An Agent-Based Implementation of the Todaro Model

Nadjia El Saadi, Alassane Bah, and Yacine Belarbi

Abstract. The problem of internal migration and its effect on urban unemployment and underemployment has been the subject of an abundant theoretical literature on economic development. However, most discussions have been largely qualitative and have not provided enough rigorous frameworks with which to analyze the mechanism of labor migration and urban unemployment. In this paper, we build up an economic behavioral model of rural-urban migration which is an agent-based version of the analytical Todaro model described by deterministic ordinary differential equations. The agent-based model allows to explore the rural-urban labor migration process and give quantitative results on the equilibrium proportion of the labor force that is not absorbed by the modern industrial economy.

1 Introduction

Economic development generates significant structural transformations such as changes in the demographic condition and in the production structure. The most important structural feature of developing economies is the distinction between rural and urban sectors and particularly, economic development is often defined in terms of the transfer of a large proportion of workers from agricultural to industrial activities [8]. The study of rural-urban labor migration has for long been an important area of research in development economics and a large body of literature has grown up in recent year around this topic in particular for less developed countries (see [1] for a survey on the theoretical literature).

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Q. Bai and N. Fukuta (Eds.): Advances in Practical Multi-Agent Systems, SCI 325, pp. 251–265.
springerlink.com

Economic historians agreed that a considerable part of the urban growth was due to rural-urban migration (see for example [11]). The towns offered new forms of employment opportunities and it was mainly the landless and the rural artisan who left and not the farmers [1]. When in the early 1950s economists turned their attention to the problems of population growth and economic development in the developing countries, it was natural to think that policies which emphasized industrialization would not only increase national incomes, but also relieve the overpopulation of the rural areas. However, during the 1960s, this view has led to a new orthodoxy in which rural-urban migration in less developed countries is viewed as "a symptom of and a contributing factor to underdevelopment". This orthodoxy is due to Todaro [10] and Harris-Todaro [6] whose models have provided a widely accepted theoretical framework for explaining the urban unemployment in less developed countries. The Todaro article [10] on urban unemployment in less-developed countries is an important advance in the study of this problem and although there has been some controversy on specific points, it has been widely applied to investigate various development issues. The key hypothesis of Todaro are that migrants react to economic incentives, earnings differentials and the probability of getting a job at the destination have influence on the migration decision. From these assumptions is deduced that the migratory dynamics in certain parametric ranges lead the economic system towards an equilibrium with urban concentration and high urban unemployment (the Todaro Paradox). The paradox is due to the assumptions that in choosing between labor markets, rural workers consider expected rather than current income differential . The expected income in the urban area is the fixed wage in the urban modern sector (formal sector) times the probability of obtaining a permanent job in this sector. This probability is defined as the number of opened jobs in the formal sector divided by the number of job seekers in the urban area. Since expected urban income is defined in terms of both wage and employment probability, in this model, it is possible to have continued migration in spite of the existence of sizeable rates of urban unemployment. Now assuming rural migrants respond to the employment probability, the Todaro model then demonstrates that an increase in urban employment may result in higher levels of urban unemployment. The repercussion of this simple set of assumptions is that contrary to received wisdom, the migration response which is factored in several policies aimed at reducing urban unemployment will raise urban unemployment rather than reduce it.

Todaro works have generated considerable discussions [2, 4, 5, 7, 9, 12] from which it has been accepted in the literature that Todaro model (1969) predicts too high an unemployment rate in developing countries compared to observed rates and accordingly additional features have been incorporated to the basic model with a view to generate lower predictions.

In this paper, we propose to revisit the Todaro dynamic model from an agent-based approach. The migration agent-based model we conceive is the simplest version derived from the basic analytic model of Todaro [10]. It is formulated too close to the analytic model so that comparisons between the two approaches results can be made. The aim of such a formulation is to reproduce explicitly the migratory dynamics at individuals scale and analyze and visualize migration process. By simulating the agent based model, we explore the behavior of the model and hence check if from an agent-based approach the Todaro paradox holds or not.

The paper is organized as follows: in section 2, we recall the theoretical Todaro model. Section 3 is devoted to the agent-based model constructed from the Todaro model. We first present the simulator model and then describe the structure of the simulator. Section 4 presents the scenarios tests and the simulations results followed by a discussion. A conclusion is presented in section 5.

2 The Todaro Model

In the Todaro model [10], migration is viewed as a two-stage phenomenon: the first stage finds the unskilled rural worker migrating to an urban area and joining a large pool of unemployed and underemployed workers who arrived in town earlier and still are waiting for a modern sector job. This pool is the so-called "urban traditional sector" or "informal sector". It is modelled as an unproductive and stagnant sector serving as a refuge for the urban unemployed and as a receiving station for newly arriving rural migrants on their way to the formal sector jobs. The second stage is reached with attainment of a permanent job in the so-called "modern sector" or "formal sector". The decision to migrate from rural to urban areas will be functionally related to two principal variables: (1) the urban-rural real income differential and (2) the probability of obtaining a permanent urban job. Hence, the equation for growth of aggregate labor supply in the urban area is:

$$\frac{dS(t)}{S(t)} = \{\beta_u + \pi(t)F(\Delta(t))\}\, dt \tag{1}$$

with

$$\Delta(t) = \frac{Y_u(t) - Y_r(t)}{Y_r(t)}$$

where $S(t)$ is the size of the total urban labor force in period t, β_u the natural rate of increase in the urban labor force and $\pi(t)$ the probability of obtaining a job in the "formal sector" in period t. $Y_r(t)$ is the net rural real income in period t while $Y_u(t)$ is the net urban real income in period t. $F(\Delta(t))$ is a function such that $dF/d\Delta > 0$. $\Delta(t)$ is the percentage urban-rural real income differential and the product $\pi(t)F(\Delta(t))$ represents the rate of urban labor force increase as a result of migration.

As mentioned earlier, the probability $\pi(t)$ of obtaining a job in the "formal sector" is defined as being equal to the ratio of new employment openings in the "formal sector" relative to the number of accumulated job seekers in the urban informal sector at date t, that is:

$$\pi(t) = \frac{\gamma N(t)}{S(t) - N(t)} \tag{2}$$

with $N(t)$ the total employment in the urban sector in period t and γ the rate of job creation in this sector. The difference $S(t) - N(t)$ measures the size of the "informal sector".

The model considers that the number of new jobs created increases at a constant exponential rate over time, specifically

$$\frac{dN(t)}{N(t)} = \gamma \; dt. \tag{3}$$

The proportion of the urban labor force employed in the formal sector at time t (the employment rate) is denoted by $E(t)$, where

$$E(t) = \frac{N(t)}{S(t)}. \tag{4}$$

and the proportionate size of the urban informal sector (unemployment rate) is denoted by $T(t)$:

$$T(t) = 1 - E(t). \tag{5}$$

Todaro [10] solves for the equilibrium rate of employment E^* in the simple case that the income differential $D(t)$ remains constant over time ie $D(t) = D$ and $F(D) = D$. He solves for:

$$\frac{dE}{E}(t) = \frac{dN}{N}(t) - \frac{dS}{S}(t) = 0. \tag{6}$$

The solution for E^* is:

$$E^* = \frac{\gamma - \beta}{\gamma \Delta + \gamma - \beta} \tag{7}$$

and alternatively, the equilibrium rate of unemployment in the urban sector is simply:

$$T^* = 1 - \frac{\gamma - \beta}{\gamma \Delta + \gamma - \beta}. \tag{8}$$

Todaro claims that this equilibrium is stable. His intuition behind the equations above can be explained as follows:
for a developing country in the very early stages of industrialization such that almost the entire population resides in rural areas, when the urbanization process is just beginning, the pool of the urban unemployed is relatively

small so that the probability of obtaining a job is high. Therefore, for a significantly positive Δ and a positive rate of urban job creation exceeding the natural rate of urban population growth (ie $\gamma > \beta$), the resulting urban expected real income induces rural-urban migration such that the urban labor force grows at a faster rate than that of job creation, that is, $\beta + \pi(t)\Delta > \gamma$. This more rapid growth of labor supply results in an increase in the size of the urban traditional sector with the result that the probability of a rural migrant finding a job in the next period is lower ($\pi(t+1) < \pi(t)$). Assuming Δ and γ remain constant, this lower probability should slow down the rate of urban labor force growth although $dS(t)/S(t)$ may continue to exceed $dN(t)/N(t)$. Eventually, $\pi(t)$ stabilizes the urban unemployment rate at some level $1 - E^*$ depending upon the values of Δ, β and γ. If the unemployment rate falls below $1 - E^*$, equilibrating forces in the form of rising π will be set in motion to restore the equilibrium.

3 The Migration Agent-Based Model

3.1 The Simulator Model

In this section, we formulate an agent-based version of the Todaro model presented above. For this purpose, we consider an economic system formed by two sectors: a rural sector with a labor force of size N_r and an urban sector with a labor force of size N_u. In turn, the urban sector is divided into a formal sector of size N_f and an informal sector of size N_i so that $N_u = N_f + N_i$. As argued in Todaro [10], individuals in the rural sector take their decisions of migrating or not by considering the differential of their real income between their present sector and the sector they intend to go (the urban formal sector), and their probability of obtaining a job in the latter. So in our agent-based formulation, each individual i in the rural sector ($1 \leq i \leq N_r$) will evaluate at date t his percentage urban-rural real income differential $\Delta_i(t) = \frac{Y_u^i(t) - Y_r^i(t)}{Y_r^i(t)}$ and his probability $\pi_i(t)$ of obtaining an urban job in period t. If both $\Delta_i(t)$ and $\pi_i(t)$ are strictly positive, the rural worker i will migrate with probability $P_i(t) = \Delta_i(t)\pi_i(t)$, elsewhere, he will remain in his sector.

Similarly to Todaro, we consider the simplifying assumptions that the percentages urban-rural real income differentials for rural individuals are assumed to be constant over time and probabilities of urban employment are not individualized, namely, $\Delta_i(t) = \Delta$ and $\pi_i(t) = \pi(t)$ for $1 \leq i \leq N_r$. This is for being able to make a comparison between Todaro and our simulations conclusions. Nevertheless, a richer agent-based model which generalizes this version will be presented in a future work.

Based on the Todaro assumption that a rural migrant might spend a certain period of time in the urban informal sector before reaching the urban formal sector, in our model we consider no direct move from rural to the formal urban sector. The dynamics of individuals between sectors are:

migration from rural sector to informal sector, transition from informal sector to formal sector or inversely from formal sector to informal sector. From this, $\pi(t)$ can be regarded as the probability of transition from informal sector to the formal one. Using our notations, formula for $\pi(t)$ given in (2) becomes

$$\pi(t) = \frac{\gamma N_f(t)}{N_i(t)} \tag{9}$$

with γ the rate of job creation considered constant along a period of simulation. Employment and unemployment rates defined in (4) and (5) are given by:

$$E(t) = \frac{N_f(t)}{N_f(t)+N_i(t)} \quad \text{and} \quad T(t) = 1 - E(t).$$

To carry out the simulation of this economic system, workers are placed in the 3 different sectors according initial chosen values for N_r, N_f and N_i. Given the values of parameters Δ , β_u (rate of natural growth) and γ , the first step in the simulation is to add to urban informal sector the new seekers for jobs as a result of natural urban labor force growth at the rate β_u. Then, the probability of finding a job $\pi(t)$ is calculated using (9). If $\pi(t)$ is strictly positive, a fraction of individuals in the urban informal sector are selected randomly to join the formal urban sector, each with probability $\pi(t)$. If $\pi(t) < 0$ (due to a negative value of γ expressing jobs suppression), a fraction of individuals in the urban formal sector would be ejected to the informal sector with probability $-\pi(t)$. When $\pi(t) = 0$, no change takes place. Then, if both Δ and $\pi(t)$ are strictly positive, every individual i in the rural sector is a potential migrant with probability $P(t) = \Delta\pi(t)$. Otherwise, individual i in the rural sector does not migrate. To conclude the transition process from one sector to another one, a random number is generated from an uniform distribution on]0,1[. If this number is less than the transition probability assessed for an individual, than the transition of the corresponding individual holds, otherwise, no change takes place. Hence, a new sectorial configuration is obtained. Knowledge of the new populations allows the system to be reset. Therefore, the state variables of the sectors have to be calculated again and the whole procedure will be repeated as many times as we set in the simulation.

3.2 Internal Structure

The simulator proposed is a tool of experiment intended to represent virtually rural-urban migration by three sectors (rural sector, urban formal sector, urban informal sector) and their interactions. We use the platform Cormas for its implementation [3]. Generally, Cormas describes models by: spatial entities to represent the geographic space, social entities to describe the actors and passive entities for the rest of the models elements. For our migration model, the classes **Space** and **Sector** are spatial entities while classes **Individual** and **Group** are social entities. **Migration** class is the model class that is used

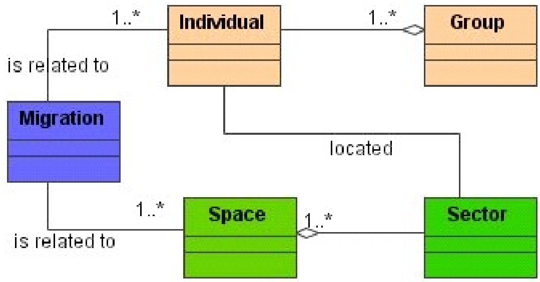

Fig. 1 Class diagram

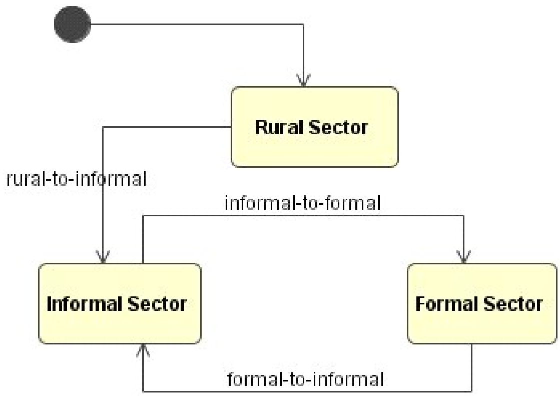

Fig. 2 The dynamic of Individuals

to manage the sequences of individual actions and the space (Fig. 1). The dynamic of individuals transitions between the three sectors is summarized in Fig. 2.

Our migration individual-based model is implemented in the Object-Oriented language Smalltalk using Visual Works Environment.Time evolves in a discrete way by steps $\triangle t$.

3.3 Interface

The simulator is provided with a three windows-interface:

(i) the first window allows to initialize, run and visualize evolution of the sectors and different indicators of migration (Fig. 3),
(ii) the second window allows the user the set up of the global variables and parameters (Fig. 4),
(iii) the third window is the simulation space, it shows the situation of the different sectors in time (Fig. 5).

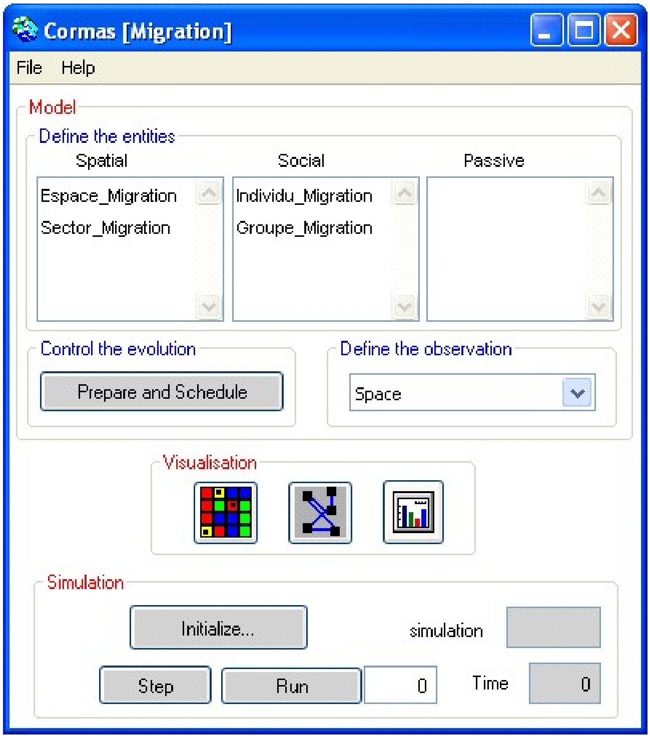

Fig. 3 The simulator interface: window 1

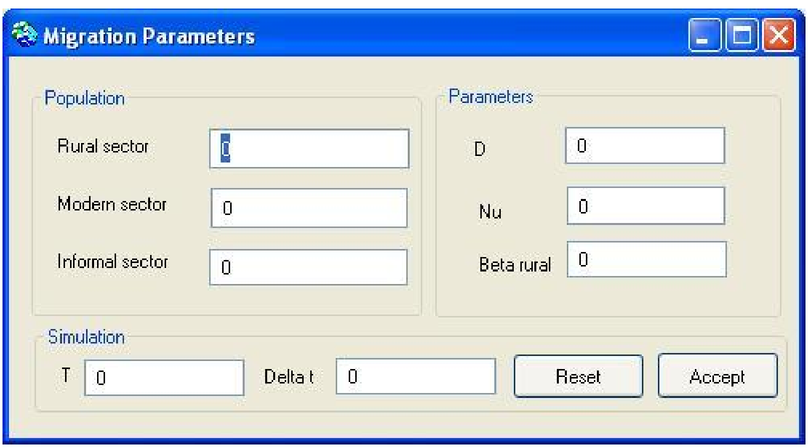

Fig. 4 The simulator interface: window 2

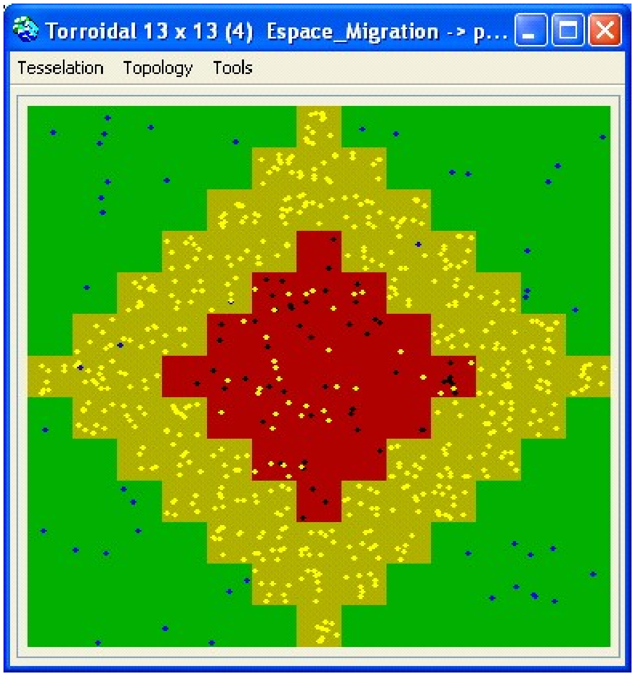

Fig. 5 The simulator interface: window 3. The rural sector is in green (its inhabitants are colored in blue), the urban informal sector is in yellow (its inhabitants are colored in yellow) and the urban formal sector is in red (its inhabitants are in black).

The simulator permits to undertake simulations for a large range of parameters. It has also been performed in order to visualize and quantify the evolution of some migration indicators (rural sector size, formal sector size, informal sector size, employment rate, unemployment rate).

4 The Simulations

4.1 The Scenarios Tested

The migration agent-based model has been conceived to perform simulations for a large parametric range. Here in this paper, we focus on the more interesting case from the economic point of view: the case of a significative positive percentage urban-rural real income differential ($\Delta > 0$) and a positive rate of urban job creation that exceeds the natural rate of urban population growth ($\gamma > \beta_u$). Our aim in simulating this case is double goal: first, we attempt to quantify the effects of the two factors Δ and γ on the migration process and unemployment and, second, we explore the behavior of the model to check if the Todaro Paradox is emergent property of our model. For these purposes, we perform simulations for different magnitudes of the parameters Δ and γ: $\Delta = 0.5$ that is $Y_u = 1.5Y_r$, $\Delta = 1$ ($Y_u = 2Y_r$) and $\Delta = 2$ ($Y_u = 3Y_r$), $\gamma = 0.1$ illustrating a high rate of urban job creation, $\gamma = 0.05$ illustrating an average urban job creation rate and $\gamma = 0.03$ an urban job creation rate proche to the natural urban labor growth β_u. These scenarios are tested for the following fixed initial condition $N_r = 5000$, $N_i = 500$, $N_f = 100$ and $\beta_u = 0.02$ a realistic value for labor natural growth rate in less developed countries. We point out that 10 runs of the simulation have been performed for each scenario to damp out randomness embodied in the simulator. Hence, each simulation result presented in the next subsection is an average of 10 repetitions.

4.2 Simulations Results

Simulations in Fig. 7 and Fig. 8 show the decrease of the initial unemployment rate and its convergence to 0. They also show the higher the rate of urban job creation, the faster the convergence. For instance, when $\gamma = 0.1$, unemployment rate reaches the equilibrium 0 after only 50 simulation steps while for $\gamma = 0.05$, the value 0 is reached after 100 simulation steps and later when $\gamma = 0.03$. Fig. 6 shows the final sectors situation when $\Delta = 1$ and $\gamma = 0.05$, it illustrates the fact that for the combination ($\Delta > 0$ and $\gamma > \beta$), the formal sector, in long run, absorbs all the labor force surplus in the economic system. Fig. 8 shows clearly the effects of the urban-rural real income

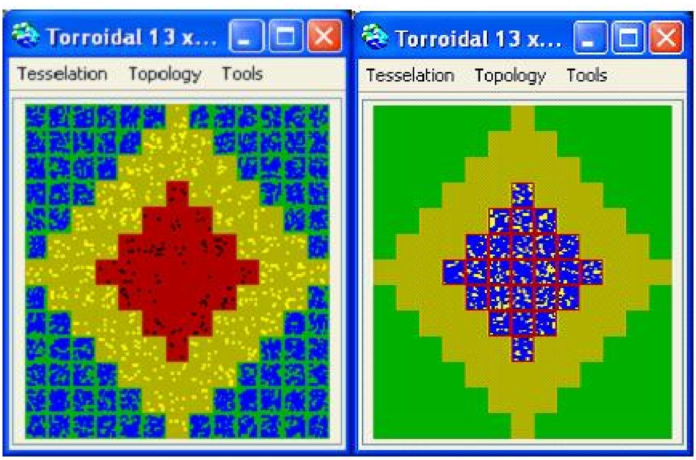

Fig. 6 Initial situation in the three sectors when $\Delta = 1$, $\gamma = 0.05$ and $N_i = 1000$ (on the left). The three sectors situation after $T = 200\Delta t$ (on the right).

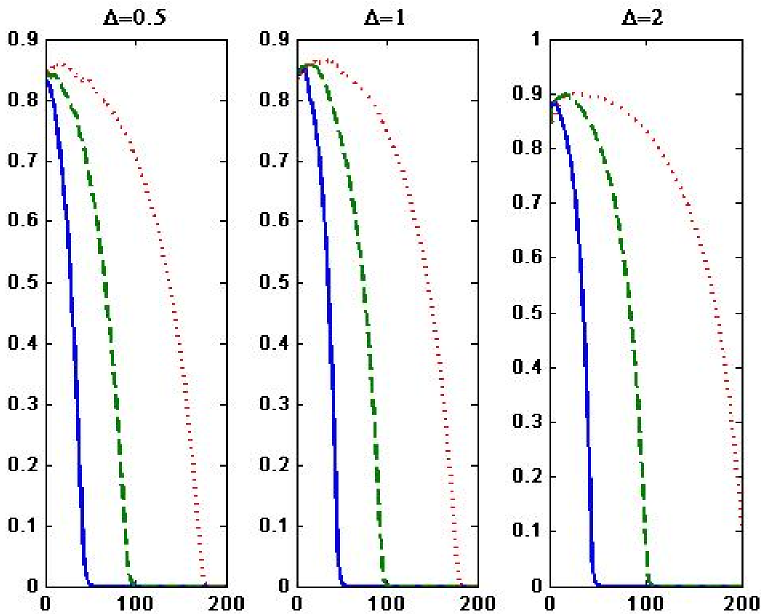

Fig. 7 Unemployment rate evolution in time for different values of γ: $\gamma = 0.1$ (solid line); $\gamma = 0.05$ (dashed line) and $\gamma = 0.03$ (dotted line).

differential Δ on the migration process, namely, an increase in Δ increases unemployment rate before converging to 0 and slows the convergence process towards equilibrium. These effects are better observed when Δ is significative and γ small. Fig. 9 shows the transitional dynamics and the long run equilibrium of the system through the three sectors evolution. Indeed, graphics in Fig. 9 show that for a positive urban-real income differential and a positive rate of urban job creation exceeding natural urban growth rate, there is a net migration towards urban sector. This migration is represented by a decline in the rural sector population and an increase in the informal sector population. It is clearly shown that the rate of decrease (respectively increase) of the rural sector size (respectively the informal sector size) is directly related to both Δ and γ. Graphics in Fig. 9 also show that in a first stage the formal sector grows but slower than the informal sector, this is due to migration which makes the urban labor force growing faster than job creation. This results in lower probabilities of migration and hence a decrease of the migration rate in the next stage. Indeed, we can observe after a certain time, the equalization of the formal and informal sectors sizes followed by a decrease in the size of the informal sector till extinction and a continuous growth of the formal sector size until it stabilizes. It is also observed that the formal sector size at equilibrium reaches a higher value and takes more time for such outcome for a higher urban-rural income and a smaller rate of urban job creation. Fig. 10

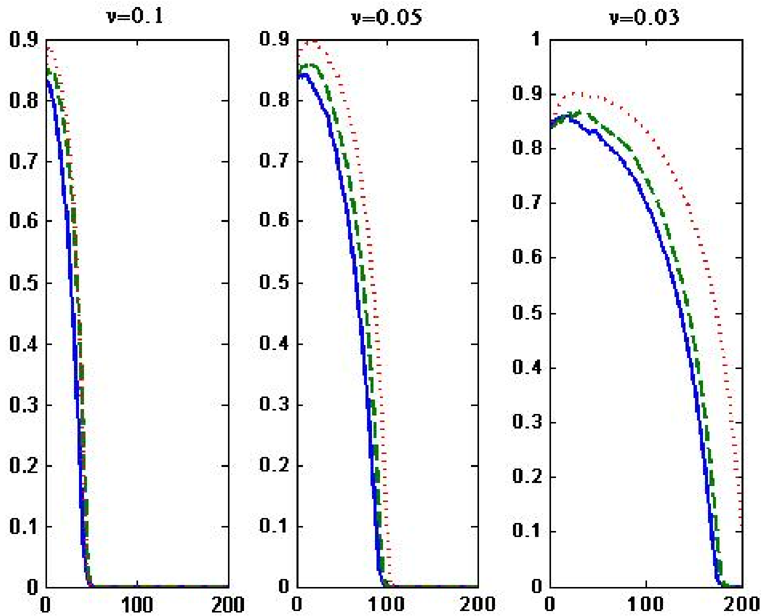

Fig. 8 Unemployment rate evolution in time for different values of $\varDelta$: $\varDelta = 0.5$ (solid line), $\varDelta = 1$ (dashed line) and $\varDelta = 2$ (dotted line).

illustrates the case of a positive percentage urban-rural real income differential $\varDelta$ with an urban job creation rate positive but less than the natural rate of growth in the urban labor force ($\gamma < \beta_u$). It is shown in this case that the unemployment rate increases progressively to converge to 1 (see (a) in Fig. 10). The graphic of the rural population size evolution shows at first a decrease in the population due to the rural-urban migration but after a certain time, migration stops and the rural population size stabilizes. Notice the large size of the informal sector in this case and the inability of the formal sector in absorbing the large labor force accumulated in the informal sector.

4.3 Discussion

The simulation of the migration agent-based model shows that a positive percentage urban-rural real income differential ($\varDelta > 0$) and a rate of urban job creation exceeding the natural rate of urban population growth ($\gamma > \beta_u$) lead the economic system to an equilibrium characterized by a null unemployment rate (full employment). Indeed, simulations show that in the first stage of the urban development, the combination ($\varDelta > 0$ and $\gamma > \beta_u$)

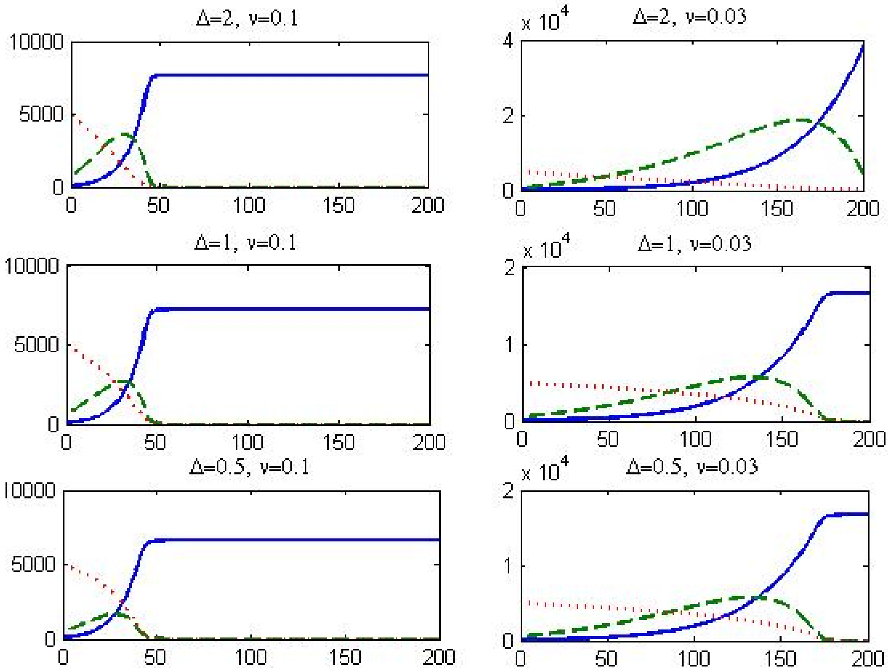

Fig. 9 Sectors evolution in time: urban formal sector (solid line); urban informal sector (dashed line) and rural urban sector (dotted line).

induces rural-urban migration such that the urban labor force grows at a faster rate than that of job creation, that is $\beta_u + \Delta\frac{\gamma N_f(t)}{N_i(t)} > \gamma$. This growth of labor supply results in an increase in the size of the informal sector with the consequence that the probability of finding a job in the next period is lower. Till now, our simulations results agree with the intuitive explanation of Todaro presented in section 2. But differently to Todaro intuition, simulations show clearly that the lower probability of obtaining an urban job slows down the migration rate and hence the rate of urban labor force such that from a certain threshold, the rate of urban job creation will irreversibly exceed the urban labor force ($\gamma > \beta_u + D\frac{\gamma N_f(t)}{N_i(t)}$) leading at long run to an equilibrium in which the formal sector absorbs all the urban labor force of the informal sector. But as soon as $\gamma < \beta_u$, the resulting equilibrium is unemployment 1.

On another hand, simulations have permitted to test implications of accelerated industrial growth and alternative rural-urban real income differentials. It has been shown that a high urban job creation rate has the effect of accelerating the convergence of the system to the full employment equilibrium while a high percentage urban-rural real income differential, through inciting to migration, has the opposite effect.

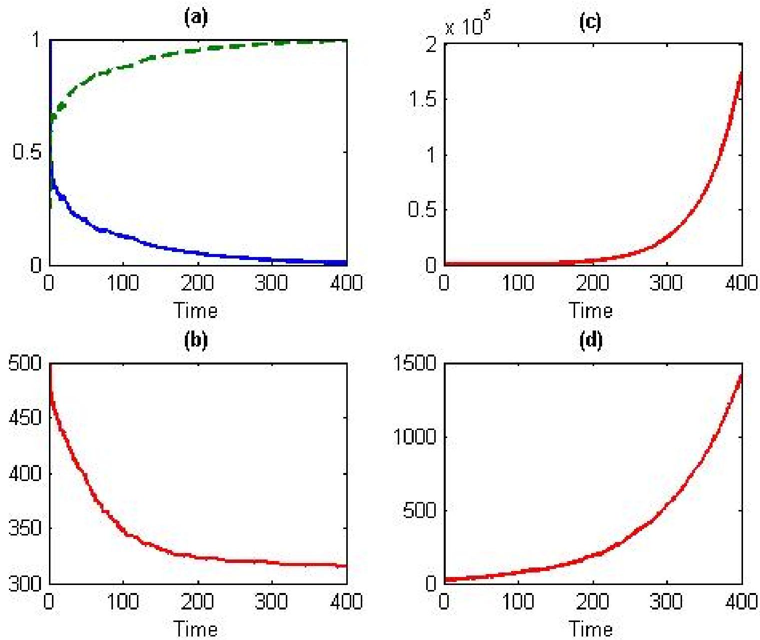

Fig. 10 (a) Employment (solid line) and unemployment (dashed line) rates evolution in time for $\Delta = 1$, $\gamma = 0.01$, $\beta_u = 0.02$, $N_r = 500$, $N_f = 20$, $N_i = 20$, (b) the rural sector evolution, (c) the urban informal sector evolution, (d) the urban formal sector evolution.

5 Conclusion

In this paper, we have conceived and implemented an agent-based model to study rural-urban labor migration in developing countries. This agent-based model has been derived from the analytic model of Todaro described by ordinary differential equations representing the links between rural-urban migration and urban unemployment. The agent-based model permits the analysis and visualization of the migration process for a large parametric range. It is also performed in order to give many quantitative results on equilibrium sectors sizes, equilibrium employment and unemployment rates.

Our main result in this paper is that the Todaro Paradox does not hold in the long run. Indeed, simulations results stemmed from our agent-based model show that as long as the urban job creation rate exceeds the natural urban labor force growth, the economic system converges to an equilibrium characterized by an unemployment rate 0 although the urban-rural real income differential is high and induces too much migration. Even if this finding is in contrast with the classical result of Todaro where inter-labor market (rural-urban) equilibrium mandates urban unemployment, it sustains many works (see for instance [7], [12]) reporting that Todaro model predicts too high an employment rate in less developed countries.

Acknowledgements. The authors would like to thank the anonymous referees for their pertinent and useful comments.

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