Mathematical Model of Blood Flow in Arteries with Porous Effects

Anil Kumar

Department of Applied Mathematics, Dronacharya College of Engineering, Greater Noida, Gautam Budhnagar UP India -201306, (Affiliated UP Technical University Lucknow UP India)

Abstract— **The present study a mathematical model for blood flow in narrow arteries in the presence of porous effect has been investigated. It is assumed that there is a lubricating layer between red blood cells and tube wall. The analysis of fluid flow between red cell and tube wall, when the cell appears to be at rest and the capillary wall moving backwards, is made. The effect of porous on the flow is examined. The computational results are shown through graphs to describe the effect of various parameters on velocity profile, leak-back flow rate and skin friction.**

Keywords— **Blood Flow, Single File Flow, Narrow Capillaries, Lubricating Zone, Porous medium.**

I. INTRODUCTION

Recently Bio- Fluid Dynamics has emerged as a new field for the study of the fluid dynamical behavior of biological fluids in the effect of porous. The control of the blood pressure has been possible by using porous effect in cases of cholesterol and related diseases. For describing the mechanics of red blood cell motion in narrow capillaries, we distinguish two situations according to the convenience with which the cells fit into the vessels. In the first case when the capillary has diameter larger than that of the cell, the cell can fit into the tube without distortion; this flow situation is called positive clearance. In the second situation called negative clearance, when the diameter of the cell is larger than that of capillary as such the cell will be deformed in order to fit into the capillary. In this case pressure must be generated in thin layer of fluid round the edge of the cell in order to deform it and depends on elastic properties of the cell. When red cell is severely deformed then in blood flow the red cell seems to plug the capillary of blood vessel and the motion of the plasma in capillary between successive red cells is called bolus flow. The significance of bolus flow was pointed out by Prothero et al. [1]. Pressure within capillaries shows periodic fluctuations; such variation can result both from local functions and from changes in central arterial and venous pressure. Chapman et al.[2] developed a mathematical model for the blood flow through the leaky neovasculature and porous interstitium of a solid tumor. Weinberg and Mofrad [3] considered a three dimensional model to see the effects of geometric factor on multiscale valve mechanics. A multiphase kinetic theory for the computation of viscosity of red blood cells and their migration from vessel walls has been discussed by Huang et al. [4]. In the above mentioned studies, no attempt has been made to study the effect of magnetic field on stenosis under porous medium together [Gupta (6)].

Various studies have been performed on rheological behavior of blood flow in narrow capillaries to establish relationship among resistance, viscosity, clearance (both positive and negative) of cell and other parameters. From earlier works, we observe that the flow resistance for plasma in narrow capillaries is greater than in large capillaries. The study of flow under the influence of a porous effect is also important so far as flow and resistance are concerned. In the present investigation, we study the motion of the blood through a very narrow capillary under the action of the porous effects. Our study is confined to microcirculation i.e. single file flow of red blood cell through narrow capillary.

II. MATHEMATICAL MODEL

In this model we consider the axially symmetric and Newtonian flow of blood in a tube of uniform radius. The blood is assumed to be homogeneous fluid, while the red blood cells are assumed to be elastic and incompressible. The single cell is fitted in the tube so as to generate a single file flow. In this investigation, we study fluid flow in lubricating zone i. e. fluid flow between red blood cell and tube wall. The effect of porous on the flow of narrow capillary is taken into account. The viscous forces are predominant in the flow of such tubes. The inertial terms are considered negligible. During passing down single red cell in narrow capillary, it deforms due to its elastic property. The viscous forces are predominant in the flow of such tubes. The inertial terms are considered negligible. During passing down single red cell in narrow capillary, it deforms due to its elastic property. The shape of red cell is bi-concave disk. [Kumar et al.(5)]

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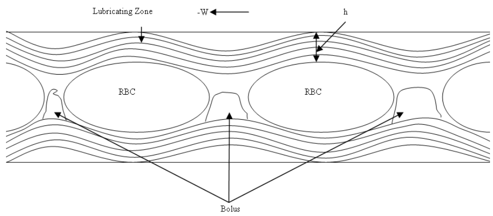


Fig. 1 Schematic diagram of fluid flow in lubricating zone

The equations governing the motion fluid flow are:

$$\frac{\partial(rv)}{\partial r}+\frac{\partial(rw)}{\partial z}=0 \tag{1}$$

$$\rho\frac{\partial w}{\partial t}=-\frac{\partial p}{\partial z}+\mu\left(\frac{\partial^2 w}{\partial r^2}+\frac{1}{r}\frac{\partial w}{\partial r}\right)-\frac{\sigma\mu}{K}w \tag{2}$$

Where u and v are the axial (z) and radial (r) velocity components, ρ is the constant fluid density eliminating fluid pressure p and μ is viscosity of the fluid.

Boundary conditions are:

$$\begin{aligned} &w=We^{-int}, v=0, \quad at\, r=h, \quad t>0 \\ &w=0, v=0, \quad at\, r=0, \quad t>0 \end{aligned} \tag{3}$$

From (1) and (3), we have:

$$\int_0^h rw\,dr=-Q_{lb} \tag{4}$$

The pressure gradient is:

$$\frac{\partial p}{\partial z}=-Pe^{-\text{int}} \tag{5}$$

Also we assume that $w(r,t)=f(r)e^{-int}$ without loss of generality.

From equation (2) and using equation (5) and w(r,t), we get

$$f''(r)+\frac{1}{r}f'(r)-\frac{B}{\mu}f(r)=-\frac{P}{\mu} \tag{6}$$

Where $B=-in\rho+\dfrac{\sigma\mu}{K}$.

The equation (6) is solve by Bessel function of series, we get

$$w=-W\frac{r^2\left(16\mu+Br^2\right)}{h^2\left(16\mu+Bh^2\right)}e^{-\text{int}} \tag{7}$$

The velocity relative to tube wall is given by:

$w_1 = w + W \cos nt$

III. Leak-back flow rate and skin friction

The leak-back flow rate Q_{lb} is given by:

$$Q_{lb}=-\int_0^h rw\,dr=W$$

The skin friction at the RBC surface is given by:

$$\tau_{rbc}=-\mu\left(\frac{1}{r}\frac{\partial w}{\partial r}\right)_{r=0}$$

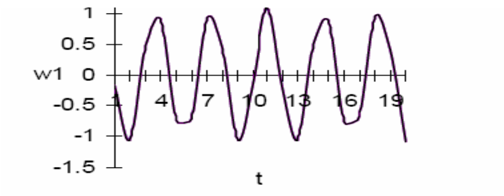
IV. Numerical results and discussion

In this section, we validate the analytical results for blood flow in very narrow capillaries by setting the default parameters as h = 0.5, μ = 3, σ = 2, μe = 1, ρ = 1.05, n = 8, w = 1.1, r = 0.05, μ = 2.5 and K=3.

Figures 2a-c shows that the relative velocity profiles for different values of K, μ and r in lubricating zone. It is noticed that the relative flow velocity along the capillary between RBC and tube wall is almost uniform and the effect of magnetic intensity on flow velocity is negligible (see Figure 2a). It is also clear from these figures that the flow is pulsatile where velocity changes periodically. It is seen in Figure 2b that the effect of viscosity (μ) on the flow velocity relative to the tube wall is almost negligible. From Figure 2c, we see that the velocity relative to the tube (remember that the tube velocity relative to RBC is-W) increases as transverse distance from highest point of RBC surface decreases and at the wall it is zero. From this we infer that the velocity relative to the tube wall declines towards wall and the velocity along the transverse distance is independent of viscosity Figures 3a-c display leak-back flow rate for different values of μ and K. It is noted that the leak-back flow rate increases as K increases. For the values of K from 10 to 100, the flow rate increases slightly but beyond these values, the flow rate is independent of K. But on decreasing the value of K(<10), the leak-back flow rate suddenly declines. For the increasing viscosity the leak-back flow rate retards significantly.

Figures 4a-c displays the effect of thickness (gap between RBC and tube wall) of fluid on leak back for different values of μ. The leak-back flow rate increases as h increases whereas on increasing viscosity, the flow rate decreases. This demonstrates that the effect of thickness and viscosity on flow rate is remarkable. Figures 5a-c exhibit skin friction vs. time for different values of K and μ the skin friction decreases (increases) as K (μ). At K = 100 and beyond this value the skin friction is zero. Thus the effects of K and viscosity on skin friction can not be ignored.

Figures 6a-c displays the profiles for skin friction for different values of h and μ. The skin friction at the surface of RBC decreases as the thickness (gap between RBC and tube wall) of lubricating zone increases.



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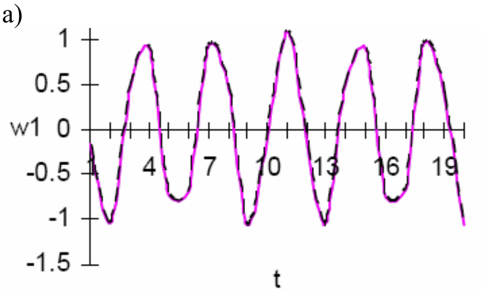

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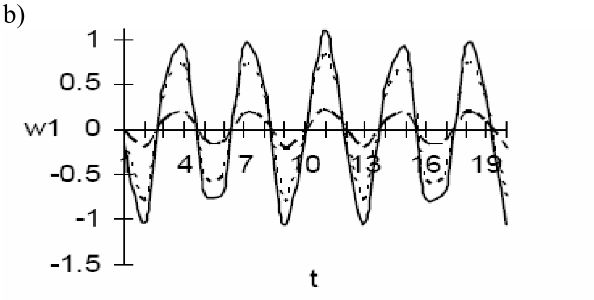
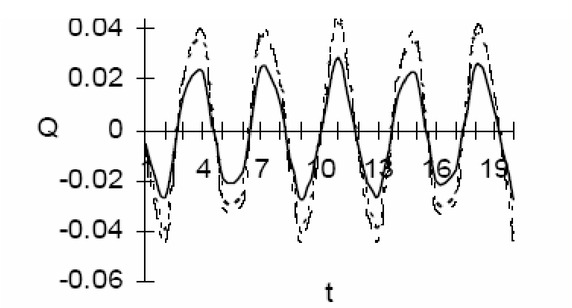


c)

Fig. 2 Velocity profiles for different values of (a) K, (b)viscosity (μ) and (c) r



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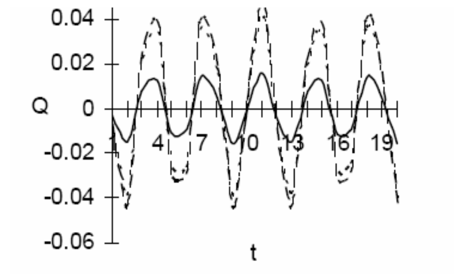

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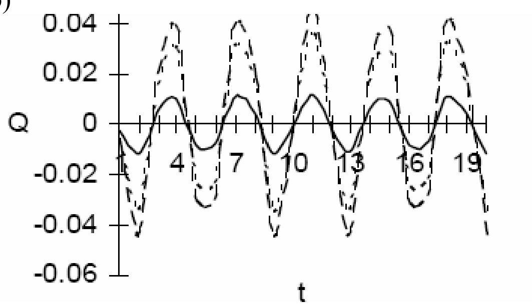

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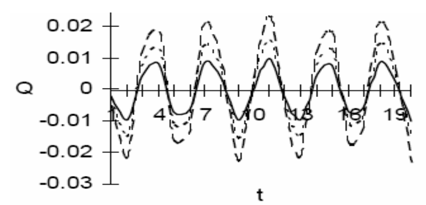
Fig. 3 Leak-back flow rate for different values of K, and for (a) μ = 2.5, (b) μ = 4.5 and (c) μ = 6



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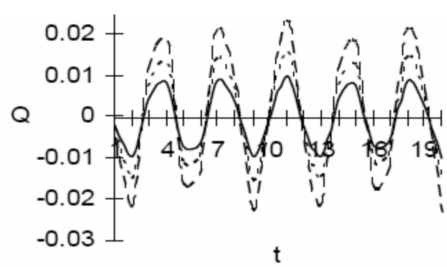

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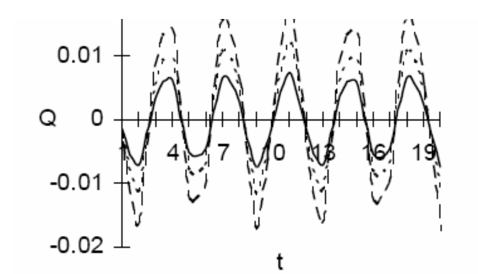
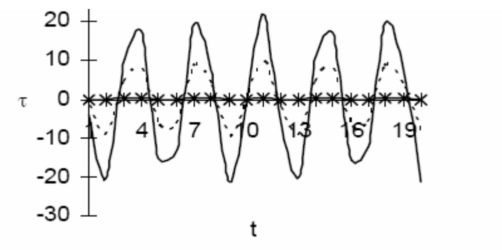


c

Fig. 4 Leak-back flow rate for different values of h, and for (a) μ = 2.5, (b) μ = 4.5 and (c) μ = 6



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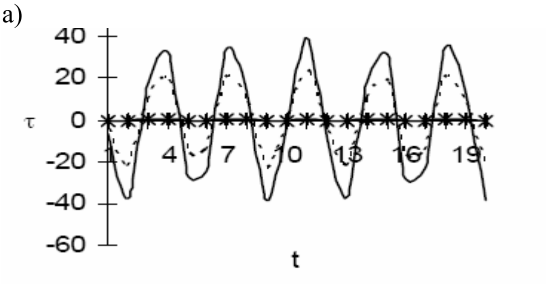

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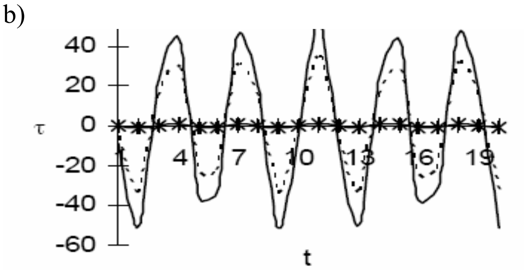

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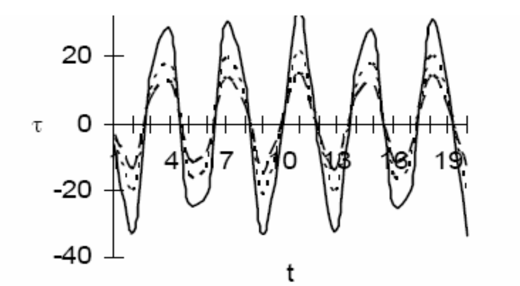
Fig. 5 Skin friction vs time for different values of K, and for (a) μ = 2.5, (b) μ = 4.5 and (c) μ = 6



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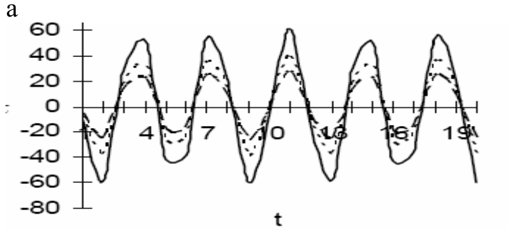

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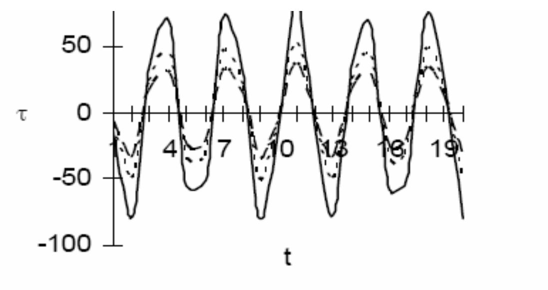

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Fig. 6 Skin friction vs time for various values of h, and for (a) μ = 2.5, (b) μ = 4.5 and (c) μ = 6

V. Concluding remarks

In this investigation, we have developed a mathematical model for blood flow in very narrow capillaries. The velocity and volumetric flow rate are calculated and validated by numerical results. The effect of porous on leak-back flow rate and skin friction seems to be significant. The viscosity effect is very much dependent on the thickness of the lubricating zone (thickness between RBC and tube wall). I hope that our investigation may be helpful for the medical practitioners and other persons in the area of bio fluid dynamics to understand the flow of blood in the presence of porous effects. The results derived may be useful for hypertension patients through magnetic therapy.

References

1. Prothero, J. and Burton, A.C., "The Physics of Blood in Capillaries", the Nature of Motion, Biophys. J., Vol. 1, (1961), 567-579.
2. Chapman, J. S., Shipley, R. J. and Jawad, R. (2008): Multiscale modeling of fluid transport in tumor, Bull. Math. Biol., Vol. 70, pp. 2334-2357.
3. Weinberg, E. J. and Mofrad, M. R. K. (2008): A multiscale computional comparison of the bicuspid and tricuspid aortic valves in relation to calcific aoretic stenosis, J. Biomech., Vol. 41, No. 16, pp. 3482-3487.
4. Huang, J., Lyczkowski, R. W. and Gidaspow, D. (2009): pulsatile flow in a coronary artery using multiphase kinetic theory, J. Biomech., Vol. 42, No. 6, pp. 743-754.
5. Kumar, Anil, Varshney C. L. and Sharma G.C. (2005). Computational technique for flow in blood vessels with porous effects. Applied Mathematics and Mechanics, 26, 63-72.
6. Gupta, Anil Kumar (2009): Finite element Galerkin's scheme for flow in blood vessels with magnetic effects, Int. J. applied systemic studies, vol.3(2), 283-293.

Anil Kumar
Dronacharya College of Engineering
B-27 Knowledge Park III,
Greater Noida Gautam Budhnagar UP-201306
India
dranilkumar73@rediffmail.com