Elimination Theory for Nonlinear Parameter Estimation

John Chiasson and Ahmed Oteafy

1 Introduction

The work presented here exploits elimination theory (solving systems of polynomial equations in several variables) [1][2] to perform nonlinear parameter identification. In particular show how this technique can be used to estimate the rotor time constant and the stator resistance values of an induction machine. Although the example here is restricted to an induction machine, parameter estimation is applicable to many practical engineering problems. In [3], L. Ljung has outlined many of the challenges of nonlinear system identification as well as its particular importance for biological systems. In these types of problems, the model developed for analysis is typically a nonlinear state space model with unknown parameter values. The typical situation is that only a few of the state variables are measurable requiring that the system be reformulated as a nonlinear input-output model. In turn, resulting the nonlinear input-output model is almost always nonlinear in the parameters. Towards that end, differential algebra tools for analysis of nonlinear systems have been developed by Michel Fliess [4][5] and Diop [6]. Moreover, Ollivier [7] as well as Ljung and Glad [8] have developed the use of the characteristic set of an ideal as a tool for identification problems. The use of these differential algebraic methods for system identification have also been considered in [9], [10]. The focus of their research has been the determination of *a priori* identifiability of a given system model. However, as stated in [10], the development of an efficient algorithm using these differential algebraic techniques is still unknown. Here, in contrast, a method for which one can actually numerically obtain the numerical value of the parameters is presented. We also point out that [11] has also done work applying elimination theory to systems problems.

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Here, using the techniques of elimination theory, it is shown that a significant class of nonlinear identification problems can be formulated as a nonlinear least-squares problem whose solution is guaranteed to be found in a finite number of steps. The proposed methodology starts with obtaining an *over-parameterized* input-output model that is linear in the parameters. It is then assumed that the relationship between the actual parameters in the over-parameterized model are *rationally* related which is not atypical of many engineering systems. After making appropriate substitutions, the problem is transformed into a *nonlinear* least-squares problem which is- not overparameterized. It is then shown how the nonlinear least-squares problem can be solved in a finite number of steps using elimination theory.

2 Mathematical Model of an Induction Machine

An induction machine is now used as a realistic application to describe the methodology. Specifically, the identification of the rotor time constant and stator resistance are considered. As background, field-oriented control provides a means to obtain high-performance control of an induction machine for use in applications such as traction drives. This field-oriented control methodology requires knowledge of the rotor flux linkages, which are not usually measured [12][13]. To get around this problem, the rotor flux linkages are usually estimated using a state observer, and this observer requires the value of the rotor time constant T_R. However, $T_R = L_R/R_R$ varies due to ohmic heating and thus it is of considerable interest to estimate its value online in order to update the flux estimator with its current value.

A standard two-phase model of the induction machine is given by ([13])

$$\begin{aligned}
\frac{di_{Sa}}{dt} &= \frac{\beta}{T_R}\psi_{Ra} + \beta n_p \omega \psi_{Rb} - \gamma i_{Sa} + \frac{1}{\sigma L_S} u_{Sa} \\
\frac{di_{Sb}}{dt} &= \frac{\beta}{T_R}\psi_{Rb} - \beta n_p \omega \psi_{Ra} - \gamma i_{Sb} + \frac{1}{\sigma L_S} u_{Sb} \\
\frac{d\psi_{Ra}}{dt} &= -\frac{1}{T_R}\psi_{Ra} - n_p \omega \psi_{Rb} + \frac{M}{T_R} i_{Sa} \\
\frac{d\psi_{Rb}}{dt} &= -\frac{1}{T_R}\psi_{Rb} + n_p \omega \psi_{Ra} + \frac{M}{T_R} i_{Sb} \\
\frac{d\omega}{dt} &= \frac{M n_p}{J L_R}\left(i_{Sb}\psi_{Ra} - i_{Sa}\psi_{Rb}\right) - \frac{\tau_L}{J}
\end{aligned} \tag{1}$$

where the state variables are the rotor angular position θ, the rotor angular speed $\omega = d\theta/dt$, the (two-phase equivalent) stator currents i_{Sa}, i_{Sb}, and the (two-phase equivalent) rotor flux linkages ψ_{Ra}, ψ_{Rb}. The controllable inputs are the (two-phase equivalent) stator voltages u_{Sa}, u_{Sb} while the disturbance input is the load torque τ_L.

The parameters of the model are the stator and rotor resistances R_S and R_R, the mutual inductance M, the stator and rotor inductances L_S and L_R, the moment of inertia J and the number of pole-pairs n_p. The symbols

$$\begin{array}{ll} T_R = L_R/R_R & \sigma = 1 - M^2/\left(L_S L_R\right) \\ \beta = M/\left(\sigma L_S L_R\right) & \gamma = R_S/\left(\sigma L_S\right) + \beta M/T_R \end{array}$$

are used to simplify the expressions where σ is referred to as the total leakage factor.

This model is transformed into a coordinate system attached to the rotor as the signals in this new (x, y) rotor frame typically vary at the slower slip frequency rather than at the stator frequency in the (a, b) frame. The current variables are transformed according to

$$\begin{bmatrix} i_{Sx} \\ i_{Sy} \end{bmatrix} = \begin{bmatrix} \cos(n_p\theta) & \sin(n_p\theta) \\ -\sin(n_p\theta) & \cos(n_p\theta) \end{bmatrix} \begin{bmatrix} i_{Sa} \\ i_{Sb} \end{bmatrix}. \tag{2}$$

This transformation does not depend on any unknown parameter in contrast to the field-oriented (or dq) transformation which requires knowledge of the rotor fluxes. The stator voltages and the rotor fluxes are transformed in the same way as the currents resulting in the following model (see [14][15])

$$\frac{di_{Sx}}{dt} = \frac{u_{Sx}}{\sigma L_S} - \gamma i_{Sx} + \frac{\beta}{T_R}\psi_{Rx} + n_p\beta\omega\psi_{Ry} + n_p\omega i_{Sy} \tag{3}$$

$$\frac{di_{Sy}}{dt} = \frac{u_{Sy}}{\sigma L_S} - \gamma i_{Sy} + \frac{\beta}{T_R}\psi_{Ry} - n_p\beta\omega\psi_{Rx} - n_p\omega i_{Sx} \tag{4}$$

$$\frac{d\psi_{Rx}}{dt} = \frac{M}{T_R} i_{Sx} - \frac{1}{T_R}\psi_{Rx} \tag{5}$$

$$\frac{d\psi_{Ry}}{dt} = \frac{M}{T_R} i_{Sy} - \frac{1}{T_R}\psi_{Ry} \tag{6}$$

$$\frac{d\omega}{dt} = \frac{Mn_p}{JL_R}\left(i_{Sy}\psi_{Rx} - i_{Sx}\psi_{Ry}\right) - \frac{\tau_L}{J}. \tag{7}$$

As explained above, the interest here is in the online estimation of T_R as it changes due to ohmic heating so that an accurate value is available to the rotor flux estimator. However, the stator resistance value R_S will also vary due to ohmic heating, therefore its variation must also be taken into account in the estimation. The electrical parameters M, L_S, σ are assumed to be known and not varying. Measurements of the stator currents i_{Sa}, i_{Sb} and voltages u_{Sa}, u_{Sb} as well as the position θ of the rotor are assumed to be available; the velocity is then computed from the position measurements. The rotor flux linkages are not assumed to be measured.

3 Input-Output Model

Standard methods for parameter estimation are based on equalities where known signals depend *linearly* on unknown parameters. However, the induction motor model described above does not fit in this category unless the rotor flux linkages are measured. As this is not the case here, the fluxes ψ_{Rx}, ψ_{Ry} and their derivatives $d\psi_{Rx}/dt, d\psi_{Ry}/dt$ must be eliminated from the final identification model. The four equations (3), (4), (5), (6) are used to solve for the four unknowns ψ_{Rx}, ψ_{Ry}, $d\psi_{Rx}/dt, d\psi_{Ry}/dt$. Further, a new set of independent equations is found by differentiating equations (3) and (4) to obtain

$$\begin{aligned}\frac{1}{\sigma L_s}\frac{du_{Sx}}{dt} &= \frac{d^2 i_{Sx}}{dt^2} + \gamma\frac{di_{Sx}}{dt} - \frac{\beta}{T_R}\frac{d\psi_{Rx}}{dt} - n_p\beta\omega\frac{d\psi_{Ry}}{dt} - n_p\beta\psi_{Ry}\frac{d\omega}{dt}\\ &\quad - n_p\omega\frac{di_{Sy}}{dt} - n_p i_{Sy}\frac{d\omega}{dt} \end{aligned} \tag{8}$$

and

$$\begin{aligned}\frac{1}{\sigma L_s}\frac{du_{Sy}}{dt} &= \frac{d^2 i_{Sy}}{dt^2} + \gamma\frac{di_{Sy}}{dt} - \frac{\beta}{T_R}\frac{d\psi_{Ry}}{dt} + n_p\beta\omega\frac{d\psi_{Rx}}{dt} + n_p\beta\psi_{Rx}\frac{d\omega}{dt}\\ &\quad + n_p\omega\frac{di_{Sx}}{dt} + n_p i_{Sx}\frac{d\omega}{dt}. \end{aligned} \tag{9}$$

To simplify the presentation we now assume that the speed is held constant as in [16][17] (this is not necessary, see [18][19]). The expressions for $\psi_{Rx}, \psi_{Ry}, d\psi_{Rx}/dt, d\psi_{Ry}/dt$ found from solving equations (3), (4), (5), (6) are substituted into equations (8) and (9) with $d\omega/dt = 0$ to obtain

$$\begin{aligned}0 &= -\frac{d^2 i_{Sx}}{dt^2} + \frac{di_{Sy}}{dt}n_p\omega + \frac{1}{\sigma L_S}\frac{du_{Sx}}{dt} - (\gamma + \frac{1}{T_R})\frac{di_{Sx}}{dt}\\ &\quad - i_{Sx}(-\frac{\beta M}{T_R^2} + \frac{\gamma}{T_R}) + i_{Sy}n_p\omega(\frac{1}{T_R} + \frac{\beta M}{T_R}) + \frac{u_{Sx}}{\sigma L_S T_R} \end{aligned} \tag{10}$$

$$\begin{aligned}0 &= -\frac{d^2 i_{Sy}}{dt^2} - \frac{di_{Sx}}{dt}n_p\omega + \frac{1}{\sigma L_S}\frac{du_{Sy}}{dt} - (\gamma + \frac{1}{T_R})\frac{di_{Sy}}{dt}\\ &\quad - i_{Sy}(-\frac{\beta M}{T_R^2} + \frac{\gamma}{T_R}) - i_{Sx}n_p\omega(\frac{1}{T_R} + \frac{\beta M}{T_R}) + \frac{u_{Sy}}{\sigma L_S T_R}. \end{aligned} \tag{11}$$

As $\gamma = R_S/\left(\sigma L_S\right) + \beta M/T_R$, it follows that

$$\begin{aligned}-\beta M/T_R^2 + \gamma/T_R &= \left(R_S/T_R\right)/\left(\sigma L_S\right)\\ \gamma + 1/T_R &= R_S/\left(\sigma L_S\right) + \left(\beta M + 1\right)/T_R\end{aligned}$$

which is used to rewrite (10) and (11) as

$$0 = -\frac{d^2 i_{Sx}}{dt^2} + \frac{di_{Sy}}{dt} n_p \omega + \frac{1}{\sigma L_S}\frac{du_{Sx}}{dt} - \Big(R_S/\left(\sigma L_S\right) + \left(\beta M + 1\right)/T_R \Big) \frac{di_{Sx}}{dt} \\ - i_{Sx}(\frac{R_S}{T_R}\frac{1}{\sigma L_S}) + i_{Sy} n_p \omega ((\beta M + 1)/T_R) + \frac{u_{Sx}}{\sigma L_S T_R} \qquad (12)$$

$$0 = -\frac{d^2 i_{Sy}}{dt^2} - \frac{di_{Sx}}{dt} n_p \omega + \frac{1}{\sigma L_S}\frac{du_{Sy}}{dt} - \Big(R_S/\left(\sigma L_S\right) + \left(\beta M + 1\right)/T_R \Big) \frac{di_{Sy}}{dt} \\ - i_{Sy}(\frac{R_S}{T_R}\frac{1}{\sigma L_S}) - i_{Sx} n_p \omega \left(\beta M + 1\right)/T_R + \frac{u_{Sy}}{\sigma L_S T_R}. \qquad (13)$$

More compactly, equations (12) and (13) are written in linear regressor form as

$$y(t) = W(t)K \qquad (14)$$

with

$$y(t) \triangleq \begin{bmatrix} \frac{d^2 i_{Sx}}{dt^2} - \frac{di_{Sy}}{dt} n_p \omega - \frac{1}{\sigma L_S}\frac{du_{Sx}}{dt} \\ \frac{d^2 i_{Sy}}{dt^2} + \frac{di_{Sx}}{dt} n_p \omega - \frac{1}{\sigma L_S}\frac{du_{Sy}}{dt} \end{bmatrix} \qquad (15)$$

and

$$W(t) \triangleq \begin{bmatrix} -\frac{di_{Sx}}{dt}\frac{1}{\sigma L_S} & (\beta M + 1)\left(-\frac{di_{Sx}}{dt} + i_{Sy} n_p \omega\right) + \frac{u_{Sx}}{\sigma L_S} & -\frac{i_{Sx}}{\sigma L_S} \\ -\frac{di_{Sy}}{dt}\frac{1}{\sigma L_S} & (\beta M + 1)\left(-\frac{di_{Sy}}{dt} - i_{Sx} n_p \omega\right) + \frac{u_{Sy}}{\sigma L_S} & -\frac{i_{Sy}}{\sigma L_S} \end{bmatrix} \qquad (16)$$

as well as

$$K = \begin{bmatrix} K_1 \\ K_2 \\ K_3 \end{bmatrix} \triangleq \begin{bmatrix} R_S \\ 1/T_R \\ R_S/T_R \end{bmatrix}. \qquad (17)$$

This model is over-parameterized in the parameters, that is, they must satisfy the constraint

$$K_3 = K_1 K_2. \qquad (18)$$

Replacing K_3 by $K_1 K_2$ in (14) results in a model that is not over-parameterized, but it is no longer linear in the parameters. This issue is considered next.

4 Nonlinear Least-Squares Identification

A discrete-time sampled version of (14) is

$$y(nT) = W(nT)K, \qquad (19)$$

where T is the sample period, nT is the time the n^{th} sample is taken, and $K = \left[K_1 \; K_2 \; K_3 \right]^T$ is the (over-parameterized) vector of unknown parameters. If the constraint (18) is ignored, then the system is a linear (but over-parameterized) least-squares problem. Theoretically, an exact unique solution for the unknown parameter vector K may be determined after several time instants. However, due to the fact that both $y(nT)$ and $W(nT)$ are measured from signals that are noisy (due to quantization and differentiation), the regressor model (19) is only approximately valid in practice. These sources of error result in an overdetermined system of equations. In order to get around this problem, the solution vector K is specified as that which minimizes a least-squares criterion. Specifically, given $y(nT)$ and $W(nT)$ where $y(nT) = W(nT)K$, one defines

$$E^2(K) = \sum_{n=1}^{N} \left| y(nT) \; - W(nT)K \right|^2 \tag{20}$$

as the *residual error* associated to a parameter vector K. Then, the least-squares estimate K^* is chosen such that $E^2(K)$ is minimized for $K = K^*$. The function $E^2(K)$ is quadratic and therefore has a unique minimum at the point where $\partial E^2(K)/\partial K = 0$ holds. Solving this expression for K^* yields the least-squares solution to $y(nT) = W(nT)K$ as

$$K^* = \left[\sum_{n=1}^{N} W^T(nT)W(nT) \right]^{-1} \left[\sum_{n=1}^{N} W^T(nT)y(nT) \right]. \tag{21}$$

However, there is no guarantee that the solution of (21) will satisfy the constraint $K_3 = K_1K_2$. Furthermore, the over-parameterized identification model consisting of (17) and (19) results in an ill-conditioned solution for K^*. That is, small changes in the data $W(nT), y(nT)$ can result in large changes in the value computed for K^*. To get around these problems, a *nonlinear* least-squares approach is taken which involves minimizing

$$E^2(K) = \sum_{n=1}^{N} \left| y(nT) - W(nT)K \;\right|^2 = R_y - 2R_{Wy}^T K + K^T R_W K \tag{22}$$

subject to the constraint $K_3 = K_1K_2$ where

$$\begin{aligned} R_y &\triangleq \sum_{n=1}^{N} y^T(nT)y(nT), R_{Wy} \triangleq \sum_{n=1}^{N} W^T(nT)y(nT) \\ R_W &\triangleq \sum_{n=1}^{N} W^T(nT)W(nT). \end{aligned} \tag{23}$$

On physical grounds, the parameters K_1, K_2 are constrained to the region

$$0 < K_1 < \infty, 0 < K_2 < \infty \tag{24}$$

and the squared error $E^2(K)$ will be minimized in this *open* region. Substituting $K_3 = K_1 K_2$ in (22), we obtain a new error function $E_p^2(K_1, K_2)$ as

$$\begin{aligned} E_p^2(K_1, K_2) &\triangleq \sum_{n=1}^{N} \Big| y(nT) \; - W(nT)K \Big|^2_{K_3=K_1K_2} \\ &= R_y - \; 2R_{Wy}^T K \Big|_{K_3=K_1K_2} + \; \left(K^T R_W K\right)\Big|_{K_3=K_1K_2} . \end{aligned} \tag{25}$$

As the minimum of (25) must occur in the region (24), it follows that the minimum is located at an extremum point. To solve for this minimum thus entails solving simultaneously the two extrema equations

$$p_1(K_1, K_2) \triangleq \frac{\partial E_p^2(K_1, K_2)}{\partial K_1} \tag{26}$$

$$p_2(K_1, K_2) \triangleq \frac{\partial E_p^2(K_1, K_2)}{\partial K_2}, \tag{27}$$

which are *polynomials* in the parameters K_1, K_2. The degrees of the polynomials p_i are given in the table below

	deg K_1	deg K_2
$p_1(K_1, K_2)$	1	2
$p_2(K_1, K_2)$	2	1

These two polynomials are rewritten in the form

$$p_1(K_1, K_2) = a_1(K_2)K_1 + a_0(K_2) \tag{28}$$

$$p_2(K_1, K_2) = b_2(K_2)K_1^2 + b_1(K_2)K_1 + b_0(K_2). \tag{29}$$

A systematic procedure to find all possible solutions to a set of polynomials is provided by elimination theory through the method of resultants [1][2]. However, in this particular example, $p_1(K_1, K_2)$ is of degree 1 in K_1 and can be solved directly. Substituting $K_1 = -a_0(K_2)/a_1(K_2)$ from $p_1(K_1, K_2) = 0$ into $p_2(K_1, K_2) = 0$ and multiplying the result through by $a_1^2(K_2)$, one obtains the (resultant) polynomial

$$r(K_2) = a_0^2(K_2)b_2(K_2) - a_0(K_2)a_1(K_2)b_1(K_2) + a_1^2(K_2)b_0(K_2), \tag{30}$$

where $\deg_{K_2}\{r\} = 5$. The roots of (30) are the only possible candidates for the values of K_2 that satisfy $p_1(K_1, K_2) = p_2(K_1, K_2) = 0$ for some K_1. In the online implementation, the coefficients of the polynomials $a_1(K_2), a_0(K_2)$, $b_2(K_2)$, $b_1(K_2), b_0(K_2)$, whose explicit expressions in terms of the elements of the matrices R_W and R_{Wy} are known a priori vis-a-vis (25), (26), and (27), are computed and stored during data collection. The coefficients of the polynomial $r(K_2)$ are then computed online according to (30). Next, the positive roots K_{2i} of $r(K_2) = 0$ are computed and substituted into $p_1(K_1, K_{2i}) = 0$ which is then solved for its positive roots K_{1j}. By this method of back solving, the finite number of possible candidate solutions (K_{1j}, K_{2i}) are found. The pair that results in the smallest squared error, i.e., the smallest value of $E_p^2(K_1, K_2)$, is chosen.

5 Simulations

The above parameter identification method was studied in simulation using a two-phase equivalent model of an induction machine under closed-loop control. The parameters of the induction machine are (see [13]): $M = 0.0117\,\mathrm{H}$, $L_R = 0.014\,\mathrm{H}$, $L_S = 0.014\,\mathrm{H}$, $R_S = 1.7\,\Omega$, $R_R = 3.9\,\Omega$, $\tau_{L0} = 0.15\,\mathrm{Nm}$, $J = 0.00011$ Kgm^2, and $n_P = 3$. The controller sets the desired rotor speed at $\omega_R = 2\pi \times 75$ rad/s, while the load torque is defined to be $\tau_L \triangleq \tau_{L0} + f\omega$ with $\tau_{L0} = 0.15$ Nm. The data was sampled at $f_S = 4$ kHz which was filtered through a 2^{nd} order low pass Butterworth filter with a cutoff frequency of 70 Hz.

To mimic the ohmic heating of the rotor and stator resistors, in the simulation of the motor model their values were increased by 50% after 3 seconds of operation with the estimator updating the value of T_R every 0.5 seconds. After the update at 3.5 secs the estimator provides the new estimates of R_S and R_R to the controller. Figure 1 below is a plot of $K_2 = 1/T_R$ and its reference versus time showing that after the update the estimator gives the value of K_2 within 2% of the correct value.

To show the importance of having an accurate value of the rotor time constant, the power consumed before and after the rotor time constant update was computed. Figure 2 shows the speed versus time for the simulation. (the transient at $t = 0$ is due to the fact that the flux in the machine is zero so that during the build up of the flux the machine has torque oscillations). Figure 3 below is a plot of the real power $P(t) = u_{Sa}i_{Sa} + u_{Sb}i_{Sb}$ vs time. As the figure shows, the real power jumps up to 66.9 W at 3 sec. After the rotor time constant value is updated to controller at 3.5 seconds, the real power comes down to 63.7 W, which is a 5% decrease. Of course these numbers are small because the simulation was done with a small (a less than kW) machine. In industry where large machines are used, the energy savings would be significant.

As explained above, the rotor time constant $T_R = 1/K_2$ is used to estimate the rotor fluxes which in turn are used to estimate the direct and quadrature currents for use in field oriented control. In field oriented control the motor torque is given by $\tau = \mu\psi_d i_q$ ($\mu = \frac{Mn_p}{JL_R}$) which at constant speed reduces to $\tau = \mu M i_d i_q$. For a given torque, the current magnitude $i_d^2 + i_q^2$ is minimized if $i_d = i_q$ [13]. Thus it is important to estimate the rotor flux angle accurately to have accurate values of the dq currents in order to achieve this minimization.

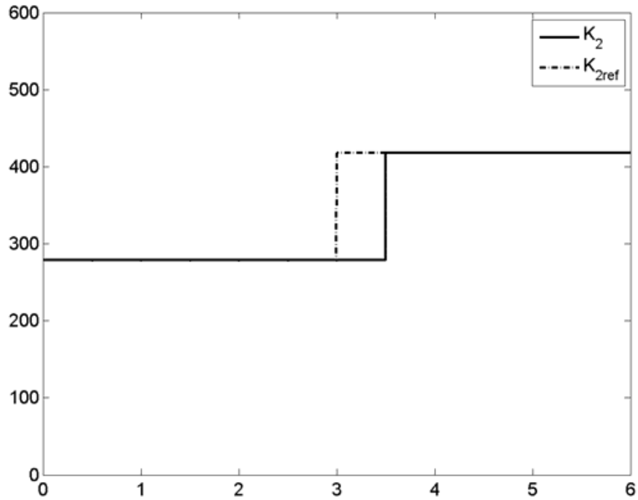

Fig. 1. $K_2 = 1/T_R$ and K_{2ref} vs. time in seconds

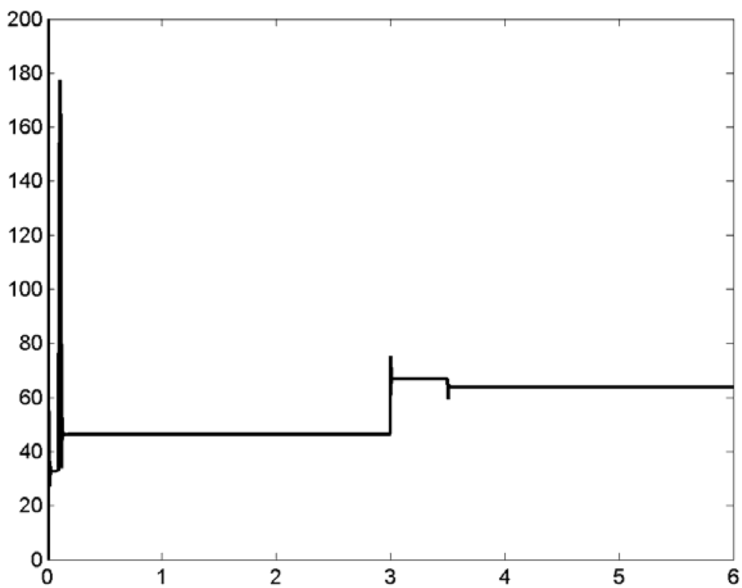

Fig. 2. Speed in radians/sec versus time in seconds

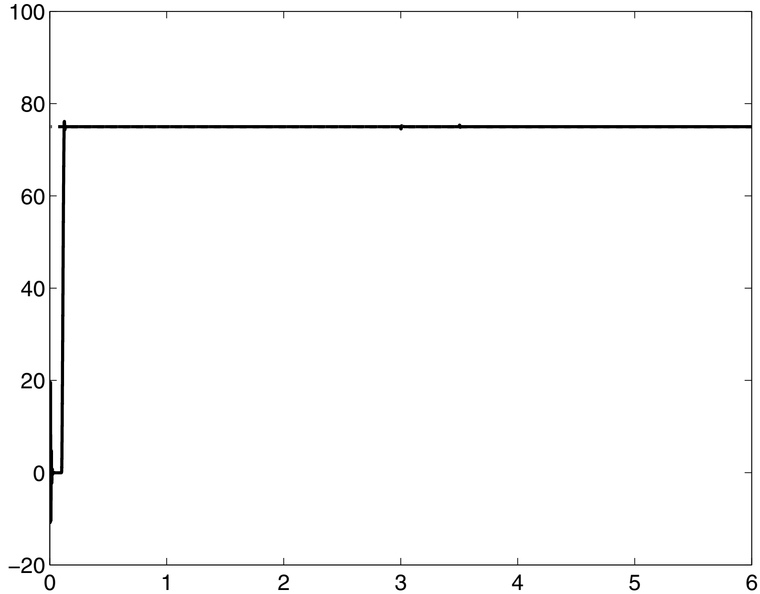

Fig. 3. Real power P in Watts versus time in seconds. (The large transient in the power at the beginning is due to the discontinuity in the acceleration - see the speed trajectory)

6 Conclusions

An approach to solving a nonlinear least-squares parameter identification problem in a *finite* number of steps was presented. This is in contrast to iterative methods which may or may not converge and, even if convergences takes place, it may be to only a local minimum. The method was presented by showing how the rotor time constant of the induction machine can be found online. In this application, the results show that an incorrect value of T_R leads to the controller commanding non-optimum values of the stator currents to the machine which in turn increases the Ohmic losses. That is, a higher power usage is required for the same torque requirement.

References

1. Cox, D., Little, J., O'Shea, D.: Ideals, Varieties, And Algorithms An Introduction to Computational Algebraic Geometry and Commutative Algebra, 2nd edn. Springer, Berlin (1996)
2. von zur Gathen, J., Gerhard, J.: Modern Computer Algebra. Cambridge University Press, Cambridge (1999)

3. Ljung, L.: Challenges of Nonlinear Identification. In: Bode Lecture, IEEE Conference on Decision and Control, Maui HI (2003)
4. Fliess, M., Glad, S.T.: An algebraic approach to linear and nonlinear control. In: Essays on Control: Perspectives in the Theory and Applications, Birkhäuser, pp. 223–267 (1993)
5. Sira-Ramírez, H., Agrawal, S.K.: Differentially Flat Systems. Marcel-Dekker, New York (2004)
6. Diop, S.: Differential-algebraic decision methods and some applications to system theory. Theor. Computer Sci. 98, 137–161 (1992)
7. Ollivier, F.: Le problème de l'identifiabilité structurelle globale: Ètude thèorique, mèthodes effective et bornes de complexè. PhD thesis, École Polytéchnique, Paris, France (1990)
8. Ljung, L., Glad, S.T.: On global identifiability for arbitrary model parameterisations. Automatica 30(2), 265–276 (1994)
9. Margaria, G., Riccomagno, E., Chappell, M.J., Wynn, H.P.: Differential algebra methods for the study of the structural identifiability of biological rational polynomial models (2004) (preprint)
10. Saccomani, M.P.: Some results on parameter identification of nonlinear systems. Cardiovascular Engineering: An International Journal 4(1), 95–102 (2004)
11. Diop, S.: Elimination in control theory. MCSS 4, 17–32 (1991)
12. Leonhard, W.: Control of Electrical Drives, 3rd edn. Springer, Berlin (2001)
13. Chiasson, J.: Modeling and High-Performance Control of Electric Machines. John Wiley & Sons, Chichester (2005)
14. Stephan, J., Bodson, M., Chiasson, J.: Real-time estimation of induction motor parameters. IEEE Transactions on Industry Applications 30(3), 746–759 (1994)
15. Stephan, J.: Real-time estimation of the parameters and fluxes of induction motors. Master's thesis, Carnegie Mellon University (1992)
16. Chiasson, J., Bodson, M.: Estimation of the rotor time constant of an induction machine at constant speed. In: Proceedings of the European Control Conference ECC 2007, Kos, Greece, pp. 4673–4678 (July 2007)
17. Oteafy, A., Chiasson, J., Bodson, M.: Online identification of the rotor time constant of an induction machine. In: Proceedings of the American Control Conference, St. Louis MO, pp. 4373–4378 (2009)
18. Wang, K., Chiasson, J., Bodson, M., Tolbert, L.: A nonlinear least-squares approach for estimation of the induction motor parameters. IEEE Transactions on Automatic Control 50(10), 1622–1628 (2005)
19. Wang, K., Chiasson, J., Bodson, M., Tolbert, L.M.: An on-line rotor time constant estimator for the induction machine. In: Proceedings of the IEEE International Electric Machines and Drives Conference, San Antonio TX, pp. 608–614 (May 2005)