An Interval Type-2 Fuzzy Neural Network for Chaotic Time Series Prediction with Cross-Validation and Akaike Test

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Abstract. A novel homogeneous integration strategy of an interval type-2 fuzzy inference system (IT2FIS) with Takagi-Sugeno-Kang reasoning (TSK IT2FIS) is presented. This TSK IT2FIS is represented as an adaptive neural network (NN) with hybrid learning (IT2FNN:BP+RLS) in order to automatically generate an interval type-2 fuzzy logic system (TSK IT2FLS). Consequent parameters are updated with recursive least-square (RLS) algorithm; antecedent parameters with back-propagation (BP) algorithm. Mackey-Glass chaotic time series forecasting results are presented (τ=17, 30, 100) with different signal noise ratio (SNR). Soundness for uncertainty, adaptability and learning and generalization capabilities is shown using 10-fold Cross Validation, Akaike Information Criteria (AIC) and F-Test.

1 Introduction

System's modeling is considered one of the most important problems in science. Models are abstractions of reality that can be applied to improve our understanding of real world phenomena. Finding functional relations between variables that intervene in phenomena is a problem that arises in several areas like economy, engineering, biology, sociology and medicine. Approximation of unknown functions from experimental data, e.g. time series prediction, is particularly important. In this case a sequence of time sample sets is used to predict the system's behavior in short and long-term. Several types of uncertainty are originated by improper data, bad modeling or wrong interpretations given to models. Also, the nature of phenomena might generate fluctuating data or variables, given thus uncertain data. Human mistakes and bad equipment calibration can generate incorrect data [1, 2,6]. Imprecision is the cause of

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language granularity. Vagueness is considered a new imperfect information cathegory. Inconsistence is originated by data redundancy that generate conflict or contradictions. Uncertainty levels vary depending on context. This uncertainty can be caused by noise in data [3,4,47,48]. Type-1 (T1FLS) and interval type-2 (IT2FLS) fuzzy logic systems, combined with techniques like neural networks have shown their ability to solve different kinds of problems like control [10-12, 14, 15, 16, 27, 32, 42, 44], prediction [12, 17, 20], signal and images processing [12, 13, 18, 28, 29], industrial processes [21, 22], etc. In many cases, success was achieved due to adding human expert knowledge. There is consensus between researchers that more intelligent systems can be developed by hybrid soft computing methodologies [41,45]. Type-1 Fuzzy Neural Network (T1FNN) [9, 40, 41] and Interval Type-2 Fuzzy Neural Network (IT2FNN) [22-26]; type-1 [31, 33, 35, 36, 38] and type-2 [32] fuzzy evolutionary systems are typical hybrid systems in softcomputing. These systems combine T1FLS generalized reasoning methods [30, 41, 42, 44, 45] and IT2FLS [19-21] with neural networks learning capabilities [41, 43, 46, 49] and evolutionary algorithms [34, 37, 39, 43], respectively. In this paper, four main architecures (IT2FNN-0, IT2FNN-1, IT2FNN-2 and IT2FNN-3) for integrating a first order TSK IT2FIS, with real consequents (A2C0) and interval consequents (A2C1), are proposed. Integration strategies for process elements of TSK IT2FIS are analised for each architecture (fuzzification, knowledge base, type reduction and defuzzification).

2 Interval Type-2 Fuzzy Neural Network (IT2FNN)

An IT2FNN combines an TSK IT2FIS and an adaptive neural network in order to take advantage of characteristics of each model. Adaptive networks [40,41] provide a theoretical frame that unifies almost all kinds of neural networks with learning capabilities, and their fundamental properties are a key element for understanding other paradigms.

An IT2FNN is characterized by a directed graph $G=(V,E)$, where fan-in zero input nodes and fan-out zero output nodes. Each node $n \in V$ represents a processing unit with an associated static function f_n, and each edge $e \in E$ indicates causal relation between connected nodes. Node set V is divided into two separate subsets, $V=(A \cup N)$. Nodes $n \in A$ are called adaptive and their outputs depend not only on their inputs, but also on internal modifiable parameters $\{\xi_1^n, \xi_2^n, \ldots\}$ internal to $n \in A$. On the other hand, nodes $n \in N$ whose functions depend only on inputs are called non adaptive. In general, when representing IT2FNN graphically, rectangles are used to represent adaptive nodes and circles to represent non-adaptive nodes. Output values of pair nodes (green color) and odd nodes (blue color) represent uncertainty intervals (Fig. 1–6). In this kind of interval type-2 neurofuzzy adaptive networks, nodes represent processing units called neurons, which can be classified into crisp and fuzzy neurons. In IT2FNN, type-1 fuzzy neuron model (T1FN) proposed by Hirota and Pedrycz [7, 8] is extended into an interval type-1 fuzzy neuron (IT1FN) and interval type-2 fuzzy neuron (IT2FN).

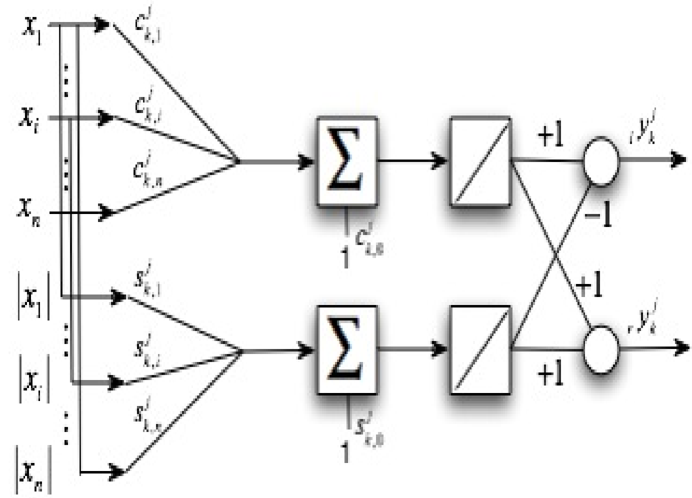

Fig. 1. Interval Type-1 Fuzzy Neuron (IT1FN)

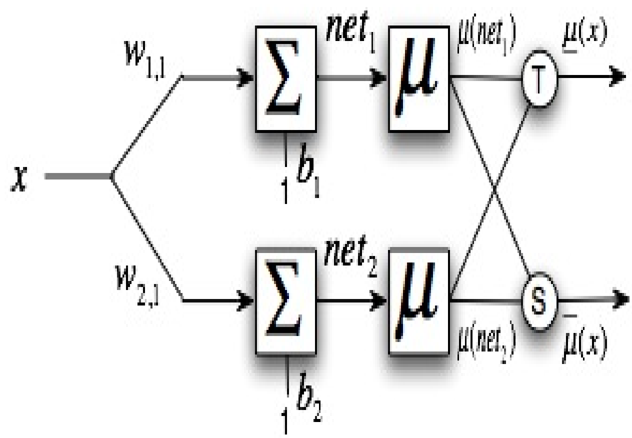

Fig. 2. Interval Type-2 Fuzzy Neuron (IT2FN)

IT2FNN-0 architecture has 7 layers. Layer 1 has adaptive nodes for fuzzifying inputs; layer 2 has non-adaptive nodes with interval fuzzy values. Layer 3 (rules) has non-adaptive nodes for generating firing strength of TSK IT2FIS rules. Layer 4, lower and upper values of rules firing strength are normalized. Adaptive nodes in layer 5 (consequent) are connected to layer 0 for generating rules consequents. Non-adaptive nodes in layer 6 evaluate values from left-right interval. Non-adaptive node in layer 7 (defuzzification) evaluates average of interval left-right values.

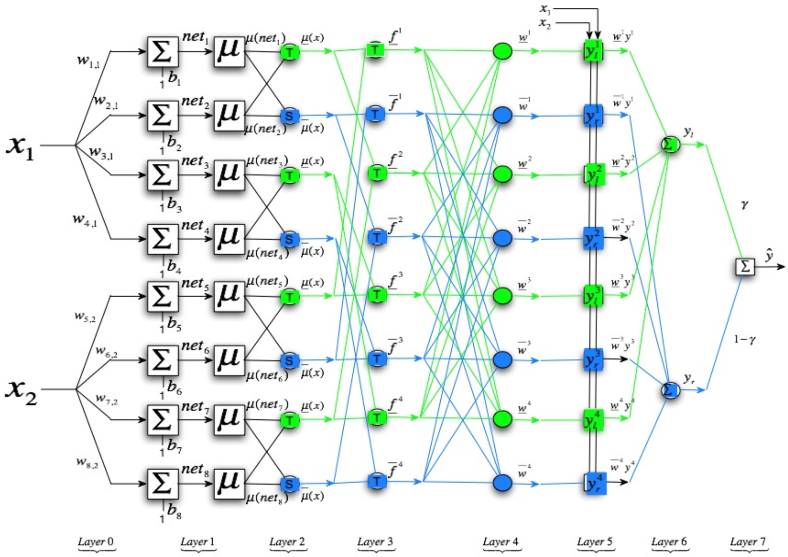

Fig. 3. IT2FNN-0 Architecture

IT2FNN-1 architecture has 5 layers, consists of adaptive nodes with equivalent function to lower-upper membership in fuzzification layer (layer 1). Non-adaptive nodes in rules layer (layer 2) interconnect with fuzzification layer (layer 1) in order to generate TSK IT2FIS rules antecedents. Adaptive nodes in consequent layer (layer 3) are connected to input layer (layer 0) to generate rules consequents. Non-adaptive nodes in type-reduction layer (layer 4) evaluate left-right values with Karnik and Mendel (KM)[19-21] algorithm. Non-adaptive node in defuzzification layer (layer 5) average left-right values.

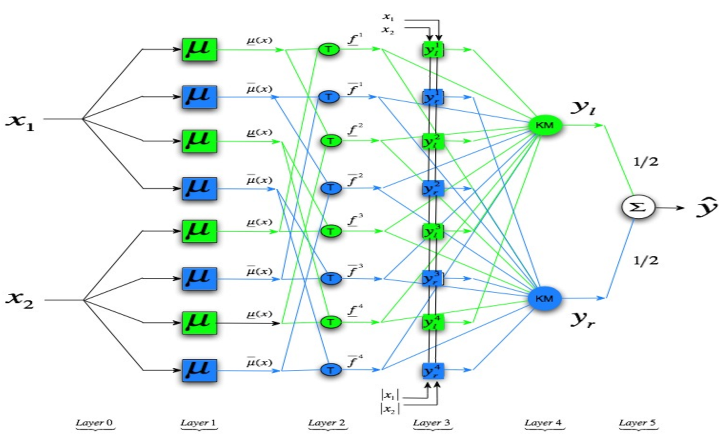

Fig. 4. IT2FNN-1 Architecture

In IT2FNN-2 case, consists on 6 layers, uses IT2FN for fuzzifying inputs (layers 1-2). Non-adaptive nodes in rules layer (layer 3) interconnect with lower-upper linguistic values layer (layer 2) to generate TSK IT2FIS rules antecedents. Non-adaptive nodes in consequents layer (layer 4) are connected with input layer (layer 0) to generate rules consequents. Non-adaptive nodes in type-reduction layer (layer 5) evaluate left-right values with KM algorithm. Non-adaptive node in defuzzification layer (layer 6) averages left-right values.

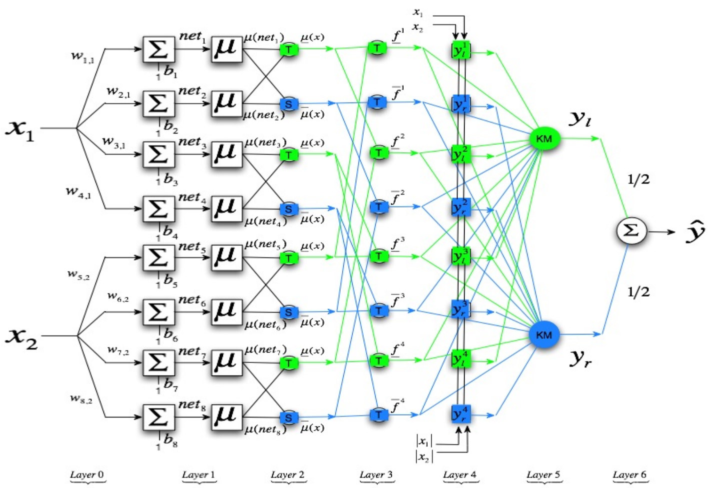

Fig. 5. IT2FNN-2 Architecture

IT2FNN-3 architecture has 8 layers, uses IT2FN for fuzzifying inputs (layers 1-2). Non-adaptive nodes in rules layer (layer 3) interconnect with lower-upper linguistic values layer (layer 2) to generate TSK IT2FIS rules antecedents. Adaptive nodes in layer 4 adapt left-right firing strenght, biasing rules lower-upper trigger forces with synaptic weights between layers 3-4. Layer 5's non-adaptive nodes normalize rules lower-upper firing strenght. Non-adaptive nodes I consequent layer (layer 6) interconnect with input layer (layer 0) to generate rules consequents. Non-adaptive nodes in type-reduction layer (layer 7) evaluate left-right values adding lower-upper product of lower-upper triggering forces normalized by rules consequent left-right values. Node in defuzzification layer is adaptive and its output $\hat{y}$ is defined as biased average of left-right values and parameter γ. Parameter γ (0.5 by default) adjusts uncertainty interval defined by left-right values $[\hat{y}_l, \hat{y}_r]$.

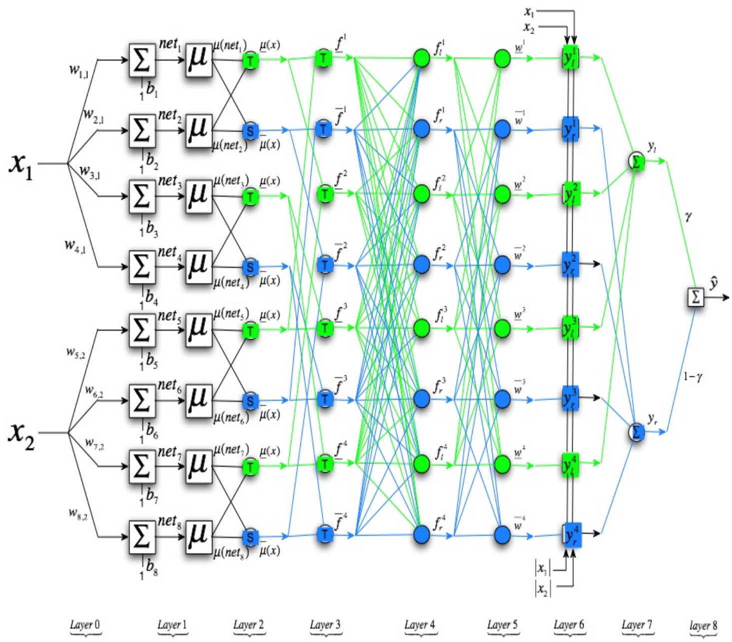

Fig. 6. IT2FNN-3 Architecture

3 Mackey-Glass Forecasting Chaotic Time Series

In this chapter, results from simulations using ANFIS, IT2FNN-0, IT2FNN-1, IT2FNN2 and IT2FNN3 are presented for forecasting Mackey-Glass chaotic time series [50] with $\tau=17$, 30, 100 and different signal noise ratio values, SNR(dB) = 0, 10, 20, 30 as uncertainty source. Levenberg-Marquardt hybrid method is used for adapting IT2FNN models parameters. Proposed IT2FNN architectures are validated using 10-fold cross-validation [53, 54] considering sum square error (***SSE***) or root mean square error (***RMSE***) in training or test phase; Akaike information criteria (AIC) and F test [51, 52]. Cross-validation procedure evaluation is done using Matlab's *crossvalind* function. Noise is added by Matlab's *awgn* function. Also, AIC and F test are evaluated using statistical Matlab's Toolbox. In Mackey-Glass chaotic time series, 1200 data sets where generated with initial conditions x(0)=1.2 and τ= 17, 30 and 100 using Runge-Kutta 4th order method and several uniform noise added. An input-output vector was taken with format: **[x(t-18), x(t-12), x(t-6), x(t); x(t+6)]**. For identifying ANFIS [40, 41] and IT2FNN models, an IT2FNN model with 4 inputs and one output is used with Mackey-Glass chaotic time series using: 16 rules, 2 igaussmtype2 IT2MF for each input, 50 epochs, 500 training data and 500 test data with 10-fold cross-validation. Tables 1-9 and figures 7-12 show RMSE (TRN and CHK) with 10-fold cross validation and the number of points (ς) out of uncertainty interval $\tilde{Y}(x) \in [\hat{y}_l(x), \hat{y}_r(x)]$ results between IT2FNN models and ANFIS. It can be seen that IT2FNN architectures forecast better than Mackey-Glass chaotic time series when noise is present.

Table 1. RMSE (TRN/CHK) and ς values determined in models ANFIS and IT2FNN with 10-fold cross-validation for forecasting Mackey-Glass chaotic time series with τ=17

SNR(dB)		0	10	20	30	free
ANFIS	TRN	0.2643	0.0902	0.0316	0.0110	0.0020
	CHK	0.2917	0.0989	0.0345	0.0118	0.0021
	ς	NA	NA	NA	NA	NA
IT2FNN-0	TRN	0.2357	0.0776	0.0289	0.0088	0.0017
	CHK	0.2576	0.0837	0.0327	0.0107	0.0019
	ς	28	23	17	13	7
IT2FNN-1	TRN	0.2089	0.0672	0.0268	0.0056	0.0016
	CHK	0.2334	0.0875	0.0318	0.0078	0.0018
	ς	25	19	15	11	6
IT2FNN-2	TRN	0.1883	0.0540	0.0189	0.0066	0.0014
	CHK	0.1947	0.0592	0.0207	0.0071	0.0015
	ς	24	18	14	10	4
IT2FNN-3	TRN	0.1558	0.0427	0.0134	0.0041	0.0011
	CHK	0.1632	0.0549	0.0148	0.0043	0.0013
	ς	20	15	11	8	3

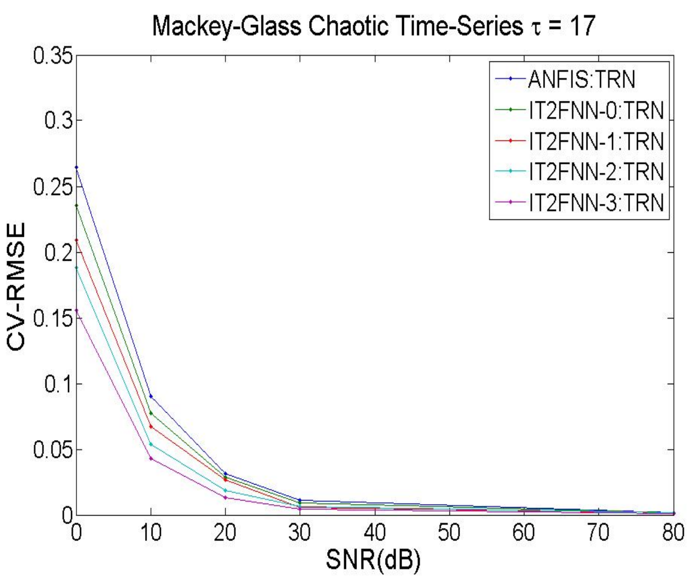

Fig. 7. RMSE (TRN) values determined in models ANFIS and IT2FNN with 10-fold cross-validation for forecasting Mackey-Glass chaotic time series with τ=17

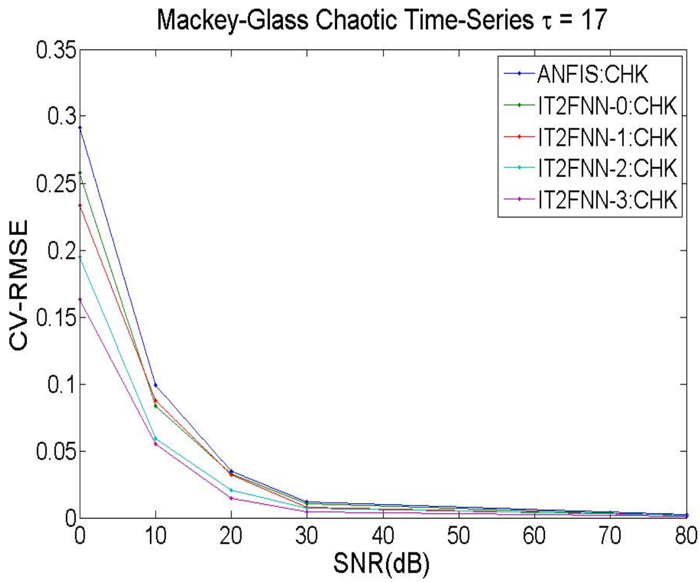

Fig. 8. RMSE (CHK) values determined in models ANFIS and IT2FNN with 10-fold cross-validation for forecastingMackey-Glass chaotic time series with τ=17

Table 2. Akaike Information Criteria (AIC) of TRN/CHK and ς values determined in models ANFIS and IT2FNN with 10-fold cross-validation for forecasting Mackey-Glass chaotic time series with τ=17

SNR(dB)		0	10	20	30	free
ANFIS	TRN	1968.63	893.58	-155.29	-1210.56	-2915.30
	CHK	2067.27	985.66	-67.49	-1140.35	-2866.51
	ς	NA	NA	NA	NA	NA
IT2FNN-0	TRN	1872.11	761.12	-226.61	-1415.70	-3059.82
	CHK	1960.96	836.79	-103.08	-1220.21	-2948.60
	ς	28	23	17	13	7
IT2FNN-1	TRN	1911.40	777.22	-142.05	-1707.68	-2960.45
	CHK	2022.30	1041.19	29.02	-1376.33	-2842.66
	ς	25	19	15	11	6
IT2FNN-2	TRN	1807.59	558.53	-491.29	-1543.38	-3093.98
	CHK	1841.01	650.47	-400.32	-1470.36	-3024.99
	ς	24	18	14	10	4
IT2FNN-3	TRN	1618.12	323.75	-835.20	-2019.46	-3335.14
	CHK	1664.53	575.06	-735.82	-1971.84	-3168.09
	ς	20	15	11	8	3

Shaded cells in Table 2 show, based on AIC, that architecture IT2FNN-3 (TRN/CHK) forecasts better Mackey-Glass chaotic series with τ=17 than ANFIS. In Table 3 it is shown that there is no significant improvement in architectures IT2FNN-1 (CHK) at SNR=10, 20 and noise-free compared with ANFIS.

Table 3. Statistic F (TRN/CHK) and ς values determined in models ANFIS and IT2FNN with 10-fold cross-validation for forecasting Mackey-Glass chaotic time series with τ=17

SNR(dB)		0	10	20	30	free
ANFIS/IT2FNN-0 $F_{0.95}(9,395)=1.9036$	TRN	11.2972	15.4097	8.5838	24.6875	16.8570
	CHK	12.3887	17.3879	4.9648	9.4877	9.7261
	ς	28	23	17	13	7
ANFIS/IT2FNN-1 $F_{0.95}(89,315)=1.3070$	TRN	2.1262	2.8374	1.3814	10.1169	1.9909
	CHK	1.9890	0.9823	0.6265	4.5609	1.2781
	ς	25	19	15	11	6
ANFIS/IT2FNN-2 $F_{0.95}(89,315)=1.3070$	TRN	3.4336	6.3359	6.3547	6.2921	3.6838
	CHK	4.4051	6.3387	6.2921	6.2368	3.3978
	ς	24	18	14	10	4
ANFIS/IT2FNN-3 $F_{0.95}(89,315)=1.3070$	TRN	6.6461	12.2542	16.1434	21.9371	8.1609
	CHK	7.7678	7.9467	15.6932	23.1138	5.6964
	ς	20	15	11	8	3

Table 4. RMSE (TRN/CHK) and ς values determined in models ANFIS and IT2FNN with 10-fold cross-validation for forecasting Mackey-Glass chaotic time series with τ=30

SNR(dB)		0	10	20	30	free
ANFIS	TRN	0.3416	0.1582	0.0876	0.0635	0.0585
	CHK	0.3800	0.1714	0.0943	0.0672	0.0619
	ς	NA	NA	NA	NA	NA
IT2FNN-0	TRN	0.2543	0.1278	0.0633	0.0512	0.0444
	CHK	0.2722	0.1385	0.0719	0.0555	0.0467
	ς	36	29	25	19	9
IT2FNN-1	TRN	0.2097	0.1123	0.0517	0.0413	0.0326
	CHK	0.2605	0.1203	0.0611	0.0429	0.0359
	ς	33	27	22	17	8
IT2FNN-2	TRN	0.1719	0.0812	0.0415	0.0301	0.0297
	CHK	0.1801	0.0892	0.0447	0.0318	0.0289
	ς	30	26	20	16	6
IT2FNN-3	TRN	0.1521	0.0733	0.0338	0.0249	0.0212
	CHK	0.1708	0.0773	0.0419	0.0302	0.0241
	ς	27	23	18	8	5

Shaded cells in Table 5 show, based on AIC, that architecture IT2FNN-3 (TRN/CHK) forecasts better Mackey-Glass chaotic series with τ=30 than ANFIS. In Table 6 it is shown that there is a significant improvement in all architectures IT2FNN (TRN/CHK) model for all SNR values with respect to ANFIS.

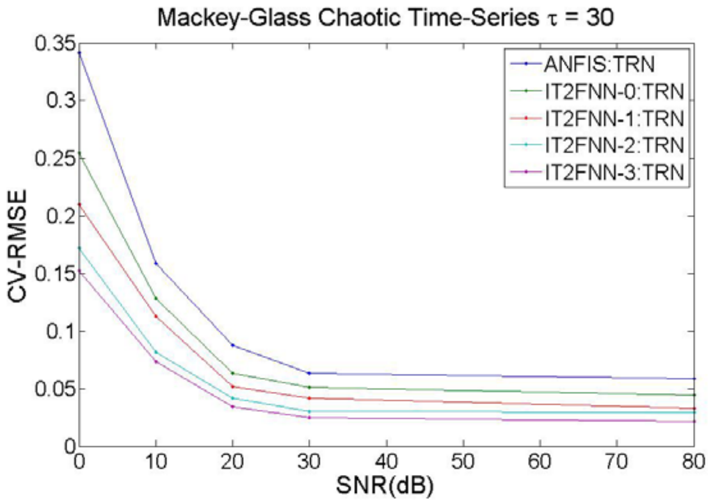

Fig. 9. RMSE (TRN) values determined in models ANFIS and IT2FNN with 10-fold cross-validation for forecasting Mackey-Glass chaotic time series with τ=30

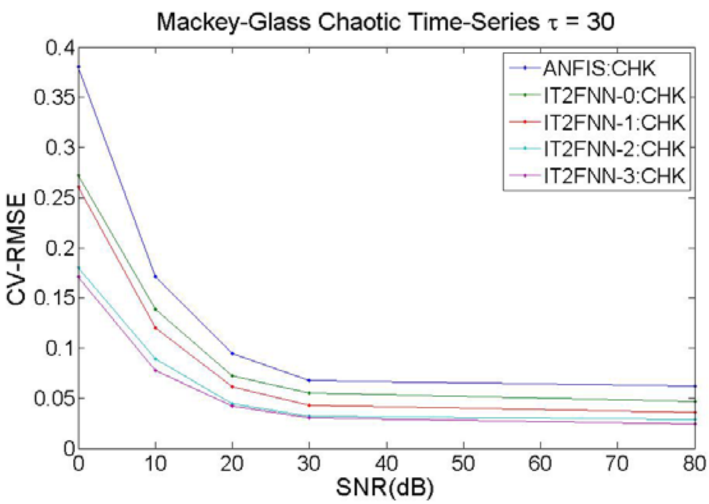

Fig. 10. RMSE (CHK) values determined in models ANFIS and IT2FNN with 10-fold cross-validation for forecasting Mackey-Glass chaotic time series with τ=30

Table 5. Akaike Information Criteria (AIC) of TRN/CHK and ς values determined in models ANFIS and IT2FNN with 10-fold cross-validation for forecasting Mackey-Glass chaotic time series with τ=30

SNR(dB)		0	10	20	30	free
ANFIS	TRN	2225.19	1455.41	864.33	542.59	460.58
	CHK	2331.72	1535.55	938.03	599.22	517.07
	ς	NA	NA	NA	NA	NA
IT2FNN-0	TRN	1948.06	1260.02	557.43	345.29	202.79
	CHK	2016.09	1340.42	684.83	425.93	253.29
	ς	36	29	25	19	9
IT2FNN-1	TRN	1915.23	1290.72	515.01	290.41	53.86
	CHK	2132.15	1359.54	682.06	328.42	150.29
	ς	33	27	22	17	8
IT2FNN-2	TRN	1716.46	966.46	295.24	25.93	39.30
	CHK	1763.06	1060.43	369.52	29.02	66.61
	ς	30	26	20	16	6
IT2FNN-3	TRN	1594.09	864.11	90.01	-215.58	-376.45
	CHK	1710.04	917.24	308.83	-22.61	-248.24
	ς	27	23	18	8	5

Table 6. Statistic F (TRN/CHK) and ς values determined in models ANFIS and IT2FNN with 10-fold cross-validation for forecasting Mackey-Glass chaotic time series with τ=30

SNR(dB)		0	10	20	30	free
ANFIS/IT2FNN-0 $F_{0.95}(9,395)=1.9036$	TRN	35.3061	23.3632	40.1645	23.6202	32.3015
	CHK	41.6464	23.3277	31.6065	20.4550	33.2196
	ς	36	29	25	19	9
ANFIS/IT2FNN-1 $F_{0.95}(89,315)=1.3070$	TRN	5.8527	3.4845	6.6219	4.8276	7.8578
	CHK	3.9920	3.6454	4.8913	5.1452	6.9830
	ς	33	27	22	17	8
ANFIS/IT2FNN-2 $F_{0.95}(89,315)=1.3070$	TRN	10.4374	9.8952	12.2307	12.2126	10.1922
	CHK	12.2172	9.5288	12.2124	12.2661	12.6977
	ς	30	26	20	16	6
ANFIS/IT2FNN-3 $F_{0.95}(89,315)=1.3070$	TRN	14.3131	12.9471	20.2343	19.4788	23.4108
	CHK	13.9798	13.8620	14.3880	13.9852	19.8096
	ς	27	23	18	8	5

Table 7. RMSE (TRN/CHK) and ς values determined in models ANFIS and IT2FNN with 10-fold cross-validation for forecasting Mackey-Glass chaotic time series with $\tau = 100$

SNR(dB)		0	10	20	30	free
ANFIS	TRN	0.4253	0.1839	0.1035	0.0706	0.0617
	CHK	0.4677	0.2035	0.1227	0.0758	0.0669
	ς	NA	NA	NA	NA	NA
IT2FNN-0	TRN	0.3017	0.1445	0.0828	0.0563	0.0415
	CHK	0.3322	0.1497	0.0898	0.0681	0.0483
	ς	50	42	38	29	13
IT2FNN-1	TRN	0.2258	0.1133	0.0687	0.0406	0.0307
	CHK	0.2976	0.1235	0.0779	0.0473	0.0397
	ς	48	40	36	27	11
IT2FNN-2	TRN	0.1751	0.0724	0.0412	0.0313	0.0189
	CHK	0.1899	0.0783	0.0570	0.0398	0.0296
	ς	46	38	34	24	8
IT2FNN-3	TRN	0.1311	0.0527	0.0258	0.0179	0.0155
	CHK	0.1377	0.0554	0.0311	0.0212	0.0133
	ς	39	31	28	19	6

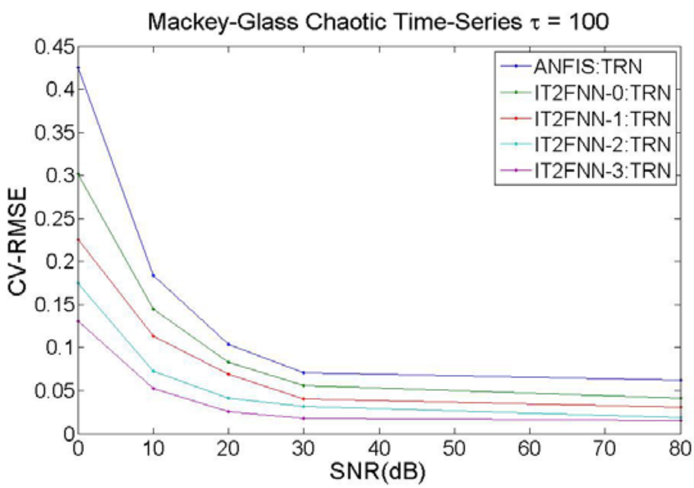

Fig. 11. RMSE (TRN) values determined in models ANFIS and IT2FNN with 10-fold cross-validation for forecasting Mackey-Glass chaotic time series with $\tau = 100$

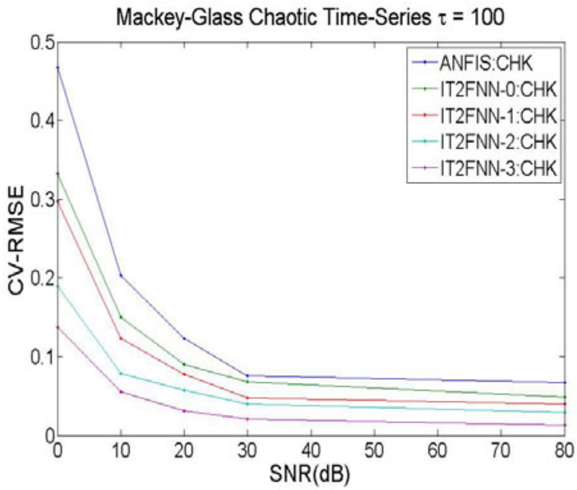

Fig. 12. RMSE (CHK) values determined in models ANFIS and IT2FNN with 10-fold cross-validation for forecasting Mackey-Glass chaotic time series with $\tau=100$

Table 8. Akaike Information Criteria (AIC) of TRN/CHK and ς values determined in models ANFIS and IT2FNN with 10-fold cross-validation for forecasting Mackey-Glass chaotic time series with $\tau=100$

SNR(dB)		0	10	20	30	free
ANFIS	TRN	2444.34	1605.94	1031.12	648.58	513.83
	CHK	2539.38	1707.21	1201.29	719.65	594.75
	ς	NA	NA	NA	NA	NA
IT2FNN-0	TRN	2118.98	1382.83	825.98	440.24	135.24
	CHK	2215.29	1418.18	907.13	630.53	286.98
	ς	50	42	38	29	13
IT2FNN-1	TRN	1989.20	1299.59	799.30	273.32	-6.19
	CHK	2265.30	1385.79	924.97	426.06	250.90
	ς	48	40	36	27	11
IT2FNN-2	TRN	1734.91	851.76	287.99	13.17	-491.29
	CHK	1816.05	930.10	612.60	253.42	-42.68
	ς	46	38	34	24	8
IT2FNN-3	TRN	1445.51	534.16	-180.08	-545.65	-689.61
	CHK	1494.63	584.13	6.76	-376.45	-842.69
	ς	39	31	28	19	6

Table 9. Statistic F (TRN/CHK) and ς values determined in models ANFIS and IT2FNN with 10-fold cross-validation for forecasting Mackey-Glass chaotic time series with τ=100

SNR(dB)		0	10	20	30	free
ANFIS/IT2FNN-0 $F_{0.95}(9,395)=1.9036$	TRN	43.3268	27.1968	24.6875	25.1267	53.1238
	CHK	43.1053	37.2147	38.0502	10.4860	40.3112
	ς	50	42	38	29	13
ANFIS/IT2FNN-1 $F_{0.95}(89,315)=1.3070$	TRN	9.0170	5.7851	4.4939	7.1630	10.7567
	CHK	5.2022	6.0705	5.2415	5.5501	6.5113
	ς	48	40	36	27	11
ANFIS/IT2FNN-2 $F_{0.95}(89,315)=1.3070$	TRN	17.3411	19.2960	18.7967	14.4677	34.1803
	CHK	17.9294	20.3677	12.8613	9.2985	14.5403
	ς	46	38	34	24	8
ANFIS/IT2FNN-3 $F_{0.95}(89,315)=1.3070$	TRN	33.7089	39.5592	53.4196	51.5191	52.5432
	CHK	37.2914	44.2169	51.5527	41.7074	86.0115
	ς	39	31	28	19	6

Shaded cells in Table 8 show, based on AIC, that architecture IT2FNN-3 (TRN/CHK) forecasts better Mackey-Glass chaotic series with τ=100 than ANFIS. In Table 9 it is shown that there is significant improvement in architectures IT2FNN-1 (CHK) model for all SNR values with respect to ANFIS.

4 Conclusions

This paper models interval type-2 fuzzy neural networks (IT2FNN) architectures for forecasting Mackey-Glass chaotic time series. Combining neural networks and interval type-2 fuzzy logic systems improves learning, uncertainty handle and discovery, which are desirable characteristics for intelligent processing of imperfect information. Results show higher efficiency due to better representation capability and scalability. Mackey-Glass results analyzed with cross-validation, AIC and F tests, show that IT2FNN is a generalization of autoregressive models in times series case.

Cross validation, AIC and F test contrast analysis with statistical models show that IT2FNN models have better results under high uncertainty or same results under low uncertainty as ANFIS models.

References

[1] Klir, G.J., Folger, T.A.: Fuzzy Sets, Uncertainty and Information. Prentice Hall, Englewood Cliffs (1988)
[2] Klir, G.J., Wierman, M.J.: Uncertainty-Based Information: Elements of Generalized Information Theory. Physica-Verlag / Springer, Heidelberg (1999)
[3] Davis, T.J., Keller, C.P.: Modelling uncertainty in natural resource analysis using fuzzy sets and Monte Carlo simulation: slope stability prediction. International Journal of Geographical Information Science 11(5), 409–434 (1997)
[4] Gil Aluja, J.: Elements for a theory of decision in uncertainty. Kluwer Academic Publishers, Boston (1999)

[5] Hagras, F., Roberts, H., Callaghan, D.: A Type-2 Fuzzy Based System for Handling the Uncertainties in Group Decisions for Ranking Job Applicants within Human Resources Systems. In: FUZZ-IEEE 2008, Hong Kong (June 2008)

[6] Hwang, C., Rhee, F.C.-H.: Uncertain Fuzzy Clustering: Interval Type-2 Fuzzy. IEEE Transaction on Fuzzy System 15(1), 107–120 (2007)

[7] Hirota, K., Pedrycz, W.: Knowledge-based networks in classification problems. Fuzzy Sets and Systems 51, 1–27 (1992)

[8] Hirota, K., Pedrycz, W.: OR/AND neuron in modeling fuzzy set connectives. IEEE Transactions on Fuzzy Systems 2, 151–161 (1994)

[9] Horikowa, S., Furuhashi, T., Uchikawa, Y.: On fuzzy modeling using fuzzy neural networks with the backpropagation algorithm. IEEE Transactions on Neural Networks 3 (1992)

[10] Chaoui, H., Gueaieb, W.: Type-2 Fuzzy Logic Control of a Flexible-Joint Manipulator. Journal Of Intelligent And Robotic Systems 51(2), 159–186 (2008)

[11] Wu, D., Tan, W.W.: A simplified type-2 fuzzy logic controller for real-time control. Isa Transactions 45(4), 503–516 (2006)

[12] Castillo, O., Melin, P., Kacprzyk, J., Pedrycz, W.: Type-2 Fuzzy Logic theory and applications. In: Proceedings of Granular Computing 2007, Silicon Valley, CA, USA, November 2007, pp. 145–150 (2007)

[13] Zeng, J., Liu, Z.-Q.: Type-2 fuzzy sets for pattern recognition: The state-of-the-art. Journal Uncertain System 1(3), 163–177 (2007)

[14] Sepulveda, R., Castillo, O., Melin, P., Rodriguez-Diaz, A., Montiel, O.: Experimental study of intelligent controllers under uncertainty using type-1 and type-2 fuzzy logic. Information Sciences 177(10), 2023–2048 (2007)

[15] Singh, M., Srivastava, S., Gupta, J.R.P., Hanmandlu, M.: A new algorithm-based type-2 fuzzy controller for diabetic patient. International Journal of Biomedical Engineering And Technology 1(1), 18–40 (2007)

[16] Astudillo, L., Castillo, O., Melin, P., Alanis, A., Soria, J., Aguilar, L.: Intelligent Control of an Autonomous Mobile Robot using Type-2 Fuzzy Logic. Journal of Engineering Letters 13(2), 93–97 (2006)

[17] Baguley, P., Page, T., Koliza, V., Maropoulos, P.: Time to market prediction using type-2 fuzzy sets. Journal of Manufacturing Technology Management 17(4), 513–520 (2006)

[18] Zeng, J., Liu, Z.-Q.: Type-2 Fuzzy Hidden Markov Models and Their Application to Speech Recognition. IEEE Transactions On Fuzzy Systems 14(3), 454–467 (2006)

[19] Mendel, J.M.: Uncertain Rule-Based Fuzzy Logic Systems: Introduction and New Directions. Prentice Hall, NJ (2001)

[20] Karnik, N.N., Mendel, J.M.: Applications of type-2 fuzzy logic systems to forecasting of time-series. Inform. Sci. 120, 89–111 (1999)

[21] Wu, D., Mendel, J.M.: A Vector Similarity Measure for Interval Type-2 Fuzzy Sets and Type-1 Fuzzy Sets. Information Sciences 178, 381–402 (2008)

[22] Lee, C.-H., Lin, Y.-C.: Type-2 Fuzzy Neuro System Via Input-to-State-Stability Approach. In: Liu, D., Fei, S., Hou, Z., Zhang, H., Sun, C. (eds.) ISNN 2007. LNCS, vol. 4492, pp. 317–327. Springer, Heidelberg (2007)

[23] Lin, Y.-C., Lee, C.-H.: System Identification and Adaptive Filter Using a Novel Fuzzy Neuro System. International Journal of Computational Cognition 5(1) (March 2007)

[24] Lee, C.H., Hong, J.L., Lin, Y.C., Lai, W.Y.: Type-2 Fuzzy Neural Network Systems and Learning. International Journal of Computational Cognition 1(4), 79–90 (2003)

[25] Wang, C.H., Cheng, C.S., Lee, T.-T.: Dynamical optimal training for interval type-2 fuzzy neural network (T2FNN). IEEE Trans. on Systems, Man, and Cybernetics Part B: Cybernetics 34(3), 1462–1477 (2004)

[26] Hagras, H.: Comments on Dynamical Optimal Training for Interval Type-2 Fuzzy Neural Network (T2FNN). IEEE Transactions on Systems Man And Cybernetics Part B 36(5), 1206–1209 (2006)

[27] Mendez, G.M., Leduc, L.: Hybrid Learning Algorithm for Interval Type-2 Fuzzy Logic Systems. Control and Intelligent Systems Journal, Special Issue on Nonlinear Adaptive PID Control Part 2 34(3), 206–215 (2006)

[28] Own, C.-M., Tsai, H.-H., Yu, P.-T., Lee, Y.-J.: Adaptive type-2 fuzzy median filter design for removal of impulse noise. Imaging Science 54(1), 3–18 (2006)

[29] Cao, X.-Q., Zeng, J., Yan, H.: Modeling Uncertain Speech Sequences Using Type-2 Fuzzy Hidden Markov Models. In: Ip, H.H.-S., Au, O.C., Leung, H., Sun, M.-T., Ma, W.-Y., Hu, S.-M. (eds.) PCM 2007. LNCS, vol. 4810, pp. 315–324. Springer, Heidelberg (2007)

[30] Pedrycz, W.: Fuzzy Modelling: Paradigms and Practice. Kluwer Academic Press, Dordrecht (1996)

[31] Wang, C.H., Liu, H.L., Lin, C.T.: Dynamic optimal Learning rate of A Certain Class of Fuzzy Neural Networks and Its Applications with Genetic Algorithm. IEEE Trans. Syst. Man, Cybern. 31(3), 467–475 (2001)

[32] Wu, D., Wan Tan, W.: Genetic learning and performance evaluation of interval type-2 fuzzy logic controllers. Engineering Applications of Artificial Intelligence 19(8), 829–841 (2006)

[33] Ascia, G., Catania, V., Panno, D.: An Integrated Fuzzy-GA Approach for Buffer Management. IEEE Trans. Fuzzy Syst. 14(4), 528–541 (2006)

[34] Bonissone, P.P., Subbu, R., Eklund, N., Kiehl, T.R.: Evolutionary Algorithms + Domain Knowledge = Real-World Evolutionary Computation. IEEE Trans. Evol Comput. 10(3), 256–280 (2006)

[35] Chiou, Y.-C., Lan, L.W.: Genetic fuzzy logic controller: an iterative evolution algorithm with new encoding method. Fuzzy Sets Syst. 152(3), 617–635 (2005)

[36] Pedrycz, W.: Fuzzy Evolutionary Computation. Kluwer Academic Publishers, Dordrecht (1997)

[37] Deb, K.: A population-based algorithm-generator for real-parameter optimization. Springer, Heidelberg (2005) (in press)

[38] Ishibuchi, H., Nozaki, K., Yamamoto, N., Tanaka, H.: Selecting fuzzy if-then rules for classification problems using genetic algorithms. IEEE Trans. Fuzzy Syst. 3, 260–270 (1995)

[39] Engelbrecht, A.P.: Fundamentals of computational swarm intelligence. John Wiley & Sons, Ltd., Chichester (2005)

[40] Jang, J.-S.R.: ANFIS: adaptive-network-based fuzzy inference system. IEEE Trans. Syst., Man, Cybern. 23(3), 665–684 (1993)

[41] Jang, J.S.R., Sun, C.T., Mizutani, E.: Neuro-fuzzy and Soft Computing. Prentice-Hall, New York (1997)

[42] Mamdani, E.H., Assilian, S.: An experiment in linguistic synthesis with a fuzzy logic controller. Int. J. Man-Mach. Stud. 7, 1–13 (1975)

[43] Russell, S., Norvig, P.: Artificial Intelligence: A Modern Approach. Prentice-Hall, NJ (2003)

[44] Takagi, T., Sugeno, M.: Fuzzy identification of systems and its applications to modeling and control. IEEE Trans. Syst., Man, Cybern. 15, 116–132 (1985)

[45] Zadeh, L.A.: Fuzzy Logic, Neural Neural Networks and Soft Computing. Comunications of the ACM 37(3), 77–84 (1994)
[46] Hagan, M.T., Demuth, H.B., Beale, M.H.: Neural Network Design. PWS Publishing, Boston (1996)
[47] Zadeh, L.A.: Towards a generalized theory of uncertainty (GTU)–an outline. Information Sciences 172, 1–40 (2005)
[48] Zadeh, L.A., Kacprzyk, J. (eds.): Computing With Words in Information/Intelligent Systems, vol. 1 & 2. Physica-Verlag, New York (1999)
[49] Haykin, S.: Adaptive Filter Theory. Prentice Hall, Englewood Cliffs (2002) ISBN 0-13-048434-2
[50] Mackey, M.C., Glass, L.: Oscillation and Chaos in Physiological Control Systems. Science 197, 287–289 (1977)
[51] Ljung, L.: System Identification: Theory for the User, pp. 278–280. Prentice-Hall, Englewood Cliffs (1987)
[52] Akaike, H.: A new look at the statistical model identification. IEEE Transactions on automatic control AC 19, 716–723 (1974)
[53] Hastie, T., Tibshirani, R., Friedman, J.: The Elements of Statistical Learning, pp. 214–216. Springer, Heidelberg (2001)
[54] Theodoridis, S., Koutroumbas, K.: Pattern Recognition, pp. 341–342. Academic Press, London (1999)