

# Individual Opinions-Based Judgment Aggregation Procedures

Farah Benamara<sup>1</sup>, Souhila Kaci<sup>2</sup>, and Gabriella Pigozzi<sup>3</sup>

<sup>1</sup> IRIT-CNRS, Toulouse, France  
benamara@irit.fr

<sup>2</sup> Université Lille-Nord de France, Artois, France  
kaci@cril.univ-artois.fr

<sup>3</sup> University of Luxembourg, Luxembourg  
gabriella.pigozzi@uni.lu

**Abstract.** Judgment aggregation is a recent formal discipline that studies how to aggregate individual judgments on logically connected propositions to form collective decisions on the same propositions. Despite the apparent simplicity of the problem, the aggregation of individual judgments can result in an inconsistent outcome. This seriously troubles this research field. Expert panels, legal courts, boards, and councils are only some examples of group decision situations that confront themselves with such aggregation problems. So far, the existing framework and procedures considered in the literature are idealized. Our goal is to enrich standard judgment aggregation by allowing the individuals to agree or disagree on the decision rule. Moreover, the group members have the possibility to abstain or express neutral judgments. This provides a more realistic framework and, at the same time, consents the definition of an aggregation procedure that escapes the inconsistent group outcome.

## 1 Introduction

Judgment aggregation is a recent formal discipline that studies how to aggregate individual judgments to form collective decisions. Examples are expert panels, legal courts, boards, and councils [7]. This field has recently attracted attention in multi-agent systems and artificial intelligence, in particular due to the relations with belief merging [13], for example for the combination of opinions of equally reliable individuals.

Judgment aggregation problems consider a group of people stating their views (in the binary form of 1 or 0) on some logically interconnected propositions. As an example, consider the board of a research funding agency whose members have to decide whether to support a research project (*conclusion*  $D$ ) on the basis of three criteria : originality ( $P$ ), quality ( $Q$ ), and applicability ( $R$ ), that is the decision rule can be expressed as  $(P \wedge Q \wedge R) \leftrightarrow D$ . As we will see, problems arise because seemingly reasonable aggregation procedures lead to paradoxical outcomes.

Clearly, the problems investigated in this new field are relevant and common to many situations. Nevertheless, the procedures considered so far in the literature are idealized. To provide a more realistic framework and to provide an

aggregation procedure that does not run into inconsistent social outcomes are the goals of the paper. More specifically, we propose to extend standard judgment aggregation to take into account two main considerations.

First, we introduce the notion of *judgment status*. It is not realistic to expect that group members state their judgments on each proposition, or that they always have a clear position on each proposition. Our model allows members to express classical binary judgment, a neutral judgment or to abstain on some or all propositions that the individuals consider as irrelevant. In our example of the project funding, suppose that the applicability criterion ( $R$ ) has been introduced only recently following some new regulation that impose all research funding agency to be evaluated on the basis of likeness to attract the interest of private funding. Suppose also that some members dissent with the criterion  $R$  because they believe that this will damage pure theoretical projects to the benefit of pure applied ones. Some members can believe that  $R$  is completely irrelevant and thus abstain to give a judgment on  $R$ . On the other hand, some other members can believe that the criterion  $R$  is relevant but they prefer to be neutral because they are not able to assess its applicability, or because they are indifferent to its value. It is worth noticing that abstention and neutral judgments denote distinct positions. The difference will be clarified later in the paper.

Second, we introduce the notion of *acceptance of the decision rule*. Our framework allows the group members to state whether or not they agree on the rule governing the decision. In our example, some members may disagree on the way the propositions are logically connected whereas some other members can accept the decision rule even if they believe that some propositions are irrelevant.

We present a flexible judgment aggregation approach, in which individuals can express 0/1 judgments as well as being neutral or abstain on some propositions, and can participate in the group decision procedure while disagreeing on the imposed decision rule. What kind of aggregation procedure does such a new framework advocate for? We suggest that the group decision procedure should be responsive of the group's opinion about the decision rule. If the majority (or a pre-fixed quota) of the individuals accepts the decision rule, the aggregation procedure will be based on the criteria of the rule. If the group rejects the rule, the only issue the group can confidently express its opinion about is the conclusion  $D$ .

The remainder of this paper is organized as follows. After necessary background on the problem of judgment aggregation, we present our general framework. We then introduce the formal representation, the aggregation procedure and show that our framework offers a solution to the judgment aggregation dilemma. Lastly, we compare our approach to some related work and then we conclude.

## 2 Judgment Aggregation

In the original problem of judgment aggregation [5,6], a court has to make a decision on whether a person is liable of breaching a contract (proposition  $D$ ). The judges have to reach a verdict following the legal doctrine. This states that

a person is liable if and only if there was a contract ( $P$ ) and there was a conduct constituting breach of such a contract ( $Q$ ). The legal doctrine can be formally expressed by the rule  $(P \wedge Q) \leftrightarrow D$ . Each member of the court expresses her judgment on the propositions  $P$ ,  $Q$  and  $D$  such that the rule  $(P \wedge Q) \leftrightarrow D$  is satisfied.

Suppose now that the three members of the court make their judgments according to Table 1.

**Table 1.** Doctrinal paradox. Premises:  $P$  = There was a contract,  $Q$  = There was conduct constituting breach of such a contract. Conclusion:  $D = (P \wedge Q)$  = There was a breach of contract.

	$P$	$Q$	$D = (P \wedge Q)$
Judge A	1	0	0
Judge B	0	1	0
Judge C	1	1	1
Majority	1	1	0

Each judge expresses a consistent judgment, i.e., she says yes to  $D$  if and only if she says yes to both  $P$  and  $Q$ . However, *proposition-wise* majority voting (consisting in the separate aggregation of the votes for each proposition  $P$ ,  $Q$  and  $D$  via majority rule) results in a majority for  $P$  and  $Q$  and yet a majority for  $\neg D$ . This is an inconsistent collective result, in the sense that  $\{P, Q, \neg D, (P \wedge Q) \leftrightarrow D\}$  is inconsistent in propositional logic. The paradox lies in the fact that majority voting can lead a group of rational agents to endorse an irrational collective judgment, i.e., to have a majority believing that the defendant should be left free while *another* majority deems there are reasons to sentence her. The literature on judgment aggregation refers to such problem as the *doctrinal paradox*. Clearly, the relevance of such aggregation problems goes beyond the specific court example and affects all collective decisions on logically interconnected propositions.

The first two ways to avoid the inconsistency that have been suggested are the *premise-based procedure* (PBP) and the *conclusion-based procedure* (CBP) [12]. According to the PBP, each member casts her judgment on each premise. The conclusion is then inferred from the judgment of the majority of the group on the premises using the rule  $(P \wedge Q) \leftrightarrow D$ . In the example above, the PBP would declare the defendant liable of breaching the contract.

According to the CBP, the members decide privately on  $P$  and  $Q$  and only express their opinions on  $D$  publicly. The judgment of the group is then inferred from applying the majority rule to the agent judgments on the conclusion. The defendant will be declared liable if and only if a majority of the judges actually believes that she is liable, and no reasons for the court decision could be supplied. In the example, contrary to the PBP, the application of the CBP would free the defendant.

### 3 General Framework

Let us first introduce some terminology from standard judgment aggregation. A set of agents  $N = \{1, 2, \dots, k\}$ , with  $k \geq 3$ , has to make judgments on logically interconnected propositions of a language  $\mathcal{L}$ . The set of propositions on which the judgments have to be made is called *agenda* (denoted by  $\mathcal{A}$ ), and this is divided between premises and conclusion. A (individual or collective) *judgment set* is the set of propositions believed by the agents or the group. An  $k$ -tuple  $(J_1, J_2, \dots, J_k)$  of agents judgment sets is called *profile*. A *judgment aggregation rule*  $F$  assigns a collective judgment set  $J$  to each profile  $(J_1, J_2, \dots, J_k)$  of agents judgment sets. A judgment set  $J$  is *consistent* if it is a consistent set in  $\mathcal{L}$ , and is *complete* if, for any  $P \in \mathcal{L}$ ,  $P \in J$  or  $\neg P \in J$  but not both.

In our framework a judgment aggregation has the form:

$$C \leftrightarrow D, \tag{1}$$

where  $C$  is a general propositional formula built on literals representing criteria  $P_i$  and  $D$  is a literal representing the final decision (or conclusion). In the following (1) is referred to as the *decision rule*. We assume that  $C$  is neither a tautology nor a contradiction, this ensures that the criteria are logically independent (as in the standard framework). Moreover, if one of the criteria is the negation of a propositional variable (e.g.  $\neg Q$ ), the members express their judgments on criteria as given, i.e., on  $P$ ,  $\neg Q$ , etc. Considering decision rules where  $C$  is a conjunction is not a limitation of our approach. Notably, our analysis extends to other truth-functional combinations of literals as well, e.g. the disjunctive decision rule  $(P \vee Q \vee R) \leftrightarrow D$ , because this rule is equivalent to  $(\neg P \wedge \neg Q \wedge \neg R) \leftrightarrow \neg D$ .

Let us now formalize the extensions we intend to give to standard judgments aggregation, namely: *judgment status* and *acceptance of the decision rule*.

#### 3.1 Judgment Status

We distinguish four possible judgments: classical binary judgment 1 (for) or 0 (against), neutral judgment and abstention. As classical binary judgment is already at work, we only detail abstention and neutral judgments.

**Neutral judgments.** Neutrality captures those situations in which members do not have a clear position on a specific issue, do not feel competent, or simply prefer not to take position on that matter. We represent a neutral judgment by a question mark “?”. For example, given  $(P \wedge Q) \leftrightarrow D$ , if a member believes  $P$  to be true but does not know about  $Q$ , then her judgment set will be  $\{(1, ?, 0)\}$ . A group member may express a neutral judgment w.r.t. some or all criteria, and on the conclusion as well.

**Abstention.** We represent an abstention by a cross “X”. The difference between abstention and a neutral judgment is that a member abstains on a criterion

$P_i$  when she deems that criterion *irrelevant* for the decision  $D$ . Consequently, abstention on criteria are ignored in the decision process. However, a member cannot abstain on the decision  $D$ . When a member takes part into a collective decision process, she is expected to express her judgment on  $D$ .

### 3.2 Acceptance/Rejection of the Decision Rule

Each member  $j$  says whether she accepts ( $Accept_j = 1$ ) or rejects ( $Accept_j = 0$ ) the decision rule.

**Acceptance of the decision rule.** This means that either, for a member  $j$ , the criteria  $P_1, \dots, P_n$  are the all and only relevant ones to make a judgment on  $D$ , or that (1) contains some irrelevant criteria together with all relevant ones. In the first case,  $j$  will give 1/0 or neutral judgments on each criterion. In the second case, she will abstain on the criteria that she deems to be irrelevant and will give a judgment only on the relevant criteria. Of course,  $D$  is computed only on the basis of the criteria on which she expressed a judgment, i.e., she did not abstain. It is worth observing that the original legal paradox of judgment aggregation is an instance of this case, where all group members (the judges) have to endorse the legal code, or behave as if this is the case.

**Rejection of the decision rule.** There are two cases under which a member can reject the decision rule. The first is when she believes that criteria  $P_i$  are not adequate, i.e., some criteria are missing. In this case,  $j$  fixes the value of  $D$  according also to the missing criteria. The intuition is that, if a member wants to have her saying in a decision process, but considers the adopted rule unable to capture the relevant criteria for the decision, she must be able to express her judgment on the conclusion while making explicit that she deems the rule to be not appropriate.

The second situation is when  $j$  agrees on the criteria  $P_i$  in the decision rule, but disagrees on the way these criteria are logically connected. For e.g. the rule is  $(P \wedge Q) \leftrightarrow D$  and according to  $j$  the decision rule should instead be  $(P \vee Q) \leftrightarrow D$ . Member  $j$  will therefore assign 0/1 or neutral judgments on the criteria while deciding on the value of  $D$  according to  $(P \vee Q) \leftrightarrow D$ .

It is important to notice that members may express judgments on the premises even when they reject the rule. The reason is that in case they are the only one to reject the rule (or in any case there is not majority rejecting the rule), the decision procedure will be PBP (see Section 4) in which case they can have their saying on the premises.

*Example 1.* Consider our running example of the board of a research funding agency whose members have to decide which research project to support on the basis of three criteria: originality ( $P$ ), quality ( $Q$ ), and applicability ( $R$ ). Suppose that the decision rule is  $(P \wedge Q \wedge R) \leftrightarrow D$ . The five members state their judgments on  $P$ ,  $Q$  and  $R$  as in Table 2.

**Table 2.** Acceptance of the general rule and individual judgments

	Acceptance	P	Q	R	D
$M_1$	1	0	0	1	0
$M_2$	1	1	X	1	1
$M_3$	1	?	0	0	0
$M_4$	0	1	X	1	?
$M_5$	0	0	1	1	1

The first three members agree with the decision rule  $(P \wedge Q \wedge R) \leftrightarrow D$  since for them  $Accept_j = 1$  (for  $j = 1, 2, 3$ ).  $M_1$  thinks that the criteria  $P$ ,  $Q$ , and  $R$  are the all and only relevant attributes for funding a project whereas  $M_2$  thinks that the criterion  $Q$  is irrelevant and decides then to abstain to give any judgement on  $Q$ . The decision of  $M_2$  is derived on the basis of  $P$  and  $R$  only. The third member also agrees on the decision rule, but unlike the first two, she is neutral on  $P$ . Since for  $M_3$ ,  $Q$  and  $R$  false and she accepts the rule,  $D$  is also false. Finally,  $M_4$  and  $M_5$  reject the decision rule.  $M_4$  thinks that  $Q$  is irrelevant and that there are missing criteria. So she abstains to give any judgement on  $Q$  and gives neutral judgement on  $D$  according to  $P$ ,  $R$  and the missing criteria.  $M_5$  does not accept the rule for other reasons: she thinks that the criteria are relevant but not correctly linked. Indeed she expresses her opinions on all the propositions but she gives her judgement to  $D$  following the rule  $(P \vee Q \vee R) \leftrightarrow D$ .

## 4 Representation and Aggregation Procedure

We represent a judgment expressed by a member  $j$  by the following tuple

$$J_j = (Accept_j, P_{1j}, \dots, P_{nj}, D_j),$$

where  $P_{ij} \in \{0, 1, ?, X\}$  and  $D_j \in \{0, 1, ?\}$ .

$D_j$  is either derived following the decision rule or fixed by the group member depending on whether she accepts the general rule or not.

Given a set of judgments  $\{J_1, \dots, J_k\}$ , the collective decision is represented as follows:

$$D = (Accept_{agg}, P_{agg_1}, \dots, P_{agg_n}, D_{agg}),$$

such that:

- $Accept_{agg}$  is the majority (or any other quota rule) of  $Accept_1, \dots, Accept_k$ . If there are as many members accepting the rule as members rejecting it, we fix  $Accept_{agg} = 0$ . In social choice theory, tie-breaking rules are usually random. Since in our approach, all members assign a value to the conclusion, it is preferable to break the tie in favour of  $Accept_{agg} = 0$  (so turning to CBP) than by a random choice (this will be detailed later in this section).
- $P_{agg_i}$  is the majority of  $P_{i1}, \dots, P_{ik}$  following proposition-wise majority voting. Abstentions on  $P_i$  are ignored when computing  $P_{agg_i}$  since those criteria

are irrelevant. Neutral judgments simply follow the majority. In case of indecision, i.e., a tie between the number of  $P_{ij} = 1$  and  $P_{ij} = 0$ , we put  $P_{agg_i} = ?$ .

- $D_{agg}$  is computed by PBP or CBP. The procedure is fixed according to  $Accept_{agg}$  as follows :
  - if  $Accept_{agg} = 0$  then we use CBP and  $D_{agg}$  is computed on the basis of  $D_1, \dots, D_k$ . This is intuitively meaningful since  $Accept_{agg} = 0$  means that the group members thought that the decision rule was not the right one for that decision, so the only reasonable thing they can say is the final conclusion.  $D_{agg}$  is calculated by simple majority voting. Neutral judgments simply follow the majority. In case of indecision, we have the following subcases:
    - \* if indecision is not allowed, then we do not accept neutrality on  $D$ . We then propose to have either a ‘pessimistic’ ( $D_{agg} = 0$ ) or an ‘optimistic’ ( $D_{agg} = 1$ ) solution on the conclusion. Such a choice is publicly stated at the beginning of the decision process and is fixed by the same person or organization that fixed the decision rule. It is reasonable to expect that the way the conclusion is decided in case of indecision depends on the context: In a legal context, for example, it is preferred to release a culprit rather than condemn an innocent. On the other hand, if we must hire a person, it is reasonable to opt for the optimistic solution, i.e., the indecision is interpreted as a positive decision.
    - \* if indecision is allowed at the beginning of the decision process then we put  $D_{agg} = ?$
  - if  $Accept_{agg} = 1$  then PBP is used and  $D_{agg}$  is derived by the collective judgments on the premises following the decision rule. If the aggregation of  $P_{agg_1}, \dots, P_{agg_n}$  results in ? following the decision rule and indecision is not allowed then we will have  $D_{agg} = 1$  in case of an optimistic reasoning or  $D_{agg} = 0$  in case of an pessimistic reasoning.
  - In case there are as many members who accept the rule as individuals who reject the rule, CBP is used. The reason is that those who reject the rule derive the value of  $D_{agg}$  using also the missing criteria or what they think are the correct logical relations among criteria. In both cases, using PBP and deriving  $D_{agg}$  by the given decision rule would not reflect their opinions. If CBP returns a tie between  $D = 0$  and  $D = 1$ , this is handled in the same way as in the case where  $Accept_{agg} = 0$ .

We now illustrate the procedure with our running example.

*Example 2.* Table 3 gives the judgments expressed by five members of our funding board. Only  $M_1$  rejects the rule because she believes that some criteria are missing. She fixes the value of  $D$  according also to the missing criteria, this is why  $D = 0$  despite the fact that we have  $P = 1$  and  $Q = 1$ . Since  $Accept_{agg} = 1$  we use premise-based procedure. We get  $D_{agg} = 1$  following the decision rule.

**Table 3.** Example of judgment aggregation with acceptance of the general rule

	Acceptance	P	Q	R	D
$M_1$	0	1	1	?	0
$M_2$	1	0	1	1	0
$M_3$	1	X	1	1	1
$M_4$	1	1	1	1	1
$M_5$	1	1	0	1	0
collective decision	1	1	1	1	1

*Example 3.* Let us now consider individuals who have to make a collective decision using the rule  $(P \wedge Q \wedge R) \leftrightarrow D$ , with a majority thinking that it is not appropriate, i.e.,  $Accept_{agg} = 0$ . Suppose that their judgments are as in Table 4.

**Table 4.** Example of judgment aggregation with rejection of the general rule

	Acceptance	P	Q	R	D
$M_1$	0	1	0	0	1
$M_2$	1	1	1	1	1
$M_3$	0	0	0	1	1
$M_4$	0	X	0	X	0
$M_5$	1	1	0	1	0
collective decision	0	1	0	1	1

A majority of members do not accept the decision rule. As we have seen, when  $Accept_{agg} = 0$  it means that the group members believe that the most important criteria for the decision are missing or that criteria are relevant but not well connected. Therefore, they express their judgments on the criteria in the rule, but their decision on the conclusion  $D$  takes into account what they believe are the missing attributes or the right decision rule. For example,  $M_3$  states that  $D = 1$  despite the fact that  $P = 0$  and  $Q = 0$  because she thinks that the rule is  $(P \vee Q \vee R) \leftrightarrow D$ . In this situation, the group will conclude  $D_{agg} = 1$  following a CBP.

Please note that letting the majority deciding on the group acceptance of the decision rule is just one possibility. Nothing forbids to fix a different quota, such as unanimity or a quota of 2/3 of the agents in order to accept or reject the decision rule at the group level. According to our framework, the original doctrinal paradox would be solved by PBP. In the court example, all judges have to give judgments according to the legal doctrine. Hence, PBP would be enforced. This would be in line with what advocated by some legal theorists, that is in a legal verdict reasons are more important than the final decision as these will form the legal *corpus* for future verdicts.



## 5 A Solution to the Dilemma

In this section we compare our approach to standard judgment aggregation. In particular, our approach can be seen not only as a more realistic and flexible framework for judgment aggregation problems, but also as an escape route from the paradoxes that trouble judgment aggregation. In order to illustrate why this is the case, we will state the first impossibility theorem [8], recall why PBP and CBP are considered escape routes from the dilemma and, finally, show that our approach is an alternative solution.

The first impossibility theorem of judgment aggregation<sup>1</sup> showed that there exists no aggregation function  $F$  that satisfies the following few desirable conditions:

**Universal Domain:** The domain of  $F$  is the set of all profiles of consistent and complete judgment sets.

**Anonymity:** For any profiles  $(J_1, \dots, J_k), (J'_1, \dots, J'_k)$  in the domain that are permutations of each other,  $F(J_1, \dots, J_k) = F(J'_1, \dots, J'_k)$ . Intuitively, this means that all agents have equal weight.

**Systematicity:** For any  $P, Q \in \mathcal{A}$  and any profiles  $(J_1, \dots, J_k), (J'_1, \dots, J'_k)$  in the domain, if  $\forall j \in N, P \in J_j \leftrightarrow Q \in J'_j$ , then  $P \in F(J_1, \dots, J_k) \leftrightarrow Q \in F(J'_1, \dots, J'_k)$ . This condition ensures that the collective judgment on each proposition depends only on the agent judgments on that proposition, and that the aggregation rule is the same across all propositions. Systematicity is clearly a very strong condition. In subsequent impossibility results, systematicity has been weakened to the independence of irrelevant alternatives:

**Independence of Irrelevant Alternatives (IIA):** For any  $P \in \mathcal{A}$  and any profiles  $(J_1, \dots, J_k), (J'_1, \dots, J'_k)$  in the domain, if  $\forall j \in N, P \in J_j \leftrightarrow P \in J'_j$ , then  $P \in F(J_1, \dots, J_k) \leftrightarrow P \in F(J'_1, \dots, J'_k)$ . In other words, IIA is systematicity without the neutrality condition, requiring that all propositions are equally treated.

It should be now clear why PBP and CBP are escape routes to the dilemmas of judgment aggregation. PBP is a procedure that relaxes the independence of irrelevant alternatives condition: The individuals are requested to express their judgments only on the premises and the collective value of the conclusion is derived by the aggregated values on the premises following the decision rule. On the other hand, CBP can never generate a paradoxical outcome as the conclusion is a literal.

However, as attractive as these procedures can appear, they leave a major open problem: When proposition-wise majority voting collapses into an inconsistent group outcome, PBP and CBP give opposite solutions, as we have seen in the court example. The question is then how we can decide between these two solutions.

<sup>1</sup> For other impossibility theorems that strengthened and expanded the original formulation, see [9].

Our approach is an attempt to provide an answer to this question. By extending the standard judgment aggregation framework and allowing the group members to accept or reject the given decision rule, we introduce a way to decide between PBP and CBP. When the majority (or any other pre-fixed quota of the voters) agrees with the decision rule, the individual judgments are aggregated by PBP. On the other hand, when the group does not agree with the rule, the only opinion the group can provide is about the final decision, so the aggregation procedure will turn to CBP.

Since our approach can always decide whether the aggregation function is PBP or CBP, we provide an escape from the paradoxes that plague standard judgment aggregation.

## 6 Related Works

In this section we refer to works that proposed to relax some of the assumptions made in the classical judgment aggregation framework. However, our model is the first that combines all these different aspects and introduces new ones.

Results in judgment aggregation usually assume complete judgment sets both at the individual and collective level. Gärdenfors [4] was the first to criticize such assumption as being too strong and unrealistic. He allows voters to abstain from expressing judgments on some propositions in the agenda. He proves that, if the judgment sets may not be complete (but logically closed and consistent), then every aggregation function that is IIA and Paretian<sup>2</sup>, must be oligarchic<sup>3</sup>. Gärdenfors' framework requires the agenda to have a very rich logical structure (with an infinite number of issues). More recently, Dokow and Holzman [3] extended Gärdenfors' result and consider finite agendas. Again, impossibility results are obtained. Hence, relaxing the completeness assumption does not avoid the impossibility results.

Nevertheless, allowing the voters to not express their judgments on some of the issues in the agenda provides a more realistic model of judgment aggregation, which is the aim of our paper. In order to avoid confusion, we must observe that we distinguish abstaining from being neutral with respect to an issue in the agenda. Abstentions in Gärdenfors and Dokow and Holzman' works correspond to what we call "neutral judgments". In our model, a voter abstains on a criterion when she deems that this criterion is irrelevant. In this case, she does not state her judgments on a criterion.

In a recent paper, Miller [10] considers judgment aggregation problems in which members have different views on how the premises are connected to

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<sup>2</sup> A *Paretian* aggregation function is such that, if all the members in the group adopt the same position on a certain issue, this position will be adopted at the collective level as well.

<sup>3</sup> An aggregation function is *oligarchic* if, for every issue in the agenda, the group adopts a position 0 (resp. 1) if and only if all the members of a subset of the group (the oligarchy) adopt position 0 (resp. 1) on that issue. Clearly, when there is only one member in the oligarchy, it corresponds to dictatorship.

the conclusion. This means that there is no imposed decision rule but, given a set of premises and a conclusion, each member expresses her judgments on the propositions in the agenda as well as providing the decision rule she has used. Miller's framework allows members to use decision rules in which only some of the premises appear. However, group members are requested to express judgments also on criteria that they deem irrelevant for the final decision. The question addressed is whether, once the members have voted following their own decision rules, it is possible to determine a group decision rule that represents how the group see the logical relations between premises and conclusion. Unless the unanimity rule is used, the answer is negative.

Another related work is [11], which considers judgment aggregation situations in which there is a gap between necessary and sufficient conditions to justify a certain decision on the conclusion. An example is the reviewing process for the publication of a paper. Suppose that the criteria for recommending publication of a manuscript ( $D$ ) are correctness of the results ( $P$ ) and originality of the ideas ( $Q$ ). We may agree that  $P$  is necessary for recommending publication ( $\neg P \rightarrow \neg D$ ), and also that  $P \wedge Q \rightarrow D$ . However, the gap between necessary and sufficient conditions is the situation in which we judge the results contained in the submission to be correct but the ideas not original:  $P \wedge \neg Q$  is consistent with both acceptance and rejection of a paper. Members may have different views on such gaps. The question posed in [11] is how to justify such individual discrepancies at the group level. Possibilities results are explored, in which majority voting on the conclusion is combined with no veto power on the premises.

Despite the similarities of the above contributions with our approach, the key feature of our proposal is to present a normative procedure: the aggregation rule is PBP or CBP depending on the members view on the given decision rule. The group will always be able to take a decision and, when most of them consider the decision rule to be appropriate, the group will also be able to provide reasons for that decision.

Another way to make the aggregation procedure reactive to the individual opinions about the decision rule is to allow the group members to assign weights to the criteria, as in [2]. The way the group decision is derived (PBP or CBP) depends on whether the final weights are above or below a fixed threshold. The problem of fixing a threshold is common to other frameworks that use similar quantitative approach (see for example, the work by (Dietrich and List 2005) using quota rules). Even if these approaches also provides a more realistic framework to judgment aggregation, problems have to be solved such as : where do the weights come from and how/who should fix the threshold.

## 7 Conclusion and Future Work

We extended standard judgment aggregation procedure in order to take into account the judgement status and the acceptance of the decision rule. Our framework allows group members to express 0/1 judgments as well as being neutral or abstain on some propositions. In addition, it allows individuals to state whether

or not they agree on the rule governing the decision. The aggregation procedure we propose is more reactive to individual opinions since we use a flexible decision rule (conclusion-based or premises-based) according to the acceptance/rejection of the rule by the group members. Our approach is more realistic and flexible compared to standard judgment aggregation procedure. In addition, it provides an escape route from the paradoxes that trouble judgment aggregation.

This work can be extended in different directions, among which we plan to investigate the relationship between acceptance of the decision rule in our framework and works on coalitions [14]. More precisely, we intend to study how group members can form coalitions and manipulate their judgments in order to drive the decision process in a particular direction. In addition, we plan to investigate the relationship with opinion aggregation in order to go beyond binary judgments [1]. A fuzzy approach would also allow to express the degree of confidence in the decision rule.

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