Cellular Automata Model for Size Segregation of Particles

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Abstract. This paper deals with the study of size segregation of particles where the size difference causes characteristic movement of particles inside granular media according to the induced vibration. In this study, segregation of particles due to the difference in size is simulated using Cellular Automata. A connected lattice automaton is introduced in the model, so that the variation of particle sizes as well as geometrical arrangement between particles can be represented. The Cellular Automata model can produce various characteristics which are observed in the actual granular systems. Furthermore, it is known from both numerical and experimental observations that the segregation progress is dependent on the amplitude of excitation as well as the particle size ratio.

1 Introduction

A mixture of particles whose diameter or the density are different, when filled in a container and exposed to a certain excitation, may exhibit segregation patterns where particles are eccentrically stabilized at positions according to each physical property. Among several types the size segregation is well known, where the smaller particles pass through voids formed between larger particles and eventually the latter rises upward to a free surface. Vibratory excitation increases the unfilled space, which then reduces friction between particles and accelerates their mobility. Consequently, particles with different properties are inhomogeneously located in space and form various types of stable segregation pattern under specific fluidization process. However, such pattern formation may cause adverse effect on product quality due to the non-uniformity in the course of mixture process. Segregation as a whole is thought to be undesirable phenomena in industries and a number of studies have been conducted to understand the mechanisms and to control processes.

Hayashi et. al. [1] and Ikeda et. al.[2] investigated experimentally the vibration induced segregation process of granular systems by testing various conditions, e. g., the way to supply granular materials and the amount of each size grain. They have shown that how the ratio of particle diameter affected the resultant segregation or the mixture state, and also revealed that the friction between container wall and particles causes particle motion be different in the vicinity

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of wall and in the midsection of the container, which then induces upward flow in the midsection and adverse flow in the peripheral region. In such a state, the peripheral smaller particles penetrate into the central relaxed region and consequently the segregation occurs. Knight[**?**] also pointed out that the convection flow intermediated by the friction of the wall is a significant factor for the issue, through similar experimental investigations.

Numerical analyses are also performed in order to understand the granular related events both from the analytical and phenomenological point of view[4]-[11]. Rather than the continuum treatment of granular systems, the discrete element method (DEM) is widely used in various studies focusing on each particle behavior for understanding the gross phenomena. DEM is built on the basis of Newtonian dynamics, where the constitutive granular system behavior is built up by respective particle motion which is modeled by a set of equation of motion. Studies include vibratory conveyance system of grains[4], Modeling of fluidized beds[5], inner convective flow dynamics[6], Vibration induced segregation model[7], etc. Taguchi[6] modeled the dynamics of vibrated bed consisting of homogenous spherical particles using DEM and simulated the typical characteristics in granular systems such as convection and heaping at the central portion of the media. He pointed out that convection is sensitive to the numerical friction coefficient setting so that the flow directions become consistent with actual observations. It should also be addressed that the computational load becomes indispensable as the number of governing equations as well as the number of particles increases. However, the time step should be kept moderately small in order to prevent the numerical instability.

Challenges have been made for modeling granular related problems using Cellular Automata and are successful by defining relatively simple rules [8]-[11]. Sakaguchi et. al.[8] introduced a lattice-connected automaton model where the single particle is associated with multiple lattice nodes, which thus lead to the alleviation of lattice constraint and the variation in movement, the particle size, the interaction between particles can be enhanced. They have also simulated the oscillating granular bed and succeeded capturing typical aspects in granular systems as mentioned above, however, the particle behavior is not explicitly associated with collision and friction properties.

In this paper, the Cellular Automata model is developed for simulating the size segregation of granular system. The model consists of a mixture of two different size particles filled in a container, where the entire system is shaken by continuous tapping excitations. Based on the lattice-connected automaton model, the local interaction rule is defined physically. Experiments are also performed for the system equivalent to the model, where the effect of the excitation amplitude and frequency to the segregation time is investigated. The Cellular Automata model can produce various characteristics which are observed in the actual granular systems. Furthermore, it is known from both numerical and experimental observations that the segregation progress is dependent on the amplitude of excitation as well as the particle size ratio.

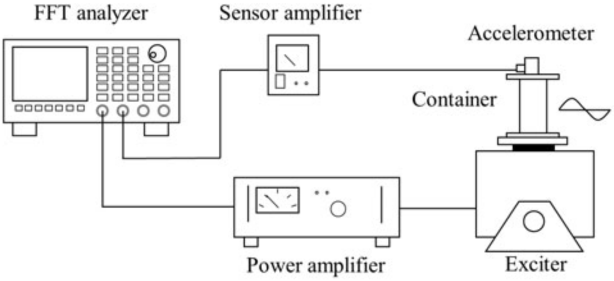


Fig. 1. Experimental setup of size-segregation of grains

2 Experiment

To capture the segregation dynamics and also to verify the Cellular Automata model as will be mentioned in the latter section, experiments are performed for granular materials consisting of two particles which are different in diameter. In this section, the overview of the experimental setup and the results are shown.

2.1 Experimental Setup

Size segregation is examined by an experimental setup shown in Fig. 1. The system consists of vertically reciprocating exciter on which a cylindrical plastic container is attached, the data analyzer, acceleration pickup, and the amplifiers. The cylinder (inner diam.: 50 [mm], height: 150 [mm]) incorporates two types of granular material made of glass beads. Two diameters are chosen out of five types, 0.6, 0.8, 1.0, 5.0 and 6.0 [mm], and five types of mixture are tested, as shown in table 1. Prior to the shaking tests, the larger particles are arranged in a layer at the bottom of the container before proceeding to pack the rest 80 [g] of smaller particles. Since the preliminary test has shown that the segregation is vulnerable to the moisture, grains are kept 24 hours inside a container with constant humidity of 80 %. Throughout experiments, the acceleration amplitude is kept 50 [m/s^2] and the plastic container is harmonically excited by the shaker within the frequency range of 20 to 200 [Hz]. In each frequency, the time required for the first larger particle to appear in a free surface is measured.

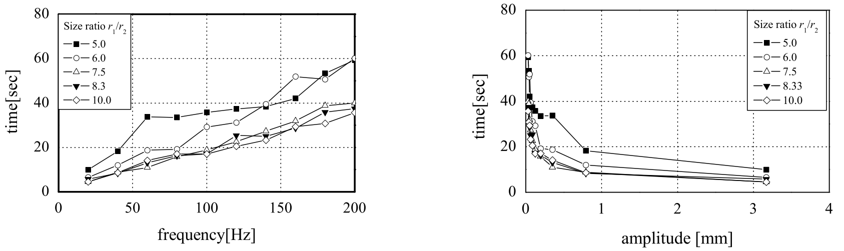
Table 1. Combination of particles

Diameter(larger)	Diameter(smaller)	Size ratio
5mm	1mm	5.0
6mm	1mm	6.0
6mm	0.8mm	7.5
5mm	0.6mm	8.33
6mm	0.6mm	10.0

2.2 Results

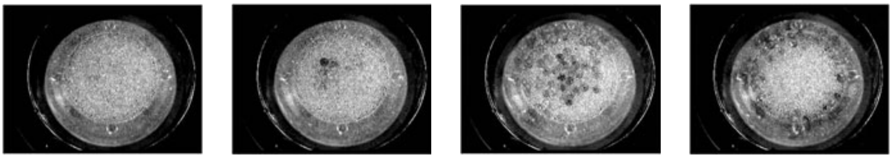
The measurement result of segregation time is shown in Fig. 2. In Fig. 2(a), the segregation time is plotted against excitation frequency. The segregation time is found to become large as the excitation frequency increases. It is also seen that the time become smaller with the increase of the particle size ratio. The expression in Fig. 2(a) is converted into the time against vibration amplitude as shown in Fig. 2(b), which signifies that the time become shorter with increase in excitation amplitude. It is known from these results that the excitation amplitude rather than frequency largely affects the vibration-induced segregation process.

In the course of excitation, photos are taken from above the container as shown in Fig. 3, for the case the size ratio of 5 and the excitation frequency of 100 [Hz]. The larger particles segregate at the free surface in the middle of the container and then move toward the peripheral wall. They remain in the vicinity of wall without sinking into the media. The upwelling of the larger particles at center occurs along with the smaller particles, however, the smaller particles plunge near the wall according to the convection flow. Fig. 4 explains schematically the segregation process mentioned above. These features are largely supported by the past studies[1]-[3] and are commonly observed for other combinations of diameters and excitations.



(a) Segregation time v.s. frequency. (b) Segregation time v.s. amplitude.

Fig. 2. Segregation time observed in experiment



(a) Initial state. (b) After 35 sec. (c) After 55 sec. (d) After 75 sec.

Fig. 3. Segregation process of larger particles

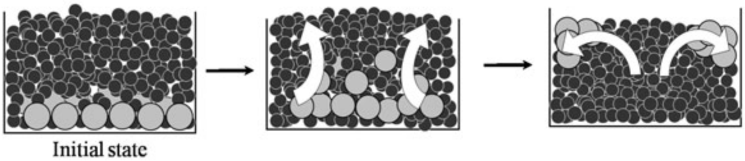


Fig. 4. Schematic of segregation process

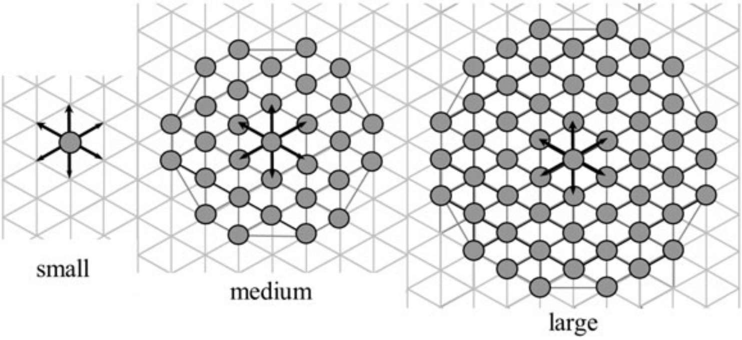


Fig. 5. Space segmentation and particle definitio

3 Cellular Automata Model of Segregation of Particles

3.1 Development of a CA Model

CA model is developed for a two-dimensional space consisting of triangular lattices. State of each lattice site is defined by three types, a segment of wall, particle and void. Additionally, apparent velocity varying within six directions is defined for particle lattices. In the present CA model, three different size particles are constituted as shown in Fig. 5. The smallest particle corresponds to single lattice, whereas the medium and the largest particles consist of 31 and 55 connected lattices, respectively. It is therefore possible to express the particle size variation and voids between particles[8]. Note that the particles are supposed to be spherical in these expressions.

3.2 Description of the Local Neighbor Rules

Three different sets of local interaction rules representing the temporal evolution are defined for each size particle. Each set consists of particle movement, collision between particles, and also collision between particle and wall.

i) Rule set for the small particle: Particle should have the vector and move toward one of the lattice direction including stationary state, as shown in Fig. 6. If the multiple particles simultaneously move into the same site, one of them is selected by equal probability with physically consistent velocity. In the case

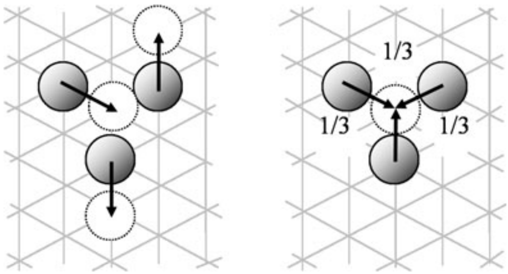


Fig. 6. Examples of rule for small particle movement, without and with conflict

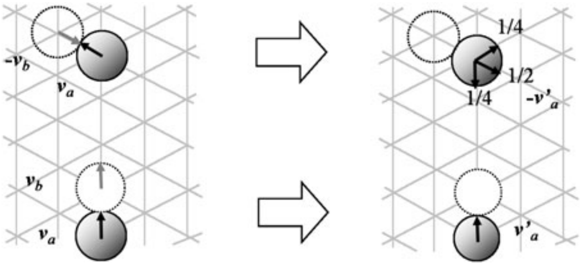


Fig. 7. Examples of rule for small particle collision

of three-body collision where the three particles are moving into the same site, equal probabilities are given for respective particles. The particles basically keep the present moving direction without collisions, however, the upward-moving particles change their moving directions to downward according to gravity if their velocity reach zero. In the present simulation model, collisions are limited to two-body cases and are evaluated between a focused and its adjacent particles. The velocity of the focused particle is changed for the colliding case. Fig. 7 shows an example of collision between small particles. In order to alleviate the lattice orientation on the particle movement after collision, the particle rebounds to the opposite direction with 1/2 probability, and 1/4 for the either oblique directions. ii) Rule set for the medium particles: Since a particle is expressed by the multiple connected lattices, the variation of relative positions and the examination of collisions become more complex than the smaller particle case, however, the basic ideas for the change in velocity and the treatment of collisions are roughly the same. A couple of examples for the movement and collision of medium particles are shown in Figs. 8 and 9. The particle can move only for the case if the multiple adjacent lattices are simultaneously vacant. In the case of collision where the expected rebound direction does not coincide with the lattice orientation as shown in Fig. 9, two adjacent directions are chosen with equal probabilities, 1/2. The change of the particle velocity after collision is determined according to the mass ratio.

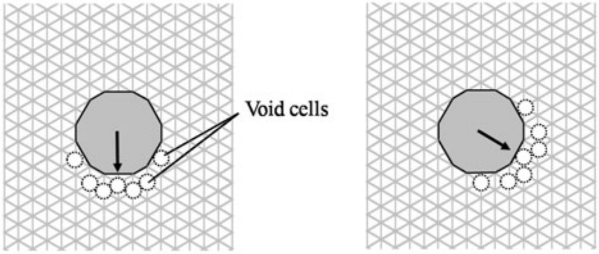


Fig. 8. Examples of rule for medium particle movement, collision between particles

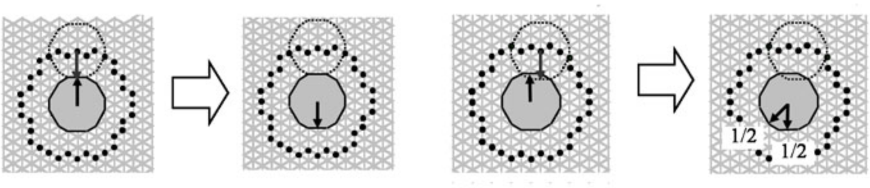


Fig. 9. Examples of rule for medium particle collision

iii) Rule set for the large particles: The rules for the large particles are basically identical to those for the medium particles, although the constitution of the reference neighbor on examining movement and collision with other is different.

3.3 Expression for the Container Excitation in Discrete System

Although the difficulty of representing continuously-changing excitation amplitude in discrete system, it is already known from the experimental observation that the segregation of particles is largely dependent on excitation amplitude rather than the excitation frequency. The vibration of the container is therefore replaced by the lift of entire granular materials at arbitrary height and the particles are let dropped freely according to gravity. Repeating this cycle apparently corresponds to steady excitation.

4 Simulation Results

Preliminary tests are first performed in order to verify the present lattice-connected Cellular Automata model for particle expression, which are followed by two cases of simulation where the particle combination is varied. Segregations are demonstrated for the granular materials consisting of small and the medium particles, and also of small and the large particles. Before starting to excite the granular media, the particles are poured randomly from above the container and are deposited inside, left over time until the count of falling particle becomes the predetermined number and the media reaches the equilibrium state. The lift of the granular media is varied from 1 to 5 grids imitating the container excitation,

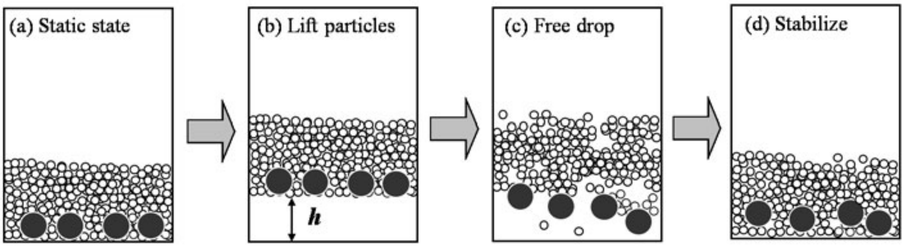


Fig. 10. Modeling of excitation process

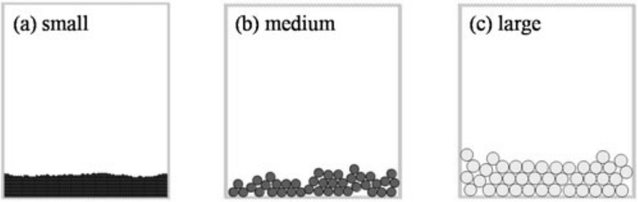


Fig. 11. Accumulation of homogeneous grains, (a) small, (b) medium, (c) large

and the results are compared for different excitation amplitude by measuring the simulation cycle until the segregation ends.

4.1 The Pile of the Homogenous Particles

Simulation results are shown in Fig.11 for respective particles with homogeneous diameter. The size of the container is given as 100 grids in both height and width. Totally 5000 particles are randomly dispersed from the opening of the container at every time step in the case of smaller particles, whereas 100 in total are dispersed for the medium and the large particles. Since the size of smaller particle corresponds to single grid, it is seen that the overall granular media is closely packed, on the other hand, in the case of medium and large particles the media is partly packed and also voids between particles are appeared. The Cellular Automata model well represents the basic profiles that are seen in the actual system related to granular materials.

4.2 Segregation of Medium Particles in Media Consisting of Smaller Particles

The segregation of medium particles submerged in a media consisting of smaller particles is simulated. Preliminary test is first conducted to verify the alternative expression of the container excitation described in Fig. 10. The system incorporates 2500 of small and 40 of medium particles, and the container is excited by lifting the gross media at $h = 4$ grids per cycle. From the results shown in Fig. 12,

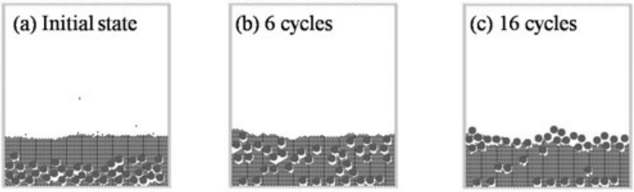


Fig. 12. Simulation result of small-medium particle segregation (2500 small, 40 medium particles)

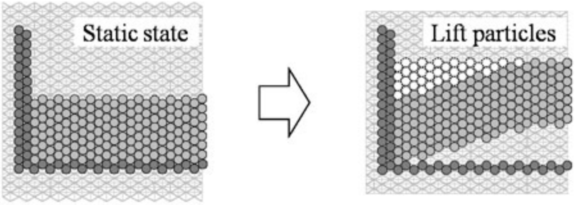


Fig. 13. The effect of friction on wall cells when lifting

the medium particles are divided into two groups, the one that segregates to the free surface, and the other remains at the bottom. The former is explained by the consequence of the insistent submerging of the smaller particles, and the latter instead is thought to be the failure of submerging due to a certain equilibrium condition between the mid and the smaller particles on lifting process. These results conform to the investigations reported by Sakaguchi, et. al.[8], however, as observed in the experiments all the larger particles segregated to the surface. Such a difference may arise from the friction between particles and the container wall, therefore, the effect is introduced in a simple manner such that the lift of the granular media is varied across the width as shown in Fig. 13 based on the experimental observation and the discussion in reference[1], that the particles in the central portion of container are less affected by the internal friction and are easy to move, whereas not for sediment adjacent to wall.

In the following simulation, the effect of the wall friction is introduced and the calculation is performed for 200 cycles. The size of the container is changed to 60 grids in height and 55 grids in width, and also the numbers of small and medium particles are 800 and 4, respectively. The four medium particles are initially located at the bottom and the maximum lift of the media is set to $h = 3$ grids at the center of the container. By 40 cycles of calculation, all of the four medium size particles emerged to the surface as shown in Fig. 14. Additionally, as observed in experiments a pile consisting of small particles is formed in the central part of the media and the segregated medium particles subsequently move toward the wall. The similar results are obtained for the combination of small and large particles as shown in Fig. 15, yet the segregation time is shortened for the larger diameter ratio. The segregation time is summarized and plotted

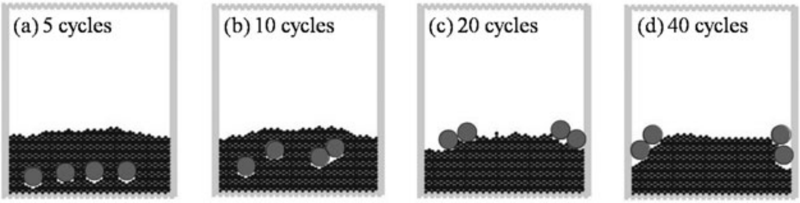


Fig. 14. Simulation result of small-medium particle segregation

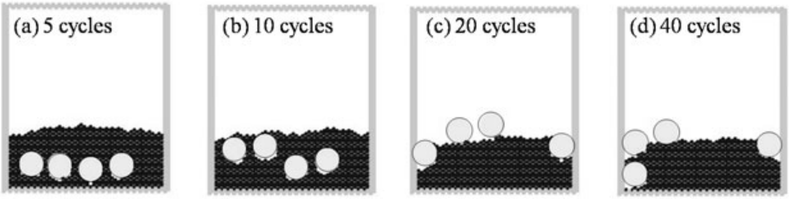


Fig. 15. Simulation result of small-large particle segregation

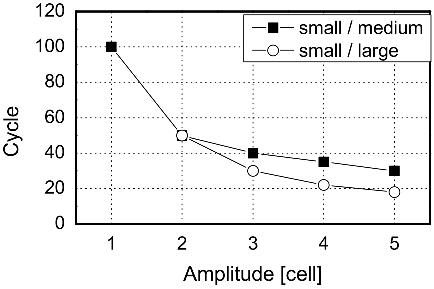


Fig. 16. Segregation time obtained by simulation

against excitation amplitude through all investigations as denoted in Fig. 16. The amount of time required for the completion of segregation becomes shorter as the excitation amplitude and the size ratio of particle increase.

5 Conclusions

In the present study, the segregation of particles where the difference in size causes characteristic movement of particles inside the granular media by the induced vibration is modeled and simulated using Cellular Automata, and the results are also compared with experimental investigations. A connected lattice automaton is introduced in the model, so that the variation of particle sizes as well as geometrical arrangement between particles can be represented. The present Cellular Automata model can produce various characteristics such as segregation, the heaping and convective pattern of particles which are observed in the actual granular systems. Furthermore, it is known from both numerical and experimental observations that the segregation progress is dependent on the amplitude of excitation as well as the particle size ratio.

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