is easy to see that $FOR(M)$ is deductively closed, i.e. $FOR(M) = Cn(FOR(M))$. Given a finite set of formulas Γ, we denote their conjunction by $\Gamma_\wedge$.

2.1 The AGM Postulates for Belief Revision

Alchourron, Gärdenfors and Makinson proposed the well-known AGM-postulates for theory revision [2]. Here revision is defined as an operator on a *belief base* Γ and a formula μ and the result of the revision is a belief base denoted $\Gamma * \mu$. AGM postulates express what properties a revision operator should have. Revision comes with another operation, expansion, that simply adds a new information to a knowledge base regardless of whether the result is inconsistent. Expansion is noted + and it holds that $\Gamma + \phi = Cn(\Gamma \cup \{\phi\})$[2]

Definition 1 (Revision operator). *Let Γ be a belief base and $\phi, \psi \in \mathcal{F}$. We call AGM revision operator every operator $*$ for which the following postulates hold*

---

[1] 2^M is the power set of set M.

[2] Expansion is characterized by another set of postulates and this equation is a representation theorem.

K∗1 *$\Gamma * \phi$ is a belief base.*
K∗2 *$\phi \in \Gamma * \phi$*
K∗3 *$\Gamma * \phi \subseteq \Gamma + \phi$*
K∗4 *If $\Gamma + \phi$ is satisfiable, then $\Gamma * \phi \supseteq \Gamma + \phi$*
K∗5 *If ϕ is satisfiable, then $\Gamma * \phi$ is satisfiable*
K∗6 *If $\phi \equiv \psi$ then $\Gamma * \phi \equiv \Gamma * \psi$* [3]
K∗7 *$\Gamma * (\phi \wedge \psi) \subseteq (\Gamma * \phi) + \psi$*
K∗8 *If $(\Gamma * \phi) + \psi$ is satisfiable then $(\Gamma * \phi) + \psi \subseteq \Gamma * (\phi \wedge \psi)$*

Postulate K∗2 tells us that revision formula ϕ belongs to the revised base $\Gamma * \phi$. Therefore, every model of $\Gamma * \phi$ is also a model of ϕ, i.e. the set of models of $\Gamma * \phi$ is a subset of the set of models of ϕ. This means that a revision operator $*$ applied to a belief base Γ selects models from the revision formula ϕ as result of the revision operation. Thinking of revision as a selection function on model sets, we can characterize AGM postulates in terms of model sets identifying a formula (or a formula set) with the set of models satisfying it. This approach, introduced by Karl Schlechta in [20] yields a rather simpler algebraic characterization of revision operators.

In the following, $\mathcal{U}$ can be considered as a set of model sets.

Definition 2 (Revision function). *Le $\mathcal{U}$ be a set of sets. $sm : \mathcal{U} \times \mathcal{U} \longrightarrow \mathcal{U}$ is a revision function on $\mathcal{U}$ iff for any $M, N, L \in \mathcal{U}$*

(S1) *$sm(M, N) \subseteq N$*
(S2) *$M \cap N \subseteq sm(M, N)$*
(S3) *if $M \cap N \neq \emptyset$ then $sm(M, N) \subseteq M \cap N$*
(S4) *if $N \neq \emptyset$ then $sm(M, N) \neq \emptyset$*
(S5) *$sm(M, N) \cap L \subseteq sm(M, N \cap L)$*
(S6) *if $sm(M, N) \cap L \neq \emptyset$ then $sm(M, N \cap L) \subseteq sm(M, N) \cap L)$*

The following correspondence theorems can only be formulated in this way when $\mathcal{P}$ is a finite set; If not, $FOR(M)$ is not always defined and -more important- the result of a revision is eventually not $\mathcal{F}$ -representable even when the knowledge base and the revision formula are. Given $M \subseteq \mathcal{M}$, we note $M^{\mathcal{F}} = FOR(M)_{\wedge}$ the conjunction of all formulas satisfied by all models in M.

The following correspondence results are obvious.

Theorem 1. *Let $*$ be an AGM-revision operator and M, N $\mathcal{F}$ -representable model sets. Then the function sm defined by $sm(M, N) = [FOR(M) * N^{\mathcal{F}}]$ is a revision function on $2^{\mathcal{M}}$.*

Proof. *S1, S2, S3, S4, S5 and S6 follow directly from K∗2, K∗3, K∗4, K∗5, K∗7 and K∗8 respectively.*

Theorem 2. *Let sm be a revision function on $2^{\mathcal{M}}$. Then the operator $*$ defined by*
$\Gamma * \mu = FOR(sm([\Gamma], [\mu]))$ is an AGM-revision operator.

[3] $\phi \equiv \psi$ iff $\models \phi \leftrightarrow \psi$

Proof. K∗2, K∗3, K∗4, K∗5, K∗7 and K∗8 follow directly from S1 through S6. K∗1 and K∗6 hold because sm is defined in terms of models and $[\psi] = [Cn(\psi)]$ *for K∗1 and* $[\psi] = [\phi]$ *iff* $\psi \equiv \phi$ *for K∗6.*

As we can consider revision as an operation that integrates new information into a knowledge base by keeping as much as possible from the knowledge base state before the revision came about, it is very natural to ask the following question: Can it be the case that the revised base $\Gamma{*}\mu$ is just $\Gamma' \cup \{\mu\}$ for some subset Γ' of Γ?

The following consequence (FG) of the AGM postulates (shown by Gärdenfors in [12]) suggests that there is such a subset, since $\Gamma \cap (\Gamma{*}\phi) \subseteq \Gamma$. (SM) is the algebraic formulation.

Fact 1. (FG) $\Gamma{*}\phi = (\Gamma \cap (\Gamma{*}\phi)) + \phi$
(SM) $sm\,(M,N) = (M \cup (sm(M,N)) \cap N$

Katsuno and Mendelson characterize revision in terms of an operator on formulas [14]. The following KM postulates specialize the AGM postulates to the case of propositional logic and rephrase them in terms of finite covers for "infinite knowledge sets". Here a belief base Γ can be represented by a formula ψ such that $\Gamma = Cn(\psi)$. Let be $\psi, \mu, \psi_1, \psi_2, \mu_1, \mu_2, \nu \in \mathcal{F}$.

R1 $\psi \circ \mu$ imply μ
R2 If $\psi \wedge \mu$ is satisfiable, then $\psi \circ \mu = \psi \wedge \mu$
R3 If ψ is satisfiable, then $\psi \circ \mu$ is satisfiable
R4 If $\psi_1 \equiv \psi_2$ and $\mu_1 \equiv \mu_2$ then $\psi_1 \circ \mu_1 \equiv \psi_2 \circ \mu_2$
R5 If $(\psi \circ \mu) \wedge \nu$ imply $\psi \circ (\mu \wedge \nu)$
R6 If $(\psi \circ \mu) \wedge \nu$ is satisfiable then $\psi \circ (\mu \wedge \nu)$ imply $(\psi \circ \mu) \wedge \nu$

Then the following theorem shown in [14] is straightforward:

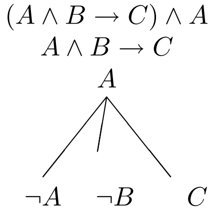
Theorem 3. *Let* Γ *be a belief base admitting a finite cover, i.e. there is* $\psi \in \mathcal{F}$ *such that* $\Gamma = Cn(\psi)$. *Then for any* $\phi, \mu \in \mathcal{F}$, $\mu \in \Gamma{*}\phi$ *iff* $\psi \circ \phi$ *implies* μ.

3 Semantic Tableaux and Implicant Revision

The method of semantic tableaux [21,11] has been introduced by Beth [4]. It is a syntactically oriented refutation system. A tableau rule T is of the form $\frac{\phi}{\psi_1, \ldots \psi_k}$ where $\phi, \psi_1, \ldots \psi_k \in \mathcal{F}$. In classical logic, $k \leq 2$. Tableaux rule T is applied to a formula set ϕ that belongs to a branch of a tree. If $k = 1$, ψ_1 is added to the branch. If $k = 2$, the result of the application are two new branches, one with ψ_1 added, the other with ψ_2 added. The following is a set of tableaux rules for classical propositional logic.

$$\frac{\neg\neg A}{A} \qquad \frac{A \wedge B}{A, B} \qquad \frac{\neg(A \wedge B)}{\neg A \mid \neg B}$$

A tableaux calculus can be considered as a function associating a formula tree to a set of formulas. As an example, consider the following tableau for the set of formulas $(A \wedge B \rightarrow C) \wedge A$



We consider the proof tree as a set (of branches) where each branch is a set of formulas. In the example above, the set of branches of the tree produced by the tableaux is $\{A \wedge B \rightarrow C, A, \neg A\}, \{A \wedge B \rightarrow C, A, \neg B\}, \{A \wedge B \rightarrow C, A, C\}$.

A tableau for formula ϕ can be associated with a disjunctive normal form (DNF) of ϕ, namely, each branch corresponds to the conjunction of the formulas on it (and this conjunction is equivalent to the conjunction of the literals on the branch). And formula ϕ is equivalent to the disjunction of the conjunction of its branches.

We conceive a tableaux system as a procedure TP that applies to a formula set and produces a set of sets of literals. TP is defined recursively as follows[4]:

$TP(\Gamma) = \{\Gamma\}$ if Γ is a set of literals.
$TP(\Gamma \cup \{\neg\neg\phi\}) = TP(\Gamma \cup \{\phi\})$.
$TP(\Gamma \cup \{\phi \wedge \psi\}) = TP(\Gamma \cup \{\phi\} \cup \{\psi\})$.
$TP(\Gamma \cup \{\phi \vee \psi\}) = TP(\Gamma \cup \{\phi\}) \cup TP(\Gamma \cup \{\psi\})$.

For the above formula, we obtain $TP(\{A \wedge B \rightarrow C) \wedge A\}) = TP(\{A \wedge B \rightarrow C), A\}) = TP(\{\neg A, A\}) \cup TP(\{\neg B, A\}) \cup TP(\{C, A\}) = \{\{\neg A, A\}, \{\neg B, A\}, \{C, A\}\}$

A branch is *closed* when it contains a literal l and its negation $\neg l$ and it is called *open* otherwise. A set of branches is closed when all of its elements are closed and it is open when at least one of its elements is open.

For the example above, we obtain the DNF $(A \wedge \neg B) \vee (A \wedge C)$ (taking into account that a closed branch is equivalent to $\bot$).

The fundamental property of tableaux for theorem proving is the following completeness theorem:

Theorem 1 *ϕ is a theorem, i.e. $\models \phi$ iff $TP(\{\neg\phi\})$ is closed. (Proof in [21]).*

Corollary 1. *ϕ is satisfiable, iff $TP(\{\phi\})$ is open.*

We consider TP not only as a theorem prover (or consistency checker) for propositional formulas but also as an operator that has useful properties for formulas and formula sets. We will use the following properties of TP.

[4] This formulation takes into account that for propositional logic, every rule is applied at most once to each formula.

Property 1. *Let $\phi, \psi \in \mathcal{F}$,*

1. $TP(\{\phi \vee \psi\}) = TP(\{\phi\}) \cup TP(\{\psi\})$
2. $TP(\{\phi \wedge \psi\}) = \{X \cup Y : X \in TP(\{\phi\})$ *and* $Y \in TP(\{\psi\})\}$
3. *for all* $t \in TP(\phi)$, $t_\wedge \rightarrow \phi$

We will use $TS(\{\phi\})$ for $TP(\{\phi\})$with closed branches suppressed, i. e. $TS(\{\phi\}) = TP(\{\phi\}) \setminus \{t : t \in TP(\{\phi\})$ and t closed$\}$.

We introduce the following operator $\otimes$ on sets of literal sets. Let S, T be sets of literal sets. Then $S \otimes T = \{s \cup t : s \in S$ and $t \in T\}$. Then a conjunction can be considered as an operation on sets of sets (of literals): $TP(\{\phi \wedge \psi\}) = TP(\{\phi\}) \otimes TP(\{\psi\})$.

The tableau of a formula ϕ converts ϕ to a disjunctive normal form (DNF), which is equivalent to a disjunction of conjunctions of literals. Each of these conjuncts is an *implicant* of the original formula, i. e. if $\phi \equiv \phi_1 \vee \ldots \phi_n$ then $\phi_i \rightarrow \phi$ for every i (property 1, 3). It should be noted that equivalent formulas might have different (equivalent) DNF.

Subsequently, we will call *implicant* a finite set of literals. Let us note $\mathcal{T}$ the set of all implicants. A tableaux prover then produces a finite set of implicants for a formula. We can think of an implicant as being a formula, namely the conjunction of its literals and of a set of implicants as of a disjunction, namely the disjunction of the conjunction of its elements. Let t be an implicant and T a set of implicants. Then, by abuse of notation, we write $[t]$ for the set of models of the implicant formula t and $[T]$ for the set of models of the disjunction of the elements of T. An implicant set T is closed iff $[T] = \emptyset$. The following holds for $\otimes$:

Fact 2. *Let $S, T \subseteq \mathcal{T}$. Then $[S \otimes T] = [S] \cap [T]$.*

In the finite case, every interpretation m can be represented by an implicant $m^I = m \cup \{\neg a : a \notin m\}$[5]. For a set of interpretations M, we set $M^I = \{m^I : m \in M\}$. Then we have $[m^I] = \{m\}$ and $[M^I] = M$.

As we have seen, belief revision is characterized by postulates that specify properties a revision operator should have. Here we will give a characterisation of revision operators in terms of implicants. The postulates we define satisfy the AGM postulates but they are stronger. They satisfy additional properties that do not follow from the AGM postulates, namely the following:

– if $\mu \in ((\Gamma * (\phi \vee \psi)) * \chi$, then $\mu \in (\Gamma * \phi) * \chi$ or $\mu \in (\Gamma * \psi) * \chi$;
– if $\mu \in (\Gamma * \phi) * \chi$ and $\mu \in (\Gamma * \psi) * \chi$ then $\mu \in (\Gamma * (\phi \vee \psi) * \chi$.

These properties are also true for distance based revision and do not follow from the AGM postulates [16].

An implicant revision operator is defined on pairs of implicant sets and its result is a new set of implicants.

Definition 3 (Implicant Revision Function). *Let be $S, T, R \in 2^{\mathcal{T}}$, where $\mathcal{T}$ is the set of all implicants. An implicant revision operator over $2^{\mathcal{T}}$ is a binary function st : $2^{\mathcal{T}} \times 2^{\mathcal{T}} \longrightarrow 2^{\mathcal{T}}$ for which the following postulates hold:*

[5] Remember that we identify an interpretation with the set of atoms evaluated to true.

(ST1) *For all* $v \in st(S,T)$ *there is* $t_1 \in S$, $t_2 \in T$ *and* $s \subseteq t_1$ *such that* $v = t_2 \cup (t_1 \setminus s)$.
(ST2) *If* $S \otimes T$ *is not closed then* $st(S,T) = S \otimes T$.
(ST3) *If* T *is not closed then* $st(S,T)$ *is not closed.*
(ST4) *If* $st(S,T) \otimes R$ *is not closed then* $st(S,T) \otimes R = st(S, T \otimes R)$.
(ST5) *If* $[S_1] = [S_2]$ *and* $[T_1] = [T_2]$ *then* $[st(S_1,T_1)] = [st(S_2,T_2)]$.

We will show that every implicant revision function defines a revision function.

Theorem 4. *Let* st *be an implicant revision function over* $2^{\mathcal{T}}$. *Then the function* $sm : 2^{\mathcal{M}} \times 2^{\mathcal{M}} \longrightarrow 2^{\mathcal{M}}$ *defined by*

$$sm(M,N) = [st(M^I, N^I)] \tag{1}$$

is a revision function.

Postulate (ST1) tells us the following. When we revise a belief base seen as a set of implicants S by a new information, given as a set of implicants T then we choose a subset among the implicants of T and eventually augment each implicant choosen by subsets of implicants in S "compatible" (not contradictory) with that implicant. In other words, we try to "keep" as many literals occurring in implicants of the original belief base as possible.

4 Specific Revision Functions Based on Distances

Implicant revision postulates suggest to consider specific revision operations that are defined as operations on implicants. They indicate how to calculate effectively results of revision operations. Given a belief base with finite cover ϕ and a revision information μ we first produce the DNF of ϕ and μ and we obtain implicant sets S and T. Then we apply operations on the implicants of ϕ and μ in order to obtain a revised base as a new implicant set. According to postulate (ST1), every of the resulting implicants contains an implicant of μ augmented by a subset of some implicant of ϕ. Here we will concentrate on a specific class of implicant revision functions based on distance functions. Various distance based approaches to belief revision (and to update) have been defined by means of the symmetric difference between valuations, e.g. [6,23,7,19,22].

The tableaux prover will be used to calculate specific revision functions, namely those based on symmetric differences between models. Our algorithm proceeds as follows: Given a belief base ϕ and a revision formula ψ, if $\phi \wedge \psi$ is satisfiable then $\phi{*}\psi = \phi \wedge \psi$. In other words, if $TS(\{\phi\}) \otimes TS(\{\psi\})$ is open then $\phi{*}\psi = \phi \wedge \psi$. If not, we have $TS(\{\phi\}) \otimes TS(\{\psi\})$ is closed. Let be $v \in TS(\{\phi\}) \otimes TS(\{\psi\})$. Then $\exists s \in TS(\{\phi\}), t \in TS(\{\psi\})$, and $v = s \cup t$, by property 1. v is closed, s and t are open and we define the *opening* of v in s by $(s \setminus \bar{t}\,) \cup t$. This means that we have suppressed in s the literals in conflict with literals in t, i. e. the literals $s \cap \bar{t}$. Then we will define the opening of the tableaux $TS(\{\phi\}) \otimes TS(\{\psi\})$ by the set of branches "minimally opened", i.e. we keep only branches within which a minimal set (or number) of literals have been suppressed. In order to determine these minimal branches we need the following definitions.

Notation. Given a literal l, we note $|l|$ the atom of l, i.e. $|a| = |\neg a| = a$ for $a \in \mathcal{P}$. For an implicant t, we note $|t| = \{|l| : l \in t\}$. If M is a set and $\leq \subseteq M \times M$ is a pre-order on M, then $Min_{<}(M) = \{m \mid m \in M$ and $\neg \exists n \in M$ and $n \leq m\}$.

We first define the symmetric difference between two implicants s and t by $|s \cap \bar{t}\ |$. It turns out that $|s \cap \bar{t}\ |$ is exactly the minimal symmetric difference between the models of s and t (lemma 1). It is easy to see that $|s \cap \bar{t}\ |$ is $\subseteq$-minimal within the symmetric differences between the models of s and t.

Lemma 1. *Let be $s, t \in \mathcal{T}$. Then*
(i)$|s \cap \bar{t}\ | = |t \cap \bar{s}\ |$ *and*
(ii)$\{|s \cap \bar{t}\ |\} = Min_{\subseteq}(\{m \div n : m \in [s]$ *and* $n \in [t]\})$

Example 1. Let be $s = (a, \neg b, c)$ and $t = (\neg a, b, d)$; $[s] = \{\{a, c\}, \{a, c, d\}\}$ and $[t] = \{\{b, d\}, \{b, c, d\}\}$. Then $s \cap \bar{t}\ = \{a, \neg b\}$, $t \cap \bar{s}\ = \{\neg a, b\}$, $|s \cap \bar{t}\ | = |t \cap \bar{s}\ | = \{a, b\}$ and $Min_{\subseteq}(\{m \div n : m \in [s]$ and $n \in [t]\}) = \{a, b\}$

Any formula ϕ is the disjunction of a finite set of implicants. This defines the set of its models as a finite union of model sets (not necessarily disjoint), namely the models of its implicants. In order to obtain the $\subseteq$-minimal symmetric differences of all the models of ϕ it is sufficient to obtain the minimal sets among the $|s \cap \bar{t}\ |$ for all $s \in S$ and $t \in T$.

We will need the following lemma to show that this minimization obtains the global minimal symmetric differences for all models of the formula.

Lemma 2. *Let M be a set that is a finite union of sets (not necessarily disjoint), $M = A_1 \cup \cdots \cup A_k$ and let $\leq$ be a pre-order on M for which the limit assumption holds (i.e. the infimum of $\mathcal{M}$is a minimum). Then*

$$Min_{\leq}(M) = Min_{\leq}(\bigcup_{i=1}^{k} Min_{\leq} A_i)$$

The following theorem shows that the set of minimal elements of the symmetric differences between two model sets can be obtained by minimizing all sets $|s \cap \bar{t}\ |$.

Theorem 5. *Let $S, T \subseteq \mathcal{T}$ and $MinI = Min_{\subseteq}(\{|s \cap \bar{t}\ | : s \in S$ and $t \in T\})$. Then $MinI = Min_{\subseteq}(m \div n : m \in [S]$ and $n \in [T])$.*
Proof due to lemma 1 and lemma 2.

Theorem 5 shows that we obtain all minimal symmetric difference sets between the models of a formula ϕ and a formula ψ as the $\subseteq$ $-$smallest sets $|s \cap \bar{t}\ |$ for implicants s of ϕ and t of ψ.

Corollary 2. *Let be $S_1, S_2, T_1, T_2 \subseteq \mathcal{T}$ and $[S_1] = [S_2]$ and $[T_1] = [T_2]$. Then $Min_{\subseteq}(\{|s \cap \bar{t}\ | : s \in S_1$ and $t \in T_1\}) = Min_{\subseteq}(\{|s \cap \bar{t}\ | : s \in S_2$ and $t \in T_2\})$*

We define the following revision function based on the symmetric difference of models.
$st\,1(S, T) = \{t \cup (s \setminus \bar{t}\ ) : t \in T, s \in S$ and $|s \cap \bar{t}\ | \in MinI\}$.

This means that $st\,1(S, T)$ is defined as the set of minimal openings of $S \otimes T$ in S.

Example 2. Let be $S = \{\{a,b,c\},\{d,e,f\}\}$ and $T = \{\{\neg a,\neg b,c,d,\neg e\}\}$. Then $|\{a,b,c\} \cap \{a,b,\neg c,\neg d,e\}| = \{a,b\}$ and $|\{d,e,f\} \cap \{a,b,\neg c,\neg d,e\}| = \{e\}$. Then $st\,1(S,T) = \{\{\neg a,\neg b,c,d,\neg e,f\},\{\neg a,\neg b,c,d,\neg e\}\}$

The following two revision functions are also based on model distance, but use the order of the cardinalities of the symmetric model differences. The first one, $st\,2$ is the well well-known distance based revision function introduced by Dalal in [7]. It uses the Hamming distance between models.

Let be $S,T \in \mathcal{T}$ and $MinH = Min_{\leq}(\{card(s \cap \bar{t}\ ) : s \in S, T \in T\})$. $st\,2(S,T) = \{t \cup (s \setminus \bar{t}\ ) : t \in T, s \in S$ and $card(s \cap \bar{t}\ ) = MinH\}$

st2 is precisely the revision operation introduced by Dalal.

The revision function we propose takes into account that for a user the different entities may have different importance. Using this function presupposes that an agent associates weights to literals that measure the "importance" of the literal in the belief base. The weight of a set of literals is simply the sum of its elements. The revision operator that uses these weights is then defined in the following way. Let be w a function from LIT to $\mathcal{N}$ (set of natural numbers). Then we extend w to a function on implicants by $w(t) = \sum_{l \in t} x(l)$. Let be $MinW = Min_{\leq}(\{w(s \cap \bar{t}\ ) : s \in S, t \in T\})$ The corresponding revision function is then:
$st\,3(S,T) = \{t \cup (s \setminus \bar{t}\ ) : t \in T, s \in S$ and $w(s \cap \bar{t}\ ) = MinW\}$

Theorem 6. *$st1, st2$, and $st3$ are implicant revision functions.*

4.1 Complexity

Eiter and Gottlob have shown that the problem of deciding whether a formula belongs to a knowledge base Γ after revision with μ, $\Gamma * \mu$, resides on the second level of the polynomial hierarchy [10]. Here we give the number of steps for calculating the set $st\,(S,T)$, where S and T are the set of implicants of formulas ϕ and μ. Let be Γ a formula representing a belief base and μ the new infomation. Let be n the lenght of the formula $\Gamma \wedge \mu$. Then, the number of conjuncts of a DNF of $\Gamma \wedge \mu$ is $O(2^n)$. To determine the $\subseteq$-smallest element within the subsets $s \cap \bar{t}$ we have to compare all branches pairwise, that gives polynomial time algorithm (in an exponential number of branches).

5 Related Work

Specific implicant revision operators defined for knowledge bases in specific syntactical forms specific have been proposed by several authors [17,18,5]. Perrussel et al. define a specific prime implicant revision method. Their system presupposes a knowledge base in clausal form and they propose to weighten a literal by counting the number of its occurrences within all clauses: the number of occurrences indicates the importance of the literal. The weight of an implicant is then the sum of the weights of the literals it contains. Their algorithm is based on prime implicants; from a belief base first the set of all its prime implicants must be calculated. But the base resulting from a revision is in general not more in form of prime implicants and those have to be recalculated after every revision step, in the case of multiple revisions. They have studied the performances of their system with many benchwork examples. Bienvenu et al. [5] propose a prime implicate-based revision operator that is a full meet operator. Their algorithm is based on the set of all prime implicates of a formula K, $\Pi(K)$. The revision operator is defined by $K *_{\Pi} \phi = \phi \wedge \bigvee(\Pi(K) \bot \neg\phi)$ where $K \bot \phi$ is the set of maximal subsets of K consistent with $\neg\phi$. Again, one problem is that $\Pi(K)$ has to be recalculated after every revision step since the revised belief base is not more the conjunction of all its prime implicates. Our approach is more general (and simpler), since we do not need a belief base in normal form and we do not need to calculate the prime implicants (or prime implicates) of the belief base and the revision formula. The tableaux prover we use produces just DNF of these formulas. While the number of branches is exponential in the length of the formula, it is always less important than the set of all prime implicants. And the algorithm to obtain prime implicants is more expensive than a tableaux algorithm. But the important difference is that in all these approaches the set of all prime implicants/implicates has to be recalculated after every revision step. In our approach the DNF is calculated once for the two formulas and the revision result is still in DNF. Therefore subsequent revision operations can directly operate on the revised base and need only to produce the DNF of the revision formula. Our approach applies to an important class of revision operators, namely those based on symmetric differences of models. But there are certainly revision operators (and also distance based revision operators) that cannot be treated by our approach without a transformation of the belief base and the revision formula to prime implicants.

6 Conclusion and Further Work

In this paper we proposed to calculate belief base revision operators by means of analytic tableaux. For that purpose we have redefined the AGM postulates in terms of implicants and we have proposed a new postulate that is more specific than the AGM postulates. We have applied our approach to two specific families of revision operators. The first defines the distance between interpretations by means of their symmetric difference. The second is based on literal weights.

Here we have only treated one type of theory change operators, namely revision. Other operators are update [13], erasure or contraction [12] but also merging [15]. Our method can also be applied to these types of operators.

Another issue is iterated revision. Darwiche and Pearl in [8] have formulated postulates which, together with the AGM postulates for revision specify properties that a knowledge base must have after subsequent revision steps. It is possible to formulate postulates for iterated revision in terms of implicants and to calculate specific iterated revision operators by means of tableaux.

Another work will be to find tableaux rules that give directly a decision procedure for the problem wether a revised knowledge base $\Gamma * \phi$ contains some formula ψ. Delgrande, Jin and Pelletier in [9] have investigated rules for calculating updates that break up the update formula. We think of tableaux rules that break up the belief base formula according to the new information. Our new implicant revision postulate ST1 suggests that the revised belief base is composed of implicants that contain implicants of the revision formula and parts of implicants of the belief base. Thus, a decision procedure for the problem whether a formula is entailed by a revised belief base must modify the original belief base according to the contradictions with elements of the revision formulas.

References

1. Proceedings of the 7th National Conference on Artificial Intelligence (1988)
2. Alchourron, C.E., Gärdenfors, V., Makinson, D.: On the logic of theory change: Partial meet contradictions and revision functions. Journal of Symbolic Logic 50(2), 510–530 (1985)
3. Allen, J.A., Fikes, R., Sandewall, E. (eds.): Proceedings of the 2nd International Conference on Principles of Knowledge Representation and Reasoning (1991)
4. Beth, E.W.: The Foundations of Mathematics. North Holland, Amsterdam (1959)
5. Bienvenu, M., Herzig, A., Qi, G.: Prime implicate-based belief revision operators. In: Ghallab, M., Spyropoulos, C.D., Fakotakis, N., Avouris, N.M. (eds.) ECAI. Frontiers in Artificial Intelligence and Applications, vol. 178, pp. 741–742. IOS Press, Amsterdam (2008)
6. Bordiga, A.: Language features for flexible handling of exceptions in information systems. ACM Transactions on Database Systems 10, 563–603 (1985)
7. Dalal, M.: Investigations into a theory of knowledge base revision. In: AAAI 1988 [1], pp. 475–479 (1988)
8. Darwiche, A., Pearl, J.: On the logic of iterated belief revision. Artificial Intelligence 89(1-2), 1–29 (1997)
9. Delgrande, J.P., Jin, Y., Pelletier, F.J.: Compositional belief update. J. Artif. Intell. Res. (JAIR) 32, 757–791 (2008)
10. Eiter, T., Gottlob, G.: On the complexity of propositional knowledge base revision, updates and counterfacts. Artificial Intelligence 57, 227–270 (1992)
11. Fitting, M.: First Order Logic and Automated Theorem Proving. Texts and Monographs in Computer Science. Springer, Heidelberg (1990)
12. Gärdenfors: Knowledge in Flux: Modeling the Dynamics of Epistemic States. Bradford Books. MIT Press, Cambridge (1988)
13. Katsuno, H., Mendelzon, A.O.: On the difference between updating a knowledge base and revising it. In: Allen et al. [3], pp. 387–394
14. Katsuno, H., Mendelzon, A.O.: Propositional knowledge base revision and minimal change. Artificial Intelligence 52, 263–294 (1991)
15. Konieczny, S., Pérez, R.P.: On the logic of merging. In: KR 1998, pp. 488–498 (1998)
16. Lehmann, D.J., Magidor, M., Schlechta, K.: Distance semantics for belief revision. J. Symb. Log. 66(1), 295–317 (2001)
17. Perrussel, L., Marchi, J., Bittencourt, G.: Une nouvelle mesure de changement minimal. In: RFIA (ed.) RFIA 2006 Proceedings Électroniques (2006)

18. Perrussel, L., Marchi, J., Bittencourt, G.: Prime implicants and belief update. In: Chad Lane, H., Guesgen, H.W. (eds.) FLAIRS Conference. AAAI Press, Menlo Park (2009)
19. Satoh, K.: Nonmonotonic reasoning by minimal belief revision. In: Proceedings of the International Conference on Fifth Generation Computer Systems, Tokyo, pp. 455–462 (1988)
20. Schlechta, K.: Coherent Systems. Studies in Logic and Practical Reasoning, vol. (2). Elsevier, Amsterdam (2004)
21. Smullyan, R.M.: First Order Logic. Springer, Heidelberg (1968)
22. Williams, M.-A.: Transmutations of knowledge systems. In: Proceedings of the 5th International Conference on Principles of Knowledge Representation and Reasoning, pp. 619–629 (1994)
23. Winslett, M.: Reasoning about action with a possible models approach. In: AAAI 1988 [1], pp. 89–93 (1988)