

# A Proximal Space Approach for Embedding Urban Geography into CA Models

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**Abstract.** In the great majority of urban models based on Cellular Automata (CA), the concept of proximity is assumed to reflect two fundamental sources of spatial interaction: (1) the accessibility of places and (2) the distance “as the crow flies”. While the geographical space defined by the latter clearly has an Euclidean representation, the former, based on the accessibility, does not admit such a regular representation. Very little operational efforts have been undertaken in CA-based urban modelling to investigate and provide a more coherent and cogent treatment of such irregular geometries, which indeed are essential and crucial feature of urban geography. In this paper, we suggest an operational approach – entirely based on cellular automata techniques – to model the complex topology of proximities arising from urban geography, and to entangle such proximity topology with a CA model of spatial interactions.

**Keywords:** urban cellular automata, land-use dynamics, proximal space, irregular neighbourhood, informational signal propagation, informational field.

## 1 Introduction

The idea of spatial interaction in CAs is strongly related to that of proximity. Indeed, a CA transition rule is always, by definition, a function describing the relation between a cell and its neighbouring, *viz.* proximal cells. And the nature of proximity depends fundamentally on the kind of geometry we are using to describe the underlying geographical space.

When modelling urban dynamics based on spatial interactions, we are always implicitly or explicitly making assumptions on the nature of proximities within an urban geography, and are hence seeking for its suitable geometrical representation.

On that account, we can start by saying that in the great majority of CA urban models, the “proximities” are assumed to reflect two fundamental sources of spatial interaction: (1) the accessibility and (2) the distance “as the crow flies”. The geographical space defined by the latter kind of proximity has clearly an Euclidean representation, and thus the proximity may be defined in a form of a *regular* spatial distance, for example as a special case of the Minkowski distance.

However, in the case of the former kind of proximity, the one based on the so called accessibility, the situation is profoundly different. The accessibility refers here to a

*measure* of how (how much, how easy, how quickly) places are mutually accessible (i.e. reachable) from one another to human beings. Such accessibility may therefore be further subdivided by the means of transportation (pedestrian, bicycle, automobile, heavy vehicle, railways and so on) as they all give rise to different accessibilities of places. Anyhow, in all these cases, the accessibility itself is deeply determined by the relevant underlying urban geography. For example, the web of pedestrian, road, railways and underground transportation networks substantially shape such geography, bringing about a highly *irregular* geometry of the accessibility-type proximity. This type of proximity manifestly does not admit any possibility of a regular representation, let alone Euclidean. Indeed, an Euclidean representation of the geometry of the accessibility-type would be to a large extent inappropriate and fundamentally flawed.

Considering these rather straightforward observations, it is remarkably surprising that very little operational efforts (e.g. [1]) have been undertaken in CA-based urban modelling to investigate and provide a more coherent and cogent treatment of such irregular geometries, which indeed are essential and crucial feature of urban geography.

The lack of treatment of this feature is for instance easily seen in two families of urban CA models which in derived, extended, specialised or inspired-by forms have been often used for CA-based urban simulation: the so called Constrained Cellular Automata (CCA) [2-4] and those based on the SLEUTH approach [5-7]. In both these two families of models, the CA does not adopt a strictly local neighbourhoods, and therefore does indeed simulate spatial interactions over greater distances, but the distance is intended exclusively in the Euclidean as-the-crow-flies sense. The same general approach is described in the attempts to comprehensively present and discuss the theory and application of urban CA (see for example [8,9]).

Being such landscape of CA applications to urban phenomena as it is, there have been, to be fair, invitations from theoretical standpoints to develop a more appropriate understanding of the concept of nearness, to give it a deeper geographical meaning, in a way, to entail it with a thick geographical theory. Indeed, such a line of reasoning may almost directly be derived from the notion of *proximal* space, coming from the research in ‘cellular geography’ [10] which set the basis for the so called *geo-algebra* approach proposed by Takeyama and Coucleis [11]. In this latter paper, the homogeneity of cells’ neighbourhoods has been questioned precisely on the ground that every cell may have different neighbourhood defined by relations of “nearness” between spatial entities, where “nearness” can mean both topological relation or generic “behavioural” (e.g. functional) influence.

In this paper, we take on the task to suggest an operational approach – entirely based on cellular automata techniques – to model the complex topology of proximities arising from urban geography, and to entangle these and such proximity topology with a CA model of spatial interactions.

## 2 Proximal Spaces as Informational Fields

The approach we take to describe the irregular geometry of the accessibility-type proximity is to assume that each cell emits an informational signal propagating throughout

the cellular space. However, the signal propagation is not uniform, but depends on the “propagation medium” which the signal encounters. This means that, starting from the emitting cell, the signal is diffused in all directions, but the decay of the signal’s intensity depends on the state (e.g. land use) of the cells crossed by the signal. As a consequence of this informational signals emission, each cell generates an *informational field* around itself, whose shape and intensity at every cell of the cellular space depends on the states of the cells along all the paths the signal propagates.

To see how these general concepts may relate and be applied to urban context, we can for example think of a model in which the above described signals propagates better (i.e. with a lower rate of decay) along the roads, and that they easily spill over to the cells surrounding the roads. Another example could be a railway transportation network. Here, the signal would propagate smoothly along the railway, but would not by model design be allowed to spill over to the surrounding cells, except starting from the cells corresponding to railway stations.

To sum up, the beforehand suggested method allows us to generate an irregular geography-based “informational field” around each cell. In other words, seen from another point of view, every cell receives a set of signals of different type and intensity from other (potentially every other) cells. Once the informational fields are generated, the CA transition rules ought to be stated in a way to combine the received signals as the input information.

The hereby suggested strategy of modelling proximities by the means of information signals propagating through cellular space is similar in spirit to the “at-a-distance interaction fields” proposed in [12]. The specific contribution of our proposal should therefore be seen in its attempt to apply and embed these concepts into *urban* CAs and to conceive a particular operational simulation approach for that purpose.

### 3 An Application to a CA Model

The experimental setting to demonstrate and discuss the above ideas was a 2D CA composed of square cells providing a raster representation of a geographical area. The state of every cell represents its land-use type, which can be of one of the following eight types: *residential, industrial, commercial, agriculture, road, railways, railway station, public services/facilities*. The latter four types are considered as static and thus cannot change nor be transformed endogenously during the simulation. Starting from a given initial configuration, the automaton evolves in discrete steps simulating the land-use dynamics of the area.

At each simulation step, the execution of the CA model is divided into two distinct phases: (1) *informational fields generation phase* and (2) *land-use dynamics phase*.

#### 3.1 Informational Fields Generation Phase

This phase of the CA execution has the task to generate the informational fields (of the kind described in section 2) around each cell. Specifically, at the first step of this phase, the cells having residential, industrial, commercial, or public services land use

are made to emit an informational signal  $\sigma$ . Each signal holds and carries the following information: (1) the ID of the *source* cell, (2) the source cell's *land use*, (3) the *propagation rule*, and (4) the signal's intensity  $\bar{\sigma}$ . During the subsequent steps, every signal held by a cell is transmitted to its Moore-neighbouring cells, provided that the signal's intensity is above a predefined threshold.

Signals are subject to a decay of intensity defined by their propagation rule. In general, a propagation rule is expressed as a function of the land uses of both the sender and the receiver cell. The functioning of the propagation rules is therefore grounded on a land-use "in-out matrix". More specifically, this matrix defines the coefficients of decay of the informational signal on the basis of the land use combination of the signal's outgoing and incoming cells. An example of such an in-out matrix is shown in Table 1. The decay coefficients in this table reflect the observations on the accessibility signal propagation exemplified above in section 2.

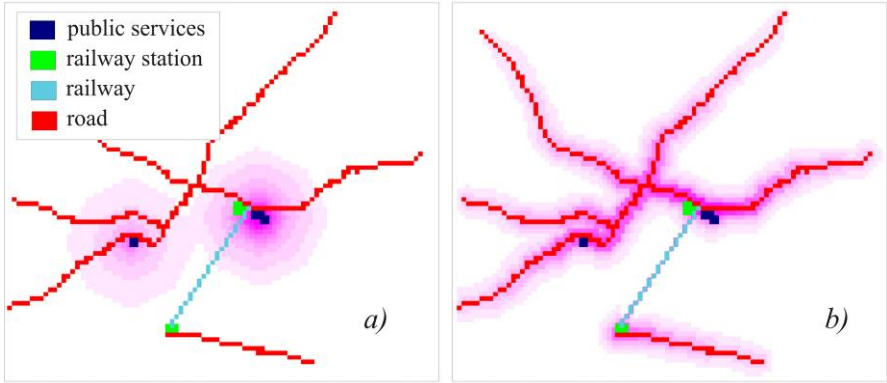
**Table 1.** Example of land-use "in-out matrix" of coefficients used for calculating the decay of a signal propagating from an outgoing to an incoming cell

Incoming cell	R	C	I	A	PS	Ro	Rw	RwS
Outgoing cell								
<b>R</b> (Residential)	0.60	0.60	0.60	0.10	0.60	0.90	0.00	0.95
<b>C</b> (Commercial)	0.60	0.60	0.60	0.10	0.60	0.90	0.00	0.95
<b>I</b> (Industrial)	0.60	0.60	0.60	0.10	0.60	0.90	0.00	0.95
<b>A</b> (Agricultural)	0.10	0.10	0.10	0.10	0.10	0.90	0.00	0.95
<b>PS</b> (Pub. services)	0.60	0.60	0.60	0.10	0.60	0.90	0.00	0.95
<b>Ro</b> (Road)	0.90	0.90	0.90	0.90	0.80	0.95	0.00	0.95
<b>Rw</b> (Railway)	0.00	0.00	0.00	0.00	0.00	0.00	0.98	0.95
<b>RwS</b> (Railway station)	0.95	0.95	0.95	0.90	0.95	0.95	0.98	0.95

To account for two relevant modes of spatial interaction discussed in the introduction (see section 1), two types of propagation rules are defined in the model:

- Regular, Euclidean-space propagation, by which the signal decay is a function of the Euclidean distance from the signal's source (and therefore does not depend on the land uses crossed by the signal);
- Irregular, Proximal-geography-space propagation, by which the decay of the signal's intensity differs depending on the land uses of the cells being crossed by the signal.

As an example, consider Fig. 1 where the intensity of the fields originated by some emitting cells is depicted in an urban-like environment characterised by the presence of a network of roads. In case (a) the signal decay does not account for the current cells' land uses: this type of propagation rule makes signals able to inform the receiving cell about the existence of the source cell and also about the level of their spatial distance in the Euclidean sense. In case (b) the signals' intensity propagates with smaller decay along the roads and railways: this propagation rule allows a receiving cell for being informed about the existence and the accessibility of the source cell.



**Fig. 1.** Comparison between the diffusion of two type of signals originating from the same cells. In case (a) signals decay does not account for the current cells' land uses; in case (b) signals' intensity propagate preferentially along cells representing roads and railways.

The informational fields generation phase ends when during a time step eventually no signal propagates further throughout the CA. This condition is satisfied when every cell has already sent all its signals and all the received signals are of the intensity below the predefined propagation threshold.

### 3.2 Land-Use Dynamics Phase

At the end of each informational fields generation phase, every cell holds a set  $\Sigma$  of signals  $\sigma$  which are used as the input information for the subsequent land-use dynamics phase.

This phase is based on the computation of the so-called *transition potentials*  $P_j \in [0, 1]$  expressing the propensity of the land to acquire the  $j$ -th land use. In [2-4], where the hereby employed concepts of transitional potentials have been developed, the cell neighbourhood is a circular region of a given radius around the cell. Therefore, we adapted the thereby presented rules to our circumstances of irregular neighbourhood patterns as drawn by informational fields. Hopefully, we succeeded in maintaining the spirit of the spatial interaction principles inherent in the original rules.

Adapting and somewhat simplifying from [2], the transition potentials of every cell in our model are computed as:

$$P_j = \begin{cases} S_j Z_j N_j & \text{if } N_j \geq 0 \\ (1 - S_j Z_j) N_j & \text{if } N_j < 0 \end{cases} \quad (1)$$

where:

- $S_j \in [0, 1]$  is the cell *physical suitability* taking into account, for each land use  $j$ , features like slope or terrain aspect.
- $Z_j$  is a Boolean value defining the exclusion of the  $j$ -th land use (for example due to zoning regulations or physical constraints)
- $N_j \in \mathbb{R}$  is the so called *neighbourhood effect*.

The latter represents the sum of all the relevant attractive and repulsive effects of land uses and land covers on the  $j$ -th land use which the current cell may assume. In the present model, and critically differently than in [2-4],  $N_j$  is computed using all the informational signals  $\Sigma$  received by the cell:

$$N_j = I_k \delta_{jk} + \sum_{\sigma \in \Sigma} f_{ij}(\sigma) \quad (2)$$

where:

- $i$  denotes the type of the signal  $\sigma$ ;
- $f_{ij}(\sigma) \in [-1, 1]$  is a function giving the influence of a signal  $\sigma$  if type  $i$  on the use  $j$  which the cell may assume;
- $\delta_{ik}$  is 1 if  $j = k$  and 0 if  $j \neq k$ , where  $k$  denotes the current land use of the cell for which the transition potential is under evaluation;
- the term  $I_k \in [0, 1]$  accounts for the effect of the cell on itself (zero-distance effect) and represents an inertia due to the costs of transformation from one land use to another

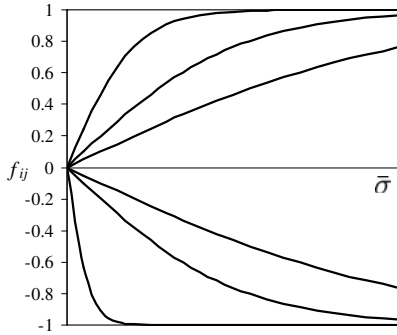
Functions  $f_{ij}(\sigma)$ , accounting for different effects of a received signal on all the potential land uses, are assumed in the following form:

$$f_{ij}(\sigma) = a_{ij} \frac{1 - e^{-s_{ij} \bar{\sigma}}}{1 + e^{-s_{ij} \bar{\sigma}}} \quad (3)$$

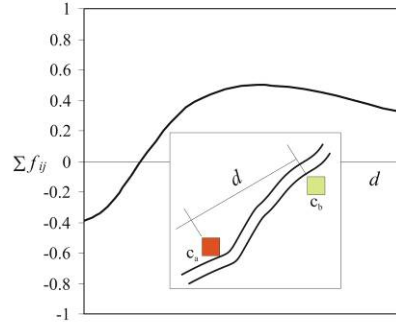
where:

- $\bar{\sigma}$  denotes the intensity of the signal  $\sigma$  of type  $i$ ;
- $a_{ij} \in [-1, 1]$  is a parameter representing the maximum influence (positive or negative) of a signal  $\sigma$  of type  $i$  on the use  $j$ ;
- $s_{ij} \in [0, 1]$  is a parameter defining the sensitivity of the use  $j$  on a signal of type  $i$ ;

Fig. 2 shows some examples of functions  $f_{ij}(\sigma)$  used in the model. It is important to note that, in spite of the simplicity of functions  $f_{ij}(\sigma)$ , the combination of all contributions given by Eq. (2), together with the different ways in which signals can propagate throughout the automaton, are able to effectively describe a variety of relevant situations. Consider for example the cell  $\mathbf{c}_a$  close to a road represented in Fig. 3 and suppose that, according to its current land use, it emits two signals, namely an Euclidean nearness signal  $\sigma_1$  and an accessibility signal  $\sigma_2$  (see also Fig. 1). Also consider a cell  $\mathbf{c}_b$  along the road, at a distance  $d$  from  $\mathbf{c}_a$ , receiving the two signals  $\sigma_1$  and  $\sigma_2$  emitted by  $\mathbf{c}_a$  and suppose that  $\sigma_1$  has a repulsive effect on a potential land use  $j$  of  $\mathbf{c}_b$  (i.e.  $a_{ij} < 0$  in Eq. 3) while  $\sigma_2$  has an attractive effect on the same use  $j$  (i.e.  $a_{ij} > 0$  in Eq. 3). The combination of the two effects, according to the model above described, leads to the transition potential contribution represented in Fig. 3 as a function of the distance  $d$ , which is characterised by an optimum distance (i.e. the position in which the contribution of  $\mathbf{c}_a$  to the potential towards the use  $j$  of  $\mathbf{c}_b$  is maximum) and a decay of the influence as  $d$  increases.



**Fig. 2.** Example graphs of the contribution to the transition potential given the signal intensity



**Fig. 3.** Total effect produced by the cell  $c_a$  on different cells  $c_b$  as a function of their distance  $d$

When all transition potential are computed, the following transition rule is applied to all cells of the automaton:

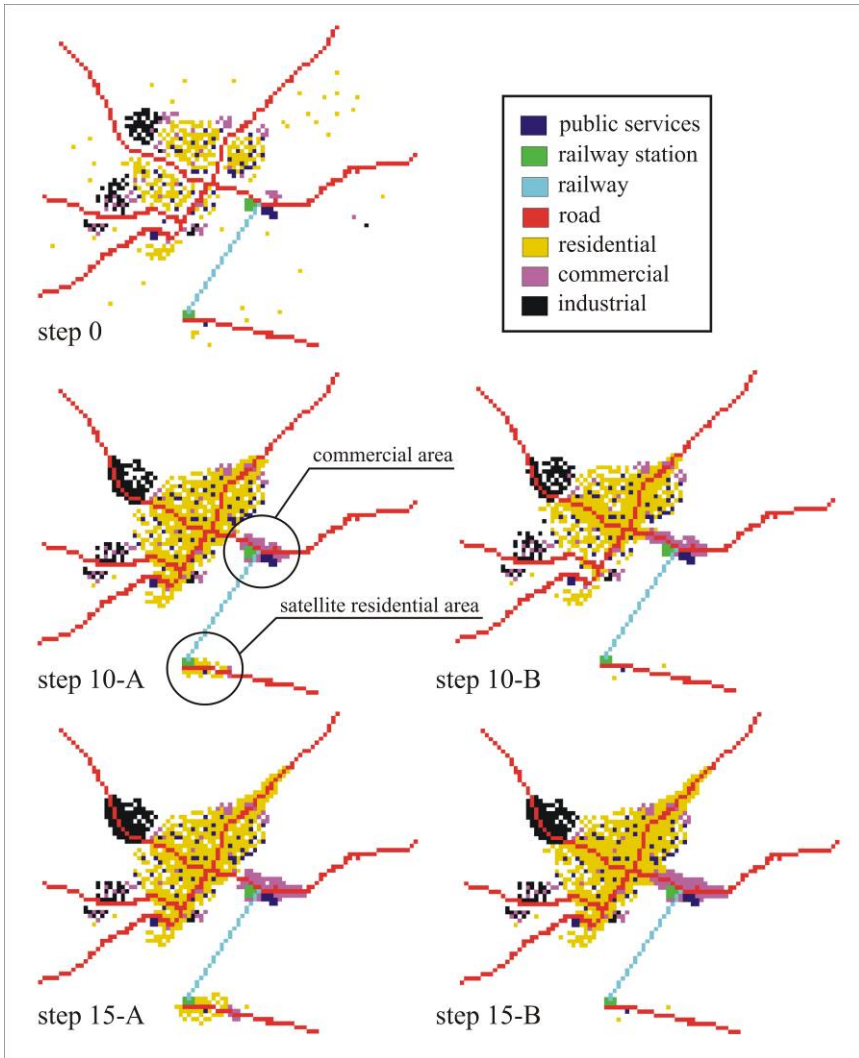
*Transition rule:* a cell of the land use  $k$  is transformed into the land use  $j$ , if  $P_j$  is the highest of all the cell's transitions potentials, and provided that  $P_j > \lambda$  and  $P_j - P_k > \epsilon$ .

$\lambda$  is a minimum threshold and  $\epsilon$  is a minimum difference threshold for a cell to be transformed into another land use. The threshold  $\epsilon$  incorporates the correction for age of the cell's land use  $k$ , namely the younger in terms of CA steps is the current land use, the greater is the threshold  $\epsilon$ .

The rationale behind this transition rule is straightforward. A cell has a transition potential to transform into every possible land use. Of all the possible land uses, it will transform into the one having the strongest transition potential, provided that it is strong in absolute terms (greater than  $\lambda$ ) and that it is strong enough to overturn the current land-use  $k$  (therefore it must hold  $P_j - P_k > \epsilon$ ). This latter threshold ( $\epsilon$ ) accounts therefore for the inherent inertia or sunk costs of urban transformations. The correction for age of the threshold  $\epsilon$  puts further consideration onto this inertia, by imposing that the required differential of transition potentials is greater had the cell just recently transformed into its current land use  $k$ . Subsequently, the threshold requirement  $\epsilon$  is lowered gradually as the age of the current land use grows.

## 4 Example Runs

In this section we show, through a preliminary simulation exercise, some typical effects of taking into account the accessibility-type proximity in a CA-based land use simulation.



**Fig. 4.** Comparison between the outcomes of two runs of the model with different models of the proximity (left: accessibility-type, right: as-the-crow-flies type)

In particular, two simulations were executed, using the model described in section 3 and assuming as initial configuration the map of a hypothetical city (“step 0” in Fig. 4.)

In the first simulation (labelled “A” in Fig. 4), the model included the computation of the relevant accessibility fields (e.g. signals of the accessibility of public services or commercial areas) while in the second run (labelled “B” in Fig. 4) only signal propagating according to their Euclidean distances were admitted. Clearly, in the second case each cell of the automaton was not informed about accessibilities, being only aware of both the existence and the distance of all the relevant land uses (roads and railways included, in the spirit of [2-4]) within a radius given by the level of isotropic decay of the signals (see Fig. 1-a).



Each simulation was executed for 15 land-use transformation steps which, including the information fields generation phases, corresponded to 321 CA steps for simulation “A” and 169 CA steps for simulation “B”.

In Fig. 4, the output maps obtained for steps 10 and 15 are depicted. In particular, as the city evolves according to the model rules, the growth of a commercial area (see the highlighting circle) can be observed at step 10 in both simulations. At the same land-use step, the map produced by simulation “A” exhibits the development of a satellite residential area in a zone (see the highlighting circle) which is relatively distant from the newly developed areas but well connected by the railway. In simulation “B” the same satellite zone, although served by the railway, remains undeveloped (i.e. in case of model “B” the cells are not informed about the existence of highly accessible public services and commercial areas of new development at the other end of the railway connection).

Comparing the outcomes at step 15, we can observe that both the commercial area and the satellite residential area had further development, but the latter continued to be completely missed by simulation B.

In spite of its simplicity, the results of the above presented preliminary example indicate that including an improved description of the proximities arising in an urban geography can lead to different, and possibly more realistic, urban patterns.

## 5 Conclusions

In this paper, our primary aim was to suggest a possible modelling of the notion of proximity and proximal space arising in urban geography, and to employ it in urban CA. The point of departure was the idea that the proximity usually held to be relevant for spatial interactions cannot be assumed to exist in and as a regular Euclidean space, since the “distortions” of urban geography bring about highly irregular and complex topology of proximities. To account for this complexity, we have thereafter suggested a description of the proximal space as a set of informational signals propagating through the space, hence generating informational fields around each cell. Every such field exhibits different strength (intensity) at different points (i.e. cells) in space, since the irregularity of its geometry is inherently dependant on the different “propagation media” (road, railways, residential, commercial or any other area) crossed by the informational signals.

A cell of the automaton is then able to “know” its proximity to another by knowing the intensity of the signal – carrying on with the wave metaphor, shall we say “the radiation”? – it receives from that cell. Finally, the combination of all the received radiations is the information input to the CA transition rules through which to model the land-use dynamics.

In the example presented, our focus was not so much on the plausibility and theoretical foundation of the transition rules and of the proximity-based land-use urban dynamics. Rather, it was a quite expedient (and probably somewhat coarse) exemplification of how even existing models based on assumptions of spatial interaction may in a reasonably convenient manner be adapted to employ our more generalised and more geography-grounded notion and description of the proximal space.

## References

1. Batty, M.: Distance in space syntax. CASA Working Papers (80). Centre for Advanced Spatial Analysis (UCL), London, UK (2004)
2. White, R., Engelen, G.: Cellular automata and fractal urban form: A cellular modeling approach to the evolution of urban land-use patterns. *Environment and Planning*, 1175–1199 (1993)
3. White, R., Engelen, G., Uljee, I.: The use of constrained cellular automata for high-resolution modelling of urban land use dynamics. *Environment and Planning B* 24, 323–343 (1997)
4. White, R., Engelen, G.: High-resolution integrated modelling of the spatial dynamics of urban and regional systems. *Computers, Environment and Urban Systems* 28(24), 383–400 (2000)
5. Clarke, K., Hoppen, S., Gaydos, L.: A self-modifying cellular automaton model of historical urbanization in the san francisco bay area. *Environment and Planning B* 24, 247–261 (1997)
6. Clarke, K.C., Gaydos, L.J.: Loose-coupling a cellular automaton model and GIS: long-term urban growth predictions for San Francisco and Baltimore. *International Journal of Geographic Information Science*, 699–714 (1998)
7. Project Gigalopolis, NCGIA (2003),  
<http://www.ncgia.ucsb.edu/projects/gig/>
8. Benenson, I., Torrens, P.M.: Geosimulation: object-based modeling of urban phenomena. *Computers, Environment and Urban Systems* 28(1-2), 1–8 (2004)
9. Torrens, P.M., Benenson, I.: Geographic Automata Systems. *International Journal of Geographical Information Science* 19(4), 385–412 (2005)
10. Tobler, W.: Cellular geography. In: Gale, S., Olsson, G. (eds.) *Philosophy in Geography*, pp. 379–386. Reidel, Dordrecht (1979)
11. Takeyama, M., Couclelis, H.: Map dynamics: integrating cellular automata and GIS through Geo-Algebra. *Intern. Journ. of Geogr. Inf. Science* 11, 73–91 (1997)
12. Bandini, S., Mauri, G., Vizzari, G.: Supporting Action-at-a-distance in Situated Cellular Agents. *Fundam. Inform.* 69(3), 251–271 (2006)