An Algorithm for Designing Controllers

Sankalp Bagaria

Centre for Development of Advanced Computing, Mumbai, India
Sankalp@cdacmumbai.in

Abstract. In control engineering, any system that takes some input signals, processes it and gives output signals, e.g. an electrical transformer, a mechanical lever, a car's hydraulic brake system, a country's economy, population of a country etc., can be treated like a black – box plant. Any given plant should be stable and also should perform well in terms of speed, accuracy and other properties. If it does not behave so, another system called controller is designed which is used in conjunction with the plant. In this paper, we propose an alternative, easy-to-use algebraic method to find the transfer function of the controller with better accuracy than graphical methods like root locus and bode plot. Our algorithm has complexity O(n^3).

Keywords: Plan; Controller; Transfer function; Design; Algorithm.

1 Introduction

A system can be electrical, mechanical, chemical, fluid, financial, biological etc. The mathematical modeling of the system, its analysis and the controller design is done using control theory in the time, complex – s or frequency domain depending on the nature of the control design problem. Control Theory is an interdisciplinary branch of engineering and mathematics. It deals with the behavior of dynamical systems. The desired output of system is called the reference. When one or more output variables of a system need to follow a certain reference over time, a controller manipulates the inputs to the system to obtain the desired effect on the output of the system.

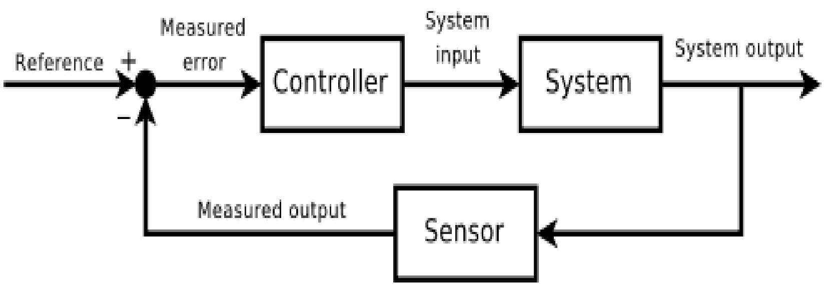

Fig. 1. A Typical Control System

N. Meghanathan et al. (Eds.): NeCoM, WiMoN, and WeST 2010, CCIS 90, pp. 235–242, 2010.

Take a fan for an example. The regulator (controller) controls the voltage sent to the motor of the fan (system). If the voltage is high, the fan has higher speed. It is an example of open – loop system. Now, take an air – conditioner. A thermometer (sensor) measures the temperature of the room and this data is used to control the time for which the cooling system is on so that the temperature desired (reference) is maintained. It is an example of on – off controller. If the difference between the reference temperature and the actual temperature is greater than zero, AC is switched on and otherwise, it is switched off.

2 Control System Design Problem

The output of the system y(t) is fed back through a sensor measurement F to the reference value r(t). The controller C then takes the error e (difference) between the reference and the output to change the inputs u to the system under control P. This is shown in the figure. This kind of controller is a closed-loop controller or feedback controller.

This is called a single-input-single-output (SISO) control system; MIMO (i.e. Multi-Input-Multi-Output) systems, with more than one input/output, are also used.

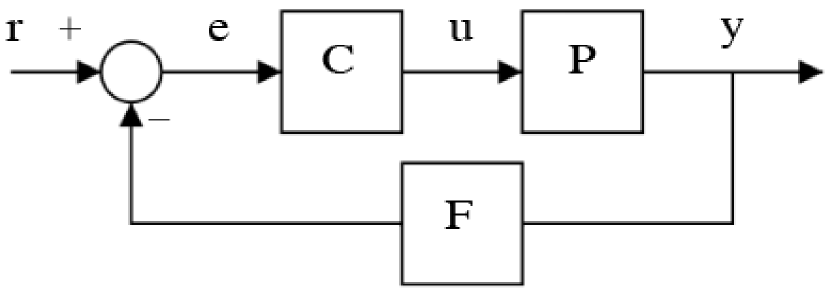

Fig. 2. Feedback Control System

If we assume the controller C, the plant P and the sensor F are linear and time-invariant (i.e.: elements of their transfer function C(s), P(s), and F(s) do not depend on time), the systems above can be analysed using Laplace transform on the variables. This gives the following relations:

$$Y(s) = P(s)U(s)$$

$$U(s) = C(s)E(s)$$

$$E(s) = R(s) - F(s)Y(s).$$

Solving for Y(s) in terms of R(s) gives:

$$Y(s) = \left(\frac{P(s)C(s)}{1 + F(s)P(s)C(s)}\right) R(s) = H(s)R(s).$$

The expression $H(s) = \dfrac{P(s)C(s)}{1 + F(s)P(s)C(s)}$ is referred to as the closed-loop transfer function of the system. The numerator is the forward (open-loop) gain from r to y, and the denominator is one plus the gain in going around the feedback loop, the so-called loop gain. If $|P(s)C(s)| \gg 1$, i.e. it has a large norm with each value of s, and if $|F(s)| \approx 1$, then Y(s) is approximately equal to R(s). This means simply setting the reference controls the output.

Most of the systems in our world, ranging from a simple lever to PID controllers in industry, can be modeled as LTIL systems. Usually such a system is expressed as a differential equation in time domain (an equation relating input and output) and is converted to an algebraic equation in s – domain using Laplace. The output is studied for knowing about system's stability, its rise time, settling time ie, how fast or slow is the response of the system to change in input. The sensitivity of the system to the disturbances and the noise is studied. If the system's response is not satisfactory, a controller is designed. Its Laplace equation is found and then used to make the system.

3 Guidelines for Designing a Stable, Useful System

Time domain specifications:

1. Peak overshoot < 10 %
2. Settling time < 5 seconds
3. Rise time a small as possible
4. Steady state error < 10 %

Frequency domain specifications:

1. Bandwidth < 10 krad/sec
2. Gain margin > 20 decibels
3. Phase margin > 45 degrees

4 Some Heuristics for Designing the Closed-Loop Transfer Function

1. For a system to be stable, all the poles of its transfer function should be in the left half of the s-plane ie, their real part should be negative.
2. Settling time is approximately 4.5 * time – constant
3. Poles nearer to the real line cause lesser overshoot but the system becomes sluggish.
4. A system should not only be absolutely stable but relatively stable too so that it is robust and should tolerate noise, disturbances and perturbations.

5 Design Problem

Let the denominator of the closed loop transfer function calculated above be ø(s). Also, let the transfer function of the plant P(s) be Np/Dp and that of the controller C(s) be Nc/Dc.

Closed - loop transfer function is then

$$\frac{\frac{Nc\,Np}{Dc\,Dp}}{1+\frac{Nc\,Np}{Dc\,Dp}} = \frac{Nc\,Np}{NcNp+DpDc}$$

Now, we have the transfer function Np/Dp of the plant and the closed – loop transfer function. We have to find suitable Nc & Dc, such that NpNc + DpDc = ø(s).

6 Algorithm for Computing Controllers for SISO System

1. Compute the desired characteristic polynomial ø(s). Degree of ø(s) must not be less than that of Np & Dp.
2. Choose degree of Nc & Dc such that it is the difference of the degrees of ø(s) and the larger of the degrees of polynomials Np and Dp.
3. Assign arbitrary variables to the unknown coefficients of polynomials Nc & Dc.
4. Equate the coefficients of various terms of NpNc + DpDc & ø(s) and form the equations in the variables representing the coefficients of Nc, Dc.
5. Solve the equations of step 4 for the unknown coefficients of Nc, Dc. If there are too many equations, Gaussian elimination method maybe used.
6. Thus the values of the coefficients of Nc, Dc will be obtained, given the desired characteristic polynomial ø(s). If the equations could not be solved for any value of the variables, the desired characteristic polynomial will have to be changed by increasing its degree (adding a single pole or a pair of conjugate poles to closed – loop transfer function) and the steps be repeated from step 2.

Examples:

1. Np=1 Nc = a

   Dp=s-1 Dc = b

   Ø=s+10 (Pole at s = -10)

   1*a + (s-1)*b = s+10

   s:b=1

   const : -b+a=10 => a=11

   $\frac{Nc}{Dc} = 11$

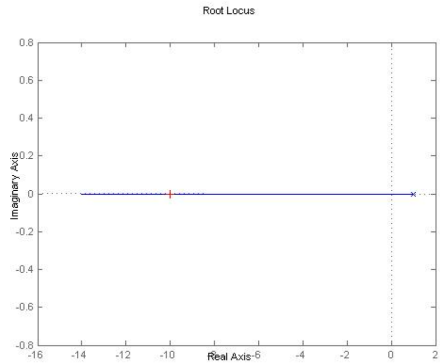

Fig. 3.

The figure 3 is the root locus of example 1. It shows how a compensator gain of 11 in a feedback system will place the pole of the overall transfer function at -10.

2. Np = 2 Nc=as+b

 Dp = s-1 Dc=cs+d

 Ø = s2 + 2s + 1 [Poles at s = -1, -1]

 2(as + b) + (s – 1)(cs + d) = s2 + 2s + 1

 => s2 : c = 1

 s1 : 2a + d – c = 2 => 2a + d = 3

 const: 2b – d = 1 => 2b – d = 1

 Choosing d as the parameter,

 $a = \frac{3-d}{2}$

 $b = \frac{1+d}{2}$

 c = 1

 d = d

 One possible choice is (d = 2)

 $\frac{0.5s + 1.5}{s+1} = \frac{s + 3}{2\ (s+2)}$

The figure 4 shows root locus of the example 2. It shows how a constant gain compensator for the above example can never yield two poles in the desired transfer function of the overall system. So, we add a first – order compensator (degree of the rational function of the compensator being 1) to the system and get the root locus shown in figure 5. We then get the desired pole – zero configuration as shown in the figure.

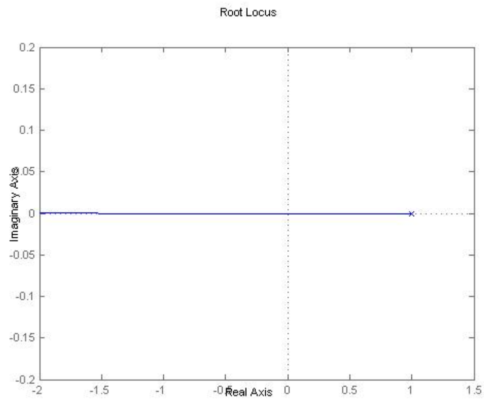

Fig. 4.

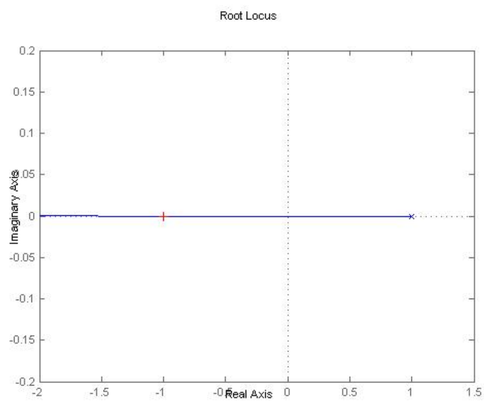

Fig. 5.

3. Np = s+2

   Dp = s2 + s + 4

   Desired poles: -2,-3 ± 3i

   Ø = (s+2)(s2+6s+18)

   = s3 + 8s2 + 30s + 36

   (as + b)(s + 2) + (cs + d)(s2 + s + 4)

   = s3 + 8s2 + 30s + 36

   s 3 : c = 1

   s 2 : a + d + c = 8 => a + d = 7

   s1: 2a + b + 4c + d = 30 => 2a + b + d = 26

   const.: 2b + 4d = 36 => b + 2d = 18

Substituting a = 7-d and b – 18-2d into 2a + b + d = 26, we get

2(7 – d) + (18 – 2d) + d = 26

⇨ 14 – 2d + 18 – 2d + d = 26

⇨ -3d = -32 + 26 = -6

⇨ d = 2.

=> a = 7 – d = 5

b = 18 – 2d = 14

$\frac{Nc}{Dc} = \frac{5s+14}{s+2}$

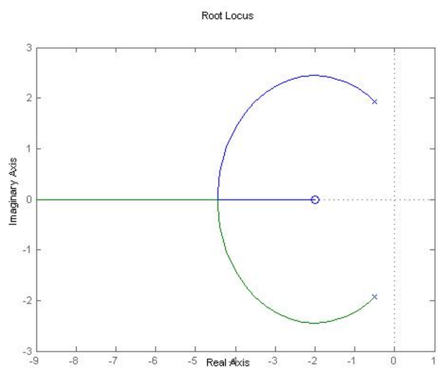

Fig. 6.

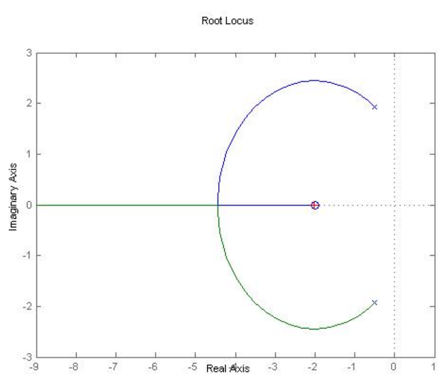

Fig. 7.

The figure 6 is the root locus of the example 3. It shows how a two – pole plant cannot give three desired poles. So, we add a first – order compensator and use our algorithm to find the coefficients of the compensator. The root locus (figure 7) is then drawn again to show how the desired pole – zero configuration is achieved. If the number of poles in the transfer function of the desired over-all system is greater than two, we can add more number of poles and zeroes in our compensator to yield the desired transfer function.

7 Conclusions

If n is the degree of the characteristic polynomial, the complexity of this algorithm is O(n^3) as Gausian elimination is used to solve the equations in step 5. This control system algorithm has been verified by comparing designs with root locus drawn using MATLAB. Further restrictions on the coefficients of controller transfer function can be integrated with this procedure to get even better suited controllers. We described the procedure for signals continuous in time and used Laplace transform to convert from time – domain to complex-s domain. As can be easily seen, the method hold for systems involving discrete – time signals, which can be mapped to complex – z domain using z – transform and the controller can be designed accordingly. As further work, this method can be extended to MIMO (Multiple Input, Multiple Output) systems by converting state space design to transfer function based pole placement design and then obtaining the coefficients of the controller matrix by the above method.

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