Estimation of a Simple Genetic Algorithm Applied to a Laboratory Experiment

Simone Alfarano, Eva Camacho, and Josep Domenech

Abstract. The aim of our contribution relies on studying the possibility of implementing a genetic algorithm in order to reproduce some characteristics of a simple laboratory experiment with human subjects. The novelty of our paper regards the estimation of the key-parameters of the algorithm, and the analysis of the characteristics of the estimator.

Keywords: Experiments, Genetic algorithm, Bounded rationality, Estimation.

1 Introduction

Nowadays, a large part of economists expresses dissatisfaction (or sometimes rejection) to the wide-spread paradigm of full or strict rationality in theorizing the behavior of economic agents. Laboratory experiments showed that, even in simple settings, human subjects are not consistent with the assumptions implied by their supposed perfect rationality. An existing alternative paradigm in economic theory considers that economic agents have limited capabilities in processing the information and in taking their decisions. Contrary to the fully rational paradigm, it does not exists a unified theory of bounded rationality. Therefore, many different models of human behavior which account for bounded rationality have been proposed in the literature (See for example [3]).

Simone Alfarano and Eva Camacho
Departamento de Economia, Universitat Jaume I, 12071 Castellón, Spain
e-mail: alfarano@eco.uji.es,camacho@eco.uji.es

Josep Domenech
Dpto. de Economía y Ciencias Sociales, Universidad Politecnica de Valencia, 46022 Valencia, Spain
e-mail: jdomenech@upvnet.upv.es

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The adaptation of genetic algorithms (GA) from the realm of optimization literature to the description of human learning is an example of the creative ability of researchers to introduce bounded rational models.[1] A number of papers are now available in the literature which apply different versions of GAs in order to reproduce the behavior of economic agents in different contexts (See, for example, [1], [2], [7], [9]). GAs have also been applied in the context of laboratory experiments in order to reproduce the human subjects' behavior in different experimental settings (See [4], [10]).

However, up to now the different contributions are almost entirely based on a rough calibration of the underlying crucial parameters. To the best of our knowledge, our paper constitutes the first attempt to estimate the underlying parameters of a genetic algorithm. In this paper we provide a method to estimate the key parameters of the GA by means of an extensive simulation-based approach, using an extremely simple experimental setting of a common-pool resources problem.The experiment exhibits, in fact, a single dominant and symmetric Nash equilibrium as illustrated in the next section. The paper is organized as follows: in Section 2 we illustrate briefly the theoretical and empirical results of the experimental setting. In Section 3 we detail the characteristics of the implementation of our GA agents. In Section 4 we present the estimation procedure. Finally, in Section 5 we conclude.

2 Experiment: Setting and Results

In this section we will summarize the experimental setting and main results used as benchmark in order to build the GA and the corresponding parameters estimation (See [5] for the details on the experiment).

Consider an industry consisting of n symmetric firms where each firm $i = \{1,...,n\}$ is characterized by both its default profit Π^0, incurred without engaging in any abatement activity, and by its abatement technology represented by an abatement cost function $C(a_i)$, where we use a_i to denote the firm's abatement level.[2] Zero abatement leads to a maximal emission level $e^{\max}$. Accordingly, the profit function of each firm can be written as $\Pi_i = \Pi^0 - C(a_i)$. Total emissions by industry are then given by $E = \sum_{i=1}^{n}(e^{\max} - a_i)$ and are evaluated by using a social damage function $D(E) = d\left[\sum_{i=1}^{n}(e^{\max} - a_i)\right]$, where $d > 0$ denotes the marginal social damage.

In this industry the regulator decides to implement the tax-subsidy mechanism, proposed by [12]. This mechanism works as follows: Whenever the aggregate abatement level falls short of (exceeds) the socially optimal aggregate abatement level A^*, the regulator charges all the firms with a tax (or pays a subsidy to all the firms) proportional to the difference between optimal and actual abatement. Note that the total tax bill (subsidy payment) is the

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[1] For more details on GA and their application to Economics see [6].

[2] The abatement cost function satisfies the following properties: $C(0) = 0$, $C' > 0$, and $C'' > 0$.

Table 1 Abatement cost schedule

Abated units	Marginal cost	Total cost
0	0	0
1	20	20
2	40	60
3	60	120
4	80	200

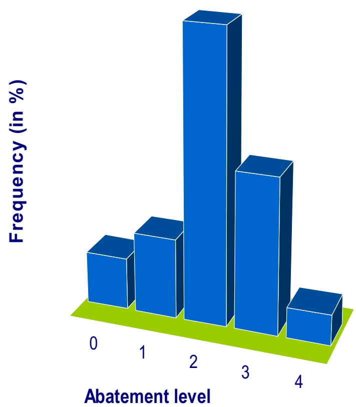


Fig. 1 Histogram of experimental subjects decisions.

same for each firm. Thus with this mechanism a typical firm's profit can be written as:

$$\Pi_i(a_i, a_{-i}) = \Pi^0 - C(a_i) - s\left[A^* - \sum_{i=1}^{n} a_i\right] , \tag{1}$$

where s denotes the tax or subsidy rate and a_{-i} the vector of the decisions by the other firms except from i. When implemented as a one-shot or finitely repeated game, the unique Nash equilibrium is characterized by the the following condition: $C'(a_i) = s$, i.e. the firms choose an abatement level with a marginal cost equaling the tax or subsidy rate. The Nash strategy is also a dominant strategy that leads to the first-best allocation, i.e. $a_i = a^*$, if s equals the marginal social damage d.[3]

In [5] they consider an industry consisting of 5 firms ($n = 5$) with a default profit $\Pi^0 = 200$ ECU (Experimental Currency Unit, which is then converted into Euros at a given exchange rate, known to the subjects at the beginning

[3] Note that the mechanism is not collusion-proof in a repeated setting as stressed by [8]. Therefore, if firms succeed in coordinating on a higher abatement level than is socially optimal, they can earn a higher profit than in the one–shot Nash equilibrium.

of the experiment), an optimal subsidy of $s = 50$ and a discrete abatement cost schedule presented in table 1. Abatement schedule and marginal damage imply a socially optimal abatement level of $a^* = 2$ for any $i = 1,...,5$, leading to an optimal aggregate abatement level of $A^* = 10$.

The mechanism was administered as a non-cooperative game and was repeated over 20 periods. In total 8 sessions with 5 subjects each were conducted. Figure 1 illustrates the aggregate results obtained in the experiments regarding the frequency of each possible abatement decision.

3 Genetic Algorithm

The basic philosophy in implementing our version of the GA is to be "as close as possible" to the laboratory setting described in the previous section. Therefore, the parameters of the algorithms and the implementations of its internal procedures are chosen, when possible, directly from the experimental design. Additionally, we do not intend to describe a general implementation of GA, neither mention all possible alternative implementations of its operators that can be found in the literature (See [6]). Instead, we directly illustrate what we have used to implement the experimental setting.

Our genetic algorithm is characterized by the following elements:

- **Strategy:** Each chromosome in the genetic algorithm represents a possible strategy that a subject can follow, that is, the abatement level decided by the subject. It is encoded as a single gene which takes integer values between 0 and 4. This is the basic element of the GA in the evolution of the algorithm. This choice follows directly the experimental setting.
- **Fitness Function:** It is associated to each strategy and accounts for the actual or potential payoff that derives from the use of a given strategy. In our setting, the GA player uses as measure of fitness the profit function that the experimental subjects face in the laboratory (as shown in Equation 1).
- **Time window:** In order to associate a fitness measure to each strategy, we compute the cumulative potential profit that a given strategy would have had if played in the past w time periods. This time window represents the time memory that the GA subjects use to evaluate each single strategy from its population.
- **Population:** Each subject is endowed with a set of P strategies. The limited size of this set bounds the sophistication of the GA subject when deciding which strategy to apply.
- **Mutation:** It implies that with a probability m one of the strategies included in the population will be randomly changed into any other strategy included in the entire set of potential strategies.
- **Choice rule:** Given the fitness measure and the population, for each single period, the GA subject chooses to play the fittest strategy available in its population.

- **Learning:** Typically there exists two different learning mechanisms: single population vs. multi population. Under the first one, each GA agent has a set of strategies that evolve independently of the strategies of the other agents. In a multi population approach, part of the genetic material is exchanged among the GA agents. This creates some sort of interaction or imitation among agents. Given that in the laboratory setting, total abatement was the only information provided to the subjects and that no communication among subjects was allowed, we decided to implement the leaning mechanism based on a single population approach.

The number of GA agents is $N = 5$, following the experimental setting. Moreover, given our limited number of possible strategies, and in order to simplify the estimation procedure, we decide not to implement the crossover operator, which is typically present in the GA (See [4]). The GA parameters we aim at estimating are population (P), time window (w) and mutation rate (m).

4 Estimation: Procedure and Results

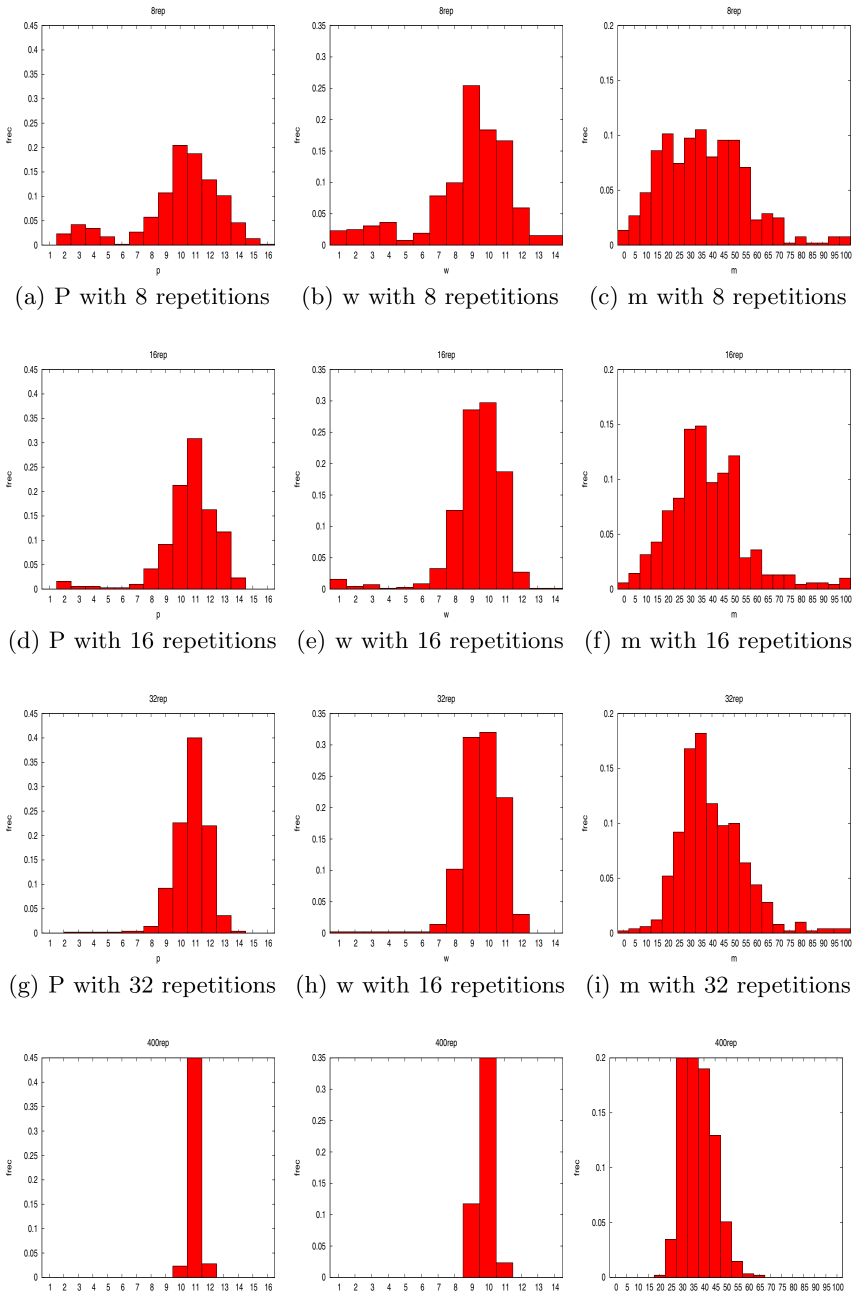
In order to estimate the key parameters of the GA described in the previous section, we fit the distribution of strategies observed in the 8 experimental sessions (See Figure 1).

Let us define as $\boldsymbol{\theta} = (P, w, m)$ the vector of the parameters to be estimated. The optimal value of $\boldsymbol{\theta}$ is calculated by minimizing the distance between the empirical histogram of the strategies from the experimental data (See Figure 1) and the histogram of the GA strategies computed using 5000 Monte Carlo simulations. The optimal value is then given by the following expression:

$$\boldsymbol{\theta}^* = \arg\min_{\boldsymbol{\theta}} \sum_{i=0}^{4} \left[f_{exp}(i) - f_{sim}(i|\boldsymbol{\theta})\right]^2, \qquad (2)$$

where $f_{exp}(i)$ is the empirical frequency of the strategy i computed from the histogram of experimental data, and $f_{sim}(i|\boldsymbol{\theta})$ is the frequency of strategy i computed from 5000 Monte Carlo simulations of the GA with parameters $\boldsymbol{\theta}$. More precisely, given a vector of parameters $\boldsymbol{\theta}$, the GA runs 5000 times for a 20 periods[4] for each realization; then the distance between the resulting simulated histogram of strategies and the empirical one is evaluated and minimized using a Nelder-Mead optimization algorithm. The Nelder-Mead method was proposed by [11] as an unconstrained optimization algorithm. It is commonly used when the derivatives of the objective function are not available. The number of Monte Carlo repetitions has been decided taking into account the computational effort and the precision of evaluation of the simulated histogram. The optimization procedure takes around one hour, which is a reasonable time. The optimal value is $\boldsymbol{\theta}^* = (11, 10, 0.36)$.

[4] The number of periods is equal to the periods conducted in the experimental sessions.



(a) P with 8 repetitions (b) w with 8 repetitions (c) m with 8 repetitions

(d) P with 16 repetitions (e) w with 16 repetitions (f) m with 16 repetitions

(g) P with 32 repetitions (h) w with 16 repetitions (i) m with 32 repetitions

(j) P with 400 repetitions (k) w with 400 repetitions (l) m with 400 repetitions

Fig. 2 Distribution of parameter values: P, w and m for different number of repetitions.

In order to evaluate the performance of the entire estimation procedure, we run a series of Monte Carlo simulations using the previously described minimizing procedure with artificially generated histograms as benchmark instead of the experimental data. The vector of parameters of the GA is $\boldsymbol{\theta}^*$. Essentially, we re-estimate the known parameters of the GA, valuating then the *ex-post* resulting distribution of the estimated $\hat{\boldsymbol{\theta}}$. The benchmark histogram is computed averaging over an increasing number of single simulations of 20 periods (See details in Figure 2). We have computed 500 Monte Carlo replications of the re-estimation procedure for each benchmark histogram. The entire process required about 60 days of computing time, although it was parallelized in a 20-node cluster to cut the simulation time to three days.

5 Conclusions

The first important result of our computational exercise is to demonstrate that it is possible to estimate the parameters of a GA using experimental data. As it turns out, the estimation of the key-parameters of GA applied to this set of experiments gives satisfactory results, considering the small data sample available and the highly complex nature of the GA algorithm. The different parameters can be, in fact, estimated with reasonable errors, as the Monte Carlo numerical re-estimation exercise shows. We have performed the re-estimation procedure with a benchmark histogram averaged over 8, 16, 32 and 400 replications of the genetic algorithm. The case using 400 repetitions was conducted as a computational exercise to see the asymptotic properties of the estimator. From an experimental point of view, our Monte Carlo exercise shows that are enough few experimental sessions to generate a sufficiently large data set in order to reliably estimate the parameters.

As final remarks, we would like to stress that our computational exercise, although promising, it is just a first step in developing a general computational approach to complement the laboratory experiments in analyzing economic phenomena. The robustness of the GAs with respect to changes in the experimental setting, the flexibility of GA under changes in its internal operators, the importance to obtain *reasonable* and *consistent* values of the parameters in describing human behavior are just few examples of open problems that we have in our research agenda.

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