Likelihood in a Possibilistic and Probabilistic Context: A Comparison

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Abstract. We provide a comparison between a probabilistic and a possibilistic likelihood both as point and set functions.

1 Introduction

We consider conditional probabilities and T-conditional possibilities (where T stands either for min or any strict t-norm). We focus on likelihood functions, regarded as coherent probability or possibility assessment on a class of conditional events $\{E|H_i\}$, with $\{H_i\}$ a finite partition of the sure event Ω. Then, we characterize their coherent extensions on the conditional events $\{E|H\}$, with H belonging to the algebra $\mathscr{H}$ spanned by the H_i's.

Our aim is to give, from a syntactic point of view, a thorough comparison of probabilistic and possibilistic likelihoods both as point and set functions. The interest arises from "bayesian-like" inferential situations where the available information is expressed by different uncertainty measures; for instance, when the prior is a possibility assessment (possibly obtained as supremum of a class of probabilities, see e.g. [1, 5, 8, 9]) and the likelihood comes from a probabilistc data base.

As the point function likelihood is concerned, we find that, from a syntactic point of view, any possibilistic likelihood is also a probabilistic likelihood, and vice versa. Moreover, both conditional possibility and conditional probability, regarded as set functions, are capacities if and only if they are not

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necessarily normalized possibilities (they are normalized if and only if at least in a point the likelihood is equal to one). An interesting difference is instead the following: in probabilistic setting no kind of monotonicity is required, while in the possibilistic one there is a local form of monotonicity (i.e. it is monotone on the elements of a suitable partition of the algebra).

2 Coherent Conditional Possibility Assessments

The concept of coherence, introduced by de Finetti in probability theory (see [7]), has a fundamental role in managing partial assessments of an uncertainty measure. In fact, coherence is a tool to check consistency, with respect to a specific measure, of a function defined on an arbitrary set of events, and to extend it to new (conditional) events, maintaining consistency.

In this paper we refer to coherent conditional probabilities (see, for instance, [3]) and coherent T-conditional possibility (with T triangular norm) starting from the following definition of T-conditional possibility (see [2]):

Definition 1. *Let $\mathscr{F} = \mathscr{B} \times \mathscr{H}$ be a set of conditional events with $\mathscr{B}$ a Boolean algebra and $\mathscr{H}$ an additive set (i.e. closed with respect to finite logical sums), such that $\mathscr{H} \subseteq \mathscr{B} \setminus \{\emptyset\}$. Let T be a t-norm, a function $\Pi : \mathscr{F} \to [0,1]$ is a T-conditional possibility if it satisfies the following properties:*

1. *$\Pi(E|H) = \Pi(E \wedge H|H)$, for every $E \in \mathscr{B}$ and $H \in \mathscr{H}$;*
2. *$\Pi(\cdot|H)$ is a possibility, for any $H \in \mathscr{H}$;*
3. *$\Pi(E \wedge F|H) = T(\Pi(E|H), \Pi(F|E \wedge H))$, for any $H, E \wedge H \in \mathscr{H}$ and $E, F \in \mathscr{B}$.*

An assessment Π on an arbitrary set $\mathscr{E}$ of conditional events is a coherent T-conditional possibility if (and only if) Π is a restriction of a T-conditional possibility (in the sense of Definition 1) defined on $\mathscr{F} = \mathscr{B} \times \mathscr{H} \supseteq \mathscr{E}$.

We recall a characterization of a coherent T-conditional possibility assessment given in [6]:

Theorem 1. *Let $\mathscr{E} = \{E_1|H_1, ..., E_n|H_n\}$ be an arbitrary set of conditional events, and $\mathscr{C}_0$ denotes the set of atoms spanned by $\{E_1, H_1, ..., E_n, H_n\}$. For a real function $\Pi : \mathscr{E} \to [0,1]$, the following statements are equivalent:*

a) Π is a coherent T-conditional possibility assessment on $\mathscr{E}$;
b) there exists a sequence of compatible systems S_α ($\alpha = 0, ..., k$), with unknowns $x_r^\alpha = \Pi_\alpha(C_r) \geq 0$ for $C_r \in \mathscr{C}_\alpha$,

$$S_\alpha \begin{cases} \max\limits_{C_r \subseteq E_i \wedge H_i} x_r^\alpha = T(\Pi(E_i|H_i), \max\limits_{C_r \subseteq H_i} x_r^\alpha) & \text{if } \max\limits_{C_r \subseteq H_i} \mathbf{x}_r^{\alpha-1} < 1 \\ x_r^\alpha \geq \mathbf{x}_r^{\alpha-1} & \text{if } C_r \in \mathscr{C}_\alpha \\ \mathbf{x}_r^{\alpha-1} = T(x_r^\alpha, \max\limits_{C_j \in \mathscr{C}_\alpha} \mathbf{x}_j^{\alpha-1}) & \text{if } C_r \in \mathscr{C}_\alpha \\ \max\limits_{C_r \in \mathscr{C}_\alpha} x_r = 1 \end{cases} \quad (1)$$

with $\alpha = 0,...,k$*, where* $\mathbf{x}^{\alpha}$ *(with r-th component* $\mathbf{x}_r^{\alpha}$*) is the solution of the system* S_{α} *and* $\mathscr{C}_{\alpha} = \{C_r \in \mathscr{C}_{\alpha-1} : \mathbf{x}_r^{\alpha-1} < 1\}$*, moreover* $\mathbf{x}_r^{-1} = 0$ *for any* C_r *in* $\mathscr{C}_o$*.*

We recall that, in possibility theory as well as in probability theory, coherence assures the extension of any (coherent) assessment to new events, by preserving coherence (see [6]). In particular, the coherent extension on any conditional event lays on a closed interval.

3 Likelihood as Point and Set Function

This session is devoted to a comparative analysis of likelihood in probabilistic and possibilistic framework. By Theorem 1 and by an analogous characterization (see e.g. [3]) of coherent conditional probability assessments the following result easily follows:

Theorem 2. *Let* $\mathscr{C} = \{E|H_i\}_{i=1,...,n}$*, be a set of conditional events, with* $\mathscr{I} = \{H_i\}_{i=1,...,n}$ *a partition of* Ω*. For any function* $f : \mathscr{C} \to [0,1]$ *such that*

$$f(E|H_i) = 0 \text{ if } E \wedge H_i = \emptyset \text{ and } f(E|H_i) = 1 \text{ if } H_i \subseteq E \tag{2}$$

the following statements hold:
i) f is a coherent conditional probability,
ii) f is a coherent T-conditional possibility.

Proof. Condition *i*) has been proved in [3]. To prove *ii*) let us consider the characterization of coherent T-conditional possibility given in Theorem 1. By the incompatibility of the events H_i, the equations of the system S_0 have different unknowns (each of them is linked only with the last equation), and so the system S_0 admits a solution assigning possibility 1 to each conditioning event H_i. Then, the assessment f is a coherent T-conditional possibility. □

The above theorem shows a syntactical coincidence between probabilistic and possibilistic (point) likelihood, so this allows to regard a probabilistic likelihood as a possibilistic one and vice versa without introducing inconsistency. Moreover, the above result puts in evidence that (in both contexts) no significant property characterizes likelihood as point function.

We are now interested on studying properties of aggregated likelihoods, that is all the coherent extensions of the assessment $f(E|H_i)$, ($H_i \in \mathscr{I}$) to the events $E|K$, with K any logical sum of the events H_i.

Theorem 3. *Let* $\mathscr{C}, \mathscr{I}$ *and f be as in Theorem 2 and let* $\mathscr{A}$ *be the algebra spanned by by* $\mathscr{I}$ *and* $\mathscr{H} = \mathscr{A} \setminus \emptyset$*. For any extension g of f on* $\{E\} \times \mathscr{H}$*, which is either a coherent conditional probability or a coherent T-conditional possibility assessment, the following condition holds for every* $H \in \mathscr{H}$*:*

$$\min_{H_i \subseteq H} f(E|H_i) \le g(E|H) \le \max_{H_i \subseteq H} f(E|H_i). \tag{3}$$

Proof. For coherent conditional probability the condition (3) is proved in [3]. Let f be a coherent T-conditional possibility assessment, then there is an extension $g = \Pi$ on $\mathscr{B} \times \mathscr{H}$, where $\mathscr{B}$ is the algebra generated by E and $\mathscr{H}$, and $\Pi(E|H) = \max_{H_i \subseteq H} T(\Pi(E|H_i), \Pi(H_i|H)) \leq \max_{H_i \subseteq H} \Pi(E|H_i)$. Moreover, by distributivity of maximum with respect to any t-norm T we have:

$$\Pi(E|H) = \max_{H_i \subseteq H} T(\Pi(E|H_i), \Pi(H_i|H)) \geq \max_{H_i \subseteq H} T(\beta, \Pi(H_i|H)) =$$
$$T\left(\beta, \max_{H_i \subseteq H} \Pi(H_i|H)\right) = T\left(\beta, 1\right) = \beta$$

where $\beta = \min_{H_i \subseteq H} \Pi(E|H_i)$. □

Remark 1. By condition (3) it follows immediately that both probability and possibility aggregated likelihood can be monotone, with respect to $\subseteq$, only if the extension can be obtained, for every H, as $\max_{H_i \subseteq H} f(E|H_i)$.

The following Theorem 4 assures that this (particular) extension is coherent. So we can conclude that both probability and possibility conditional probabilities, when they are regarded as function of the conditioning events, are capacities if and only if they are obtained through the maximum.

Theorem 4. *Let $\mathscr{C}$, $\mathscr{I}$, f and $\mathscr{H}$ be as in Theorem 3. For any extension g of f on $\{E\} \times \mathscr{H}$, and such that*

$$g(E|H \vee K) = \max\{g(E|H), g(E|K)\} \tag{4}$$

for every $H, K \in \mathscr{H}$ the following conditions hold:
i) g is a coherent conditional probability,
ii) g is a coherent T-conditional possibility.

Proof. The proof of *i)* is in [3]. To prove *ii)* consider as solution of system S_0 in Theorem 1 the possibility $\Pi_0(H_i) = 1$, for any $H_i \in \mathscr{C}$. □

Remark 2. Note that in Theorem 4 we state that g is a coherent T-conditional possibility, but $g(E|.)$ is not necessarily a possibility even if condition (4) holds for every $H, K \in \mathscr{H}$. In fact $g(E|\Omega)$ can be strictly less than 1.

Actually, $g(E|\Omega)$ is 1 if and only if there is H_i with $f(E|H_i) = 1$: this requirement could seem natural since it claims the existence of an event H_i supporting the evidence E.

In order to deepen the comparison of possibilistic and probabilistic aggregated likelihoods we need to introduce the notion of *scale* and then a relevant local form of monotonicity.

Definition 2. *Let $\mathscr{I} = \{H_1, ..., H_n\}$ be a partition of the sure event and $\mathscr{H} = \mathscr{A} \setminus \emptyset$ with $\mathscr{A}$ the algebra spanned by by $\mathscr{I}$. A scale of $\mathscr{H}$ is any partition $\{\mathscr{H}^0, ..., \mathscr{H}^k\}$ of $\mathscr{H}$, such that every $\mathscr{H}^i$ (with $i = 0, ..., k$) is an additive set and it contains at least one element $H_j \in \mathscr{I}$ and any $K \supseteq H_j$, with $K \in \mathscr{H} \setminus \bigcup_{k<i} \mathscr{H}^k$.*

Definition 3. *Let $\mathscr{H}$ as in Definition 2, a function $\varphi : \mathscr{H} \to [0,1]$ is scale monotone, with respect to a scale $\mathscr{H}^0,...,\mathscr{H}^k$ of $\mathscr{H}$, if φ, restricted to any $\mathscr{H}^i$ with $i = 0,...,k$, is monotone, with respect to the inclusion $\subseteq$.*

We give an example of a scale and a scale monotone function:

Example 1. Let $\mathscr{I} = \{H_1, H_2, H_3\}$ be a partition of the sure event and denote by $\mathscr{H}$ the algebra spanned by $\mathscr{I}$ less the impossible event. Consider the set $\mathbb{K} = \{\mathscr{H}^0, \mathscr{H}^1\}$ with $\mathscr{H}^0 = \{H_2, H_1 \vee H_2, H_2 \vee H_3, \Omega\}$ and $\mathscr{H}^1 = \{H_1, H_3, H_1 \vee H_3\}$. It is easy to check that $\mathbb{K}$ is a scale.

Consider now the assessment $\varphi(H_1) = 0.3$, $\varphi(H_2) = 0.5$, $\varphi(H_3) = 0.8$, $\varphi(H_1 \vee H_2) = 0.5$, $\varphi(H_1 \vee H_3) = 0.8$, $\varphi(H_2 \vee H_3) = 0.7$ and $\varphi(\Omega) = 0.75$.

For every $K \in \mathscr{H}$, condition (3) holds, so $\varphi(\cdot)$ is scale monotone with respect to $\mathbb{K}$.

Theorem 5. *Let $\mathscr{A}$ be a finite algebra, $\mathscr{H} = \mathscr{A} \setminus \emptyset$ and $\varphi(\cdot) = \Pi(E|\cdot)$ be a coherent T-conditional possibility on $\{E\} \times \mathscr{H}$. Then, there exists a scale $\{\mathscr{H}^0,...,\mathscr{H}^k\}$ of $\mathscr{H}$ such that φ is scale monotone with respect to it.*

Proof. If $\Pi(E|\cdot)$ is a coherent T-conditional possibility, then there is an extension (that we continue to denote by Π) on $\mathscr{B} \times \mathscr{H}$, where $\mathscr{B}$ is the algebra generated by E and $\mathscr{H}$. Put $\mathscr{H}^0 = \{H \in \mathscr{H} : \Pi(H|\Omega) = 1\}$ and define, for any $j = 1,...,k$, the set $\mathscr{H}^j = \{H \in \mathscr{H} : \Pi(H|H_0^{j-1}) = 1\}$ where $\mathscr{C}^{j+1} = \mathscr{H} \setminus \bigcup_{i=0}^{j}(\mathscr{H}^i)$ and $H_0^j = \bigvee_{H \in \mathscr{C}^j} H$. The class $\{\mathscr{H}^0,...,\mathscr{H}^k\}$ is a partition of $\mathscr{H}$ and each element is an additive set. Let $\mathscr{I} = \{H_1,...,H_n\}$ be the set of atoms of $\mathscr{H}$, since $\Pi(\cdot|H_0^{j-1})$ is a possibility, there is at least an atom H_i of $\mathscr{I}$ such that $\Pi(H_i|H_0^{j-1}) = 1 = \Pi(K|H_0^{j-1})$ for any $K \supseteq H_i$. Then, $\{\mathscr{H}^0,...,\mathscr{H}^k\}$ is a scale and, by construction, Π is monotone with respect to it: in fact, for any $H, H \vee K \in \mathscr{H}^i$ it follows $\Pi(E|H) = T(\Pi(E|H), \Pi(H|H \vee K)) = \Pi(E|H \vee K)$ since $\Pi(H|H \vee K) = 1$. □

We notice that for coherent conditional probabilities a similar result does not hold as the following example shows:

Example 2. Let $\mathscr{I} = \{H_1, H_2, H_3\}$ be a partition of the sure event, and consider the likelihood assessment: $P(E|H_1) = \frac{1}{2}, P(E|H_2) = \frac{1}{4}, P(E|H_3) = \frac{1}{8}$.

It is easy to prove that the following assessment

$P(E|H_1 \vee H_2) = \frac{3}{8}, P(E|H_1 \vee H_3) = \frac{5}{16}, P(E|H_2 \vee H_3) = \frac{3}{16}, P(E|\Omega) = \frac{7}{24}$

is a coherent conditional probability extending the above likelihood (the extension is obtained by giving $P(H_i) = 1/3$, for i=1,...,3).

Nevertheless, this probabilistic aggregated likelihood is not scale monotone. In fact, by definition, the first element $\mathscr{H}_0$ of any scale must contain an atom and all its supersets, so the thesis follows immediately by the following inequalities: $P(E|H_1) > P(E|H_1 \vee H_2)$, $P(E|H_2) > P(E|H_2 \vee H_3)$ and $P(E|H_2 \vee H_3) > P(E|H_1 \vee H_2 \vee H_3)$.

We notice that the necessary conditions given in Theorem 3 and Theorem 5 are not sufficient to characterize possibilistic aggregated likelihood as coherent extensions of a point likelihood, as shown in the following example:

Example 3. Let us consider again the scale $\mathbb{K}$ and the function φ of Example 1. We show that $\varphi(\cdot) = \Pi(E|\cdot)$ cannot be seen as a coherent conditional possibility. Let $C_i = E \wedge H_i$ and $C_{i+3} = E^c \wedge H_i$ $(i = 1, \ldots, 3)$ be the atoms spanned by $\{E, H_1, H_{2,3}\}$ and consider the following system with unknowns $x_r^0 = \Pi(C_r) \geq 0$ for $r = 1, \ldots, 6$

$$S_0^m = \begin{cases} x_1^0 = \min\{0.3, \max\{x_1^0, x_2^0\}\} \\ x_3^0 = \min\{0.5, \max\{x_3^0, x_4^0\}\} \\ x_5^0 = \min\{0.8, \max\{x_5^0, x_6^0\}\} \\ \max\{x_1^0, x_3^0\} = \min\{0.5, \max\{x_1^0, x_2^0, x_3^0, x_4^0\}\} \\ \max\{x_1^0, x_5^0\} = \min\{0.8, \max\{x_1^0, x_2^0, x_5^0, x_6^0\}\} \\ \max\{x_3^0, x_5^0\} = \min\{0.7, \max\{x_3^0, x_4^0, x_5^0, x_6^0\}\} \\ \max\{x_1^0, x_3^0, x_5^0\} = \min\{0.75, \max\{x_1^0, x_2^0, x_3^0, x_4^0, x_5^0, x_6^0\}\} \\ \max\{x_1^0, x_2^0, x_3^0, x_4^0, x_5^0, x_6^0\} = 1 \end{cases}$$

The systems S_0^m admits no solution: in fact, only x_2^0, x_4^0 and x_6^0 can assume value 1, but the seventh equation forces to be $x_2^0 < 1$ and $x_6^0 < 1$ in the fifth one, while the sixth equation implies $x_4^0 < 1$ in the seventh one.

Similarly, it is possible to prove that φ is not a coherent T-conditional possibility, for any strict t-norm T.

The following result characterizes all the coherent extensions of a likelihood $f(E|\cdot)$ as coherent T-conditional possibility (with T either strict t-norm or min). This allows an easy comparison.

Theorem 6. *Let $\mathscr{C}$, $\mathscr{I}$, f and $\mathscr{H}$ be as in Theorem 3. For any extension Π of f on $\{E\} \times \mathscr{H}$, and for every strict t-norm T the following two statements are equivalent:*

i) Π is a coherent T-conditional possibility extending f to $\mathscr{K} = \{E\} \times \mathscr{H}$;

ii) there exist a class of subfamilies $\mathscr{H}_\alpha$ $(\alpha = 0, ..., k \leq n-1)$, with $\mathscr{H}_\alpha \supset \mathscr{H}_{\alpha+1}$, and sets of coefficients $\lambda_i^\alpha \geq 0$ with $\max_i \lambda_i^\alpha = 1$, $\lambda_i^{-1} = 0$ for any i, and $H_i \in \mathscr{H}_\alpha$ if and only if $\lambda_i^{\alpha-1} = 0$, such that for every $H \in \mathscr{H}$ the value $x = \Pi(E|H)$ is a solution of

$$T(x, \max_{H_i \subseteq H} \lambda_i^\alpha) = \max_{H_i \subseteq H} T(\lambda_i^\alpha, \Pi(E|H_i)) \tag{5}$$

for every α such that $H_i \in \mathscr{H}_\alpha$ when $H_i \subseteq H$.

Proof. Since f is a coherent T-conditional possibility then there is at least a T-conditional possibility Π on $\mathscr{B} \times \mathscr{H}$ extending it. So, by Theorem 1 there exists a sequence of compatible systems and for any $H \in \mathscr{H}$ there is a j such that $\max_{C_r \subseteq H} \mathbf{x}_r^j = 1$ and $\Pi(E|H)$ is solution of the following equation for any $i = 0, ..., j$

$$T(\Pi(E|H), \max_{C_r \subseteq H} \mathbf{x}_r^i) = \max_{C_r \subseteq E \wedge H} \mathbf{x}_r^i.$$

That implies the existence of λ_r^i with $H_r \in \mathscr{H}_i$ such that

$$T(\Pi(E|H), \max_{H_r \subseteq H} \lambda_r^i(H)) = \max_{H_r \subseteq H} T(\pi(E|H_r), \lambda_r^i). \qquad \square$$

Note that in the case of product t-norm the aggregate likelihood is the maximum of a **weighted combination** of the $\Pi(E|H_i)$'s (with weights λ_i^j) over the maximum of the weights.

A similar characterization can be given for the t-norm minimum:

Theorem 7. *Let $\mathscr{C}, \mathscr{I}$, f and $\mathscr{H}$ be as in Theorem 3. For any extension Π of f on $\{E\} \times \mathscr{H}$, the following two statements are equivalent:*

i) Π is a coherent conditional possibility extending f to $\mathscr{K} = \{E\} \times \mathscr{H}$;

ii) there exist a class of subfamilies $\mathscr{H}_\alpha$ ($\alpha = 0,...,k \leq n-1$), with $\mathscr{H}_\alpha \supset \mathscr{H}_{\alpha+1}$, and sets of coefficients $\lambda_i^\alpha \geq 0$ with $\max_i \lambda_i^\alpha = 1$, $\lambda_i^{-1} = 0$ for any i, and $H_i \in \mathscr{H}_\alpha$ if and only if $\lambda_i^{\alpha-1} \leq f(E|H_i)$, such that for every $H \in \mathscr{H}$ the value $x = \Pi(E|H)$ is a solution of

$$\min\{x, \max_{H_i \subseteq H} \lambda_i^\alpha\} = \max_{H_i \subseteq H} \min\{\lambda_i^\alpha, \Pi(E|H_i)\} \tag{6}$$

for every α such that $H_i \in \mathscr{H}_\alpha$ when $H_i \subseteq H$.

We recall the quoted characterization for the probabilistic aggregated likelihood.

Theorem 8. *Let $\mathscr{C}, \mathscr{I}$, f and $\mathscr{H}$ be as in Theorem 3. For any extension P of f on $\{E\} \times \mathscr{H}$, the following two statements are equivalent:*

i) P is a coherent conditional probability extending f to $\mathscr{K} = \{E\} \times \mathscr{H}$;

ii) there exist a class of subfamilies $\mathscr{H}_\alpha$ ($\alpha = 0,...,k \leq n-1$), with $\mathscr{H}_\alpha \supset \mathscr{H}_{\alpha+1}$, and sets of coefficients $\lambda_i^\alpha \geq 0$ with $\max_i \lambda_i^\alpha = 1$, $\lambda_i^{-1} = 0$ for any i, and $H_i \in \mathscr{H}_\alpha$ if and only if $\lambda_i^{\alpha-1} = 0$, such that for every $H \in \mathscr{H}$ the value $x = P(E|H)$ is a solution of

$$x \sum_{H_i \subseteq H} \lambda_i^\alpha = \sum_{H_i \subseteq H} \lambda_i^\alpha P(E|H_i) \tag{7}$$

for every α such that $H_i \in \mathscr{H}_\alpha$ when $H_i \subseteq H$.

Note that while in the probabilistic context the aggregated likelihood is obtained as **weighted mean** of the $P(E|H_i)$'s, in the possibilistic one the aggregated likelihood is obtained as solution of equation (5) for strict t-norms and (6) for the minimum. However in the case of product t-norm the aggregate likelihood, as recalled above, is the maximum of a weighted combination of the $\Pi(E|H_i)$'s. In both probabilistic and possibilistic cases weights equal to zero or one are allowed and in the case where the likelihood is scale monotone with respect to a scale with the maximum number n of elements (n equal to the number of elementary events), then the extensions coincides.

Remark 3. Notice that if we can assign $\lambda_i^0 = 1$ for every i, then we obtain only one class of λ_i^α, and we get a coherent T-conditional possibility taking only the values assumed by the likelihood. Moreover, the above aggregated likelihood is obtained (for every α) by giving value 1 to only one $\lambda_{i^*}^\alpha$, and value 0 to all others. So in this case the aggregated likelihood is also a coherent conditional probability.

A particular case of this situation is when at any step α the value 1 corresponds, for $E|H_i \in \mathscr{H}_\alpha$, to the *maximum* (or *minimum*) value of $\Pi(E|H_i)$.

4 Conclusions

We compare the likelihood function in probabilistic and possibilistic contexts both as point and set functions. For this aim we provide also a characterization of coherent extensions of a possibilistic likelihood. The results can be extended to any conditional decomposable measure (for the definition see e.g. [3]): this extension is immediate for Theorems 2, 3, 4.

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