Implementing Prioritized Merging with ASP

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Abstract. The paper addresses the extension of the removed sets framework to prioritized removed sets fusion (PRSF). It discusses the links between PRSF and iterated removed sets revision and shows that PRSF satisfy most of the postulates proposed for prioritized merging. An implementation of this new syntactic prioritized fusion operator is proposed thanks to answer sets programming.

Keywords: fusion, iterated revision, ASP reasoning under inconsistency.

1 Introduction

In the last decade, multiple sources belief bases merging has been widely discussed [11,29,3]. Postulates characterizing the rational behavior of merging operations have been proposed [22]. Several merging operations have been proposed that can be divided into two families. The semantic (or model-based) ones which select interpretations that are the "closest" to the original belief bases [22,23,20,12,30,21,10] and the syntactic (or formula-based) ones which select some formulas from the initial bases [25,33,17,13,5].

In some situations, the belief bases are not flat and the beliefs are stratified or equipped with priority levels, in other cases the belief bases are flat but the sources are not equally reliable and there exists a preference relation between sources. In such cases, prioritized merging consists of combining belief bases taking into account the stratification of the belief bases or the preference relation [6]. Prioritized merging has been studied within the framework of propositional logic [12,33,18] as well as within the possibilistic logic one [7,5,8]. The links between iterated revision and prioritized merging has been discussed and rational postulates for prioritized merging have been proposed in [12]. Different iterated revision operations have been studied and it has been shown that the DMA approach [9] then the Qi's approach [27] can be characterized by a lexicographic strategy.

This paper addresses the extension of the Removed Sets framework to prioritized merging where preferences are expressed between belief bases. A new syntactic prioritized fusion operation is proposed and can be expressed in terms

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of iterated removed sets revision. This operation satisfies most of the postulates for prioritized merging. An implementation stemming from the Answer Sets Programming is described.

The rest of the paper is organized as follows. Section 1 gives a refresher on prioritized merging, removed sets revision, removed sets fusion and logic programming with answer sets semantics. Section 2 presents the prioritized removed sets fusion (PRSF). Section 3 discusses the links between PRSF and iterated removed sets revision and shows that PRSF satisfies the proposed postulates for prioritized merging. Section 4 presents an implementation of PRSF based on logic programming with answer sets semantics before concluding.

2 Background

2.1 Notations

Throughout the paper we consider a propositional language $\mathcal{L}$ over a finite alphabet $\mathcal{P}$ of atoms. A literal is an atom or the negation of an atom. The usual propositional connectives are denoted by $\neg$, $\wedge$, $\vee$, $\rightarrow$, $\leftrightarrow$ and Cn denotes the logical consequence. A *belief base* K is a finite set of propositional formulas over $\mathcal{L}$. Let $E = \{K_1, \ldots, K_n\}$ be a multi-set of n consistent belief bases to be merged, E is called a *belief profile*. The n belief bases $K_1, \ldots, K_n$ are not necessarily different and the union of belief bases, taking repetitions into account, is denoted by $\sqcup$ and their conjunction (resp. disjunction) are denoted by $\bigwedge$ (resp. $\bigvee$).

2.2 Removed Sets Revision

We briefly recall the Removed Sets Revision (RSR) approach. RSR [32] deals with the revision of a set of propositional formulas by a set of propositional formulas[1]. Let K and A be finite sets of clauses. Removed Sets Revision (RSR) focuses on the minimal subsets of clauses to remove from K, called *removed sets*, in order to restore the consistency of $K \cup A$. More formally: let K and A be two consistent sets of clauses such that $K \cup A$ is inconsistent. Let R be a subset of clauses of K, R is a removed set of $K \cup A$ iff (i) $(K \backslash R) \cup A$ is consistent; (ii) $\forall R' \subseteq K$, if $(K \backslash R') \cup A$ is consistent then $\mid R \mid \leq \mid R' \mid$[2]. Let denote by $\mathcal{R}(K \cup A)$ the collection of removed sets of $K \cup A$, RSR is defined as follows: let K and A be two consistent sets of clauses, $K \circ_{RSR} A =_{def} \bigvee_{R \in \mathcal{R}(K \cup A)} (K \backslash R) \cup A$.

2.3 Removed Sets Fusion

The Removed Sets Revision approach has been extended to the Removed Sets Fusion (RSF) for merging belief bases consisting of well-formed formulas. The key idea of the approach is to remove subsets of well-formed formulas from

[1] We consider propositional formulas in their equivalent conjunctive normal form (CNF).

[2] $\mid R \mid$ denotes the number of clauses of R.

the union of the belief bases, according to some strategy P, in order to restore consistency. Let $E = \{K_1, \ldots, K_n\}$ be a belief profile such that $K_1 \sqcup \cdots \sqcup K_n$ is inconsistent and let IC be integrity constraints. Removed Sets Fusion (RSF) provides, as a result of merging, subsets of formulas of $K_1 \sqcup \cdots \sqcup K_n$ which are consistent with IC.

Definition 1. *Let $E = \{K_1, \ldots, K_n\}$ be a belief profile and IC the integrity constraints such that $K_1 \sqcup \cdots \sqcup K_n \sqcup IC$ is inconsistent, $X \subseteq K_1 \sqcup \cdots \sqcup K_n$ is a* potential Removed Set *of E iff $((K_1 \sqcup \cdots \sqcup K_n) \backslash X) \sqcup IC$ is consistent.*

In order to select the most relevant potential Removed Sets according to a strategy P, a total preorder $\leq_P$ over the potential Removed Sets is defined ($X \leq_P Y$ means that X is preferred to Y according to the strategy P). The associated strict preorder is denoted by $<_P$.

Definition 2. *Let $E = \{K_1, \ldots, K_n\}$ be a belief profile and IC the integrity constraints such that $K_1 \sqcup \cdots \sqcup K_n \sqcup IC$ is inconsistent, $X \subseteq K_1 \sqcup \cdots \sqcup K_n$ is a* Removed Set *of E according to P iff* (i) *X is a potential removed set of E;* (ii) *there is no $Y \subseteq K_1 \sqcup \cdots \sqcup K_n$ such that $Y <_P X$.*

We denote by $\mathcal{F}_{P,IC}\mathcal{R}(E)$ the collection of removed sets of E constrainted by IC according to P. The definition of Removed Sets Fusion is:

Definition 3. *Let $E = \{K_1, \ldots, K_n\}$ be a belief profile and IC the integrity constraints such that $K_1 \sqcup \cdots \sqcup K_n \sqcup IC$ is inconsistent. The fusion operation $\Delta^P_{IC}(E)$ is defined by: $\Delta^P_{IC}(E) = \bigvee_{X \in \mathcal{F}_P\mathcal{R}(E)} \{((K_1 \sqcup \cdots \sqcup K_n) \backslash X) \sqcup IC\}$.*

The usual merging strategies ($Card, \Sigma, Max, GMax$) are captured within our framework by encoding the preference relation between potential removed sets as given in the following table. For the $GMax$ strategy, let X be a potential removed set and K_i be a belief base, we define $p^i(X) = |X \cap K_i|$. Let L^E_X be the sequence composed with every $(p^i(X))_{1 \leq i \leq n}$ in decreasing order. We denote by $\leq_{lex}$ the lexicographic ordering.

P	$X \leq_P Y$
$Card$	$\|X\| \leq \|Y\|$
Σ	$\Sigma_{1 \leq i \leq n} \mid X \cap K_i \mid \leq \Sigma_{1 \leq i \leq n} \|Y \cap K_i\|$
Max	$max_{1 \leq i \leq n} \mid X \cap K_i \mid \leq max_{1 \leq i \leq n} \mid Y \cap K_i\|$
$GMax$	$L^E_X \leq_{lex} L^E_Y$

2.4 Answer Set Programming

In the following, $c, a_i (1 \leq i \leq n), b_j (1 \leq j \leq m)$ are propositional atoms and the symbol *not* stands for *negation as failure*. A *normal logic program* is a set of rules of the form $c \leftarrow a_1, \ldots, a_n, not\ b_1, \ldots, not\ b_m$ where Let r be a rule, we introduce $head(r) = c$ and $body(r) = \{a_1, \cdots, a_n, b_1, \cdots, b_m\}$. Furthermore, let $body^+(r) = \{a_1, \cdots, a_n\}$ denotes the set of positive body atoms and $body^-(r) = \{b_1, \cdots, b_m\}$ the set of negative body atoms, it follows $body(r) = body^+(r) \cup$

$body^-(r)$. Moreover, r^+ denotes the rule $head(r) \leftarrow body^+(r)$, obtained from r by deleting all negative atoms in the body of r.

A set of atoms X is *closed under* a basic program Π iff for any rule $r \in \Pi$, $head(r) \in X$ whenever $body(r) \subseteq X$. The smallest set of atoms which is closed under a basic program Π is denoted by $CN(\Pi)$. The *reduct* or Gelfond-Lifschitz transformation [15], Π^X of a program Π relatively to a set X of atoms is defined by $\Pi^X = \{r^+ \mid r \in \Pi \text{ and } body^-(r) \cap X = \emptyset\}$. A set of atoms X is an *answer set* of Π iff $CN(\Pi^X) = X$.

2.5 ASP Solvers

In the last decade, answer set programming has been considered as a convenient tool to handle non-monotonic reasoning. Moreover, several efficient systems, called ASP solvers, have been developed for computing answer sets, Smodels [26], XSB [28], DLV [14], NoMore [1], ASSAT [24], CMODELS [16], CLASP [2].

In order to extend the expressivity and the efficiency of ASP solvers, logic programs have been extended with new statements [31]:

- *domain definitions* allow for compactly encoding the possible values in a given domain, e.g. the declarations $\#domain\ possible(X)$, $possible(1..n)$. ensure that every occurrence of the variable X will take a value from 1 to n.
- *domain restrictions* can be added in some rules. For example, $short(X) \leftarrow size(Y), X < Y$ is only instantiated for X and Y such that $X < Y$.
- *cardinality optimization* make possible to express that at most, respectively at least, some atoms should appear in the answer sets. For example the rule $h \leftarrow k\ \{a_1, \ldots, a_n\}\ l$ expresses that at least k atoms and at most l atoms among $\{a_1, \ldots, a_n\}$ should appear in the answer sets.
- *optimization statements* allow for selecting among the possible answer sets, the ones that satisfy statements like $minimize\{.\}$ or $maximize\{.\}$. For example, the statement $minimize\{a_1, \cdots, a_n\}$ allows for selecting the answer sets with as few of the atoms in $\{a_1, \cdots, a_n\}$ as possible.

3 Prioritized Removed Sets Fusion

We now present a merging operation which respects the preferences expressed over belief bases. In the context of Removed Sets Fusion, the preferences can be interpreted as a strategy which removes as few formulas as possible in high-ranked belief bases in order to restore consistency.

Let $B = \{B_1, \ldots, B_m\}$ be a belief profile where B_i, $1 \leq i \leq m$ is a belief base. B is equipped with a reliability or preference relation (a total pre-order) denoted by $\leq_B$ such that B_i is preferred to B_j iff $B_i \leq_B B_j$. Since several belief bases may be equally reliable, we rearrange the profile B in order to regroup the equally reliable belief bases according to a ranking function, denoted by r. This ranking is such that if $B_i <_B B_j$ then $r(B_i) < r(B_j)$ and if $B_i =_B B_j$ then $r(B_i) = r(B_j)$. In the following, we consider the belief profile $E = \{K_1, \ldots, K_n\}$, where n, $n \leq m$ is the number of ranks, and $\forall i,\ 1 \leq i \leq n$, $K_i = \cup_j B_j$ such that $r(B_j) = i$. When dealing with integrity constraint, $r(IC) < r(K_1)$.

In order to define a merging strategy which takes into account the ranking of E, we define the following total preorder to compare the potential Removed Sets defined in 1.

Definition 4. *Let* $(p^1_X, \ldots, p^n_X)$ *be the sequence where* $p^i_X = |X \cap K_i|$ *is the number of formulas removed from* K_i *by a potential Removed Set* X *of* E. *Let* X *and* X' *be two potential Removed Sets of* E. *The* $\leq_{lexipref}$ *pre-order is defined by:* $X \leq_{lexipref} X'$ *iff* $(p^1_X, \ldots, p^n_X) \leq_{lex} (p^1_{X'}, \ldots, p^n_{X'})$.

We can now define the Removed Sets of E according to the $lexipref$ strategy:

Definition 5. *Let* X *and* X' *be potential Removed Sets of* E, X *is a Removed Set of* E *according to* $lexipref$ *iff* ***(i)*** X *is a potential Removed Set of* E; ***(ii)*** *There does not exist* X' *such that* $X' \subset X$; ***(iii)*** *There does not exist* X' *such that* $X' <_{lexipref} X$.

We denote by $\mathcal{F}_{lexipref,IC}\mathcal{R}(E)$ the set of Removed Sets of E according to $lexipref$ and the new merging operation, denoted by $\Delta^{RSF}_{lexipref,IC}(E)$, is the following.

Definition 6. *The merging operation of* E *constrainted by* IC *according to the* $lexipref$ *strategy is such that:*

$$\Delta^{RSF}_{lexipref,IC}(E) = \bigvee_{X \in \mathcal{F}_{lexipref,IC}\mathcal{R}(E)} \{((K_1 \sqcup \cdots \sqcup K_n) \backslash X) \sqcup IC\}.$$

Example 1. We consider the belief profile $E = \{K_1, K_2, K_3\}$ s.t. $K_1 < K_2 < K_3$ and $IC = \top$ with $K_1 = \{a\}$, $K_2 = \{\neg a \vee b, \neg a \vee c\}$ and $K_3 = \{\neg b, \neg c\}$. Table 1 presents some potential Removed Sets of E, among which the minimal ones according to inclusion, as well as the corresponding $(p^1_X, \ldots, p^n_X)$ sequence.

The potential Removed Set which is minimal according to $\leq_{lexipref}$ is $\{\neg b, \neg c\}$. It is even preferred to $\{a\}$ which removes less formulas and would be preferred according to the Σ strategy. So $\Delta^{RSF}_{lexipref,IC}(E) = \{a, \neg a \vee b, \neg a \vee c\}$.

As shown in [12], merging when preferences are expressed can be dealt with as an iterated revision problem.

Table 1. Potential removed sets of example 1

potential Removed Sets X	p^1_X	p^2_X	p^3_X	potential Removed Sets X	p^1_X	p^2_X	p^3_X
$\{a\}$	1	0	0	$\{\neg a \vee b, \neg a \vee c\}$	0	2	0
$\{a, \neg c\}$	1	0	1	$\{\neg a \vee b, \neg c\}$	0	1	1
$\{a, \neg b\}$	1	0	1	$\{\neg a \vee c, \neg b\}$	0	1	1
$\{a, \neg b, \neg c\}$	1	0	2	$\{\neg b, \neg c\}$	0	0	2

4 Prioritized Merging as an Iterated Revision Operation

We now propose two iterated revision operations based on Removed Sets Revision (RSR) in order to deal with prioritized merging according to two directions. The first version, denoted by $\Delta^{PRSF}_{\alpha,IC}$, performs the revision starting from the least preferred bases to the most preferred ones. The second version, denoted by $\Delta^{PRSF}_{\beta,IC}$, performs in the opposite way, revising the belief bases from the most preferred to the least preferred.

Definition 7. *Let $E = \{K_1, \ldots, K_n\}$ be a belief profile and IC the integrity constraints.*
$\Delta^{PRSF}_{\alpha,IC}(E) = (((K_n \circ_{RSR} K_{n-1}) \circ_{RSR} \cdots \circ_{RSR} K_1) \circ_{RSR} IC)$;
$\Delta^{PRSF}_{\beta,IC}(E) = (K_n \circ_{RSR} (K_{n-1} \circ_{RSR} \cdots \circ_{RSR} (K_1 \circ_{RSR} IC)))$.

The behaviour of the $\Delta^{PRSF}_{\alpha,IC}(E)$ operation is not satisfactory as illustrated by the example 2. On the contrary, the same example shows that the behaviour of the $\Delta^{PRSF}_{\beta,IC}$ operation is closer to our excpectations.

Example 2. We come back to the example 1. In this case, we have $\Delta^{PRSF}_{\alpha,IC}(E) = (K_3 \circ K_2) \circ K_1$. Since $K_3 \circ K_2 = K_3 \sqcup K_2$, the removed sets are $R_1 = \{\neg a \vee b, \neg a \vee c\}$, $R_2 = \{\neg a \vee b, \neg c\}$, $R_3 = \{\neg a \vee c, \neg b\}$, $R_4 = \{\neg b, \neg c\}$, and $(K_3 \circ K_2) \circ K_1 = \{a, \neg a \vee c, \neg b\} \vee \{a, \neg a \vee b, \neg c\} \vee \{a, \neg a \vee b, \neg a \vee c\} \vee \{a, \neg b, \neg c\}$. The result is not satisfactory because the origin of the formulas as well as the preferences attached to the bases are lost during the iteration of the revision process. This operation does not correctly reflect the preferences for prioritized merging.

On the other side, $\Delta^{PRSF}_{\beta,IC}(E) = K_3 \circ (K_2 \circ K_1)$. Since $K_2 \circ K_1 = K_2 \sqcup K_1$, the only removed set is $R = \{\neg b, \neg c\}$, and $K_3 \circ (K_2 \circ K_1) = \{a, \neg a \vee b, \neg a \vee c\}$. This operation gives the same result than the one provided by $\Delta^{RSF}_{lexipref,IC}(E)$.

More generally, the $\Delta^{PRSF}_{\beta,IC}$ and the $\Delta^{RSF}_{lexipref,IC}$ operations lead to the same result and the prioritized merging can be expressed in terms of iterated revision.

Proposition 1. $\Delta^{PRSF}_{\beta,IC}(E) = \Delta^{RSF}_{lexipref,IC}(E)$.

Remark 1. In the context of fusion, considering a belief profile $E = \{K_1, \ldots, K_n\}$, the belief bases K_i are supposed to be consistent. However when grouping the equally reliable belief bases, some belief bases could be inconsistent. This does not affect the $\Delta^{RSF}_{lexipref,IC}$ operation since for each removed set X_i, $|X_i \cap K_k| \neq 0$ where K_k is an inconsistent belief base. This does not affect the $\Delta^{PRSF}_{\beta,IC}$ operation since the iteration of the revision process starts by the revision by IC. On contrast for the $\Delta^{PRSF}_{\alpha,IC}$ operation if K_k is an inconsistent belief base then $K_{k+1} \circ_{RSR} K_k =_{def} K_{k+1} \cup K_k$.

We rephrase within our framework the rational postulates for prioritized merging [12].

$(PMon)$ for $i < n$, $\Delta^{PRSF}_{\beta,IC}(K_1, \cdots K_{i+1}) \vdash \Delta^{PRSF}_{\beta,IC}(K_1, \cdots K_i)$.
$(Succ)$ $\Delta^{PRSF}_{\beta,IC}(K_1, \cdots K_n) \vdash \Delta^{PRSF}_{\beta,IC}(K_1)$.
$(Cons)$ $\Delta^{PRSF}_{\beta,IC}(K_1, \cdots K_n)$ is consistent.
$(Taut)$ $\Delta^{PRSF}_{\beta,IC}(K_1, \cdots K_n, \top) \equiv \Delta^{PRSF}_{\beta,IC}(K_1, \cdots K_n)$.
(Opt) if $\bigwedge E$ is consistent then $\Delta^{PRSF}_{\beta,IC}(K_1, \cdots K_n) = \bigwedge E$.
(RA) $\Delta^{PRSF}_{\beta,IC}(K_1, \cdots K_i) = \Delta^{PRSF}_{\beta,IC}(\Delta^{PRSF}_{\beta,IC}(K_1, \cdots K_{i-1}), K_i)$.

Proposition 2. *$\Delta^{RSF}_{lexipref,IC}$ satisfies the $(PMon)$, $(Succ)$, $(Cons)$, $(Taut)$, (Opt), (RA) postulates* [3]*.*

$\Delta^{PRSF}_{\alpha,IC}$ satisfies the $(Succ)$, $(Cons)$, $(Taut)$, (Opt) postulates.

5 Implementation of Prioritized Removed Sets Fusion

We now propose an implementation of the $\Delta^{RSF}_{lexipref,IC}$ operation stemming from the translation of the merging problem into a logic program with stable model semantics.

This implementation builds a logic program $\Pi_{E,IC}$ which consists of two parts: the first one computing the potential Removed Sets, the second one selecting among them the potential Removed Sets according to the *lexipref* strategy.

The first part was presented in [19]. The generation of Potential Removed Set is based on the generation of the interpretations over the atoms of E. It introduces new atoms called rule atoms. For a formula f, the rule atom r_f is deduced if the formula is not satisfied by the interpretation. The logic program generating all the interpretations and the corresponding sets of rule atoms is denoted $\Pi_{E,IC}$.

1. For every atom $a \in E$, the first step introduces the rules: $a \leftarrow not\ a'$ and $a' \leftarrow not\ a$. These rules build a correspondence between interpretations over the atoms of E and answer sets of the logic program $\Pi_{E,IC}$.
2. The second step introduces the rule atoms. For every formula $f \in K_i$, the following rules are introduced according to the syntax of f : **(i)** If $f =_{def} a$, the corresponding rule is $r^i_f \leftarrow not\ a$; **(ii)** If $f =_{def} \neg f^1$, the corresponding rule is $r^i_f \leftarrow not\ \rho_{f^1}$; **(iii)** If $f =_{def} f^1 \vee \cdots \vee f^j$, the corresponding rule is $r^i_f \leftarrow \rho_{f^1}, \ldots, \rho_{f^j}$; **(iv)** If $f =_{def} f^1 \wedge \cdots \wedge f^j$, the corresponding rules are $r^i_f \leftarrow \rho_{f^1}$, $r^i_f \leftarrow \rho_{f^2}$ to $r^i_f \leftarrow \rho_{f^j}$.

It has been shown in the article cited *supra* that there is a one-to-one correspondence between stable models of $\Pi_{E,IC}$ and the potential Removed Sets of E constrainted by IC. Based on this result, we can translate the notion of preference between potential Removed Sets into a preference between stable models.

Definition 8. *Let X and X' be two stable models of $\Pi_{E,IC}$. The $\leq_{lexipref}$ total preorder between stable models is defined as follows: $X \leq_{lexipref} X'$ iff $(p^1_{(X\cap R^+)}, \ldots, p^n_{(X\cap R^+)}) \leq_{lex} (p^1_{(X'\cap R^+)}, \ldots, p^n_{(X'\cap R^+)})$.*

The potential Removed Sets are compared according to the number of formulas removed in each belief base. The stable models can be compared according to the number of rule atoms representing those formulas. This is the usefulness of rule atoms. It leads to the definition of preferred stable models of $\Pi_{E,IC}$ according to the *lexipref* strategy.

[3] Obviously $\Delta^{RSF}_{lexipref,IC}$ does not satisfy the (IS) postulate since it is a syntactic prioritized merging operation.

Definition 9. *Let X be a set of atoms of E and X' be a stable model of $\Pi_{E,IC}$, X is a preferred stable model of $\Pi_{E,IC}$ according to the lexipref strategy iff the following conditions hold:* ***(i)*** *X is a stable model of $\Pi_{E,IC}$;* ***(ii)*** *there does not exist X' such that $X' \subset X$;* ***(iii)*** *there does not exist X'such that $X' <_{lexipref} X$.*

The problem consisting in determining among the stable models those which are the preferred ones is solved through a set of logic programming statement. The predicate $size(I, J)$ [4] represents the fact that J formulas are coming from K_I in the potential Removed Set. $size(I, J)$ is computed by the following rule which is introduced for every base K_I and every possible U from 1 to m which is the maximum cardinality of a belief base in the profile E.

$$\Pi_E^{lexipref,size} = \left\{\gamma_1 \ : \ size(V,U) \leftarrow U \ \{f_1^V, \ldots, f_m^V\} \ U.\right\}$$

Therefore the complete program computing the result of $\Delta^{RSF}_{lexipref,IC}(E)$ is the following: $\Pi^{lexipref}_{E,IC} = \Pi_{E,IC} \cup \Pi_E^{lexipref,size} \cup minimize[\ size(1,1) = 1 \times (m+1)^{n-1}, size(1,2) = 1 \times (m+1)^{n-1}, \ldots, size(i,1) = 1 \times (m+1)^{n-i}, size(i,2) = 2 \times (m+1)^{n-i}, \ldots, size(n,m) = m\ ]$. The stable models of $\Pi^{lexipref}_{E,IC}$ are the preferred stable models $\Pi_{E,IC}$ according to the *lexipref* strategy. Moreover, it computes exactly the expected Removed Sets.

Proposition 3. *The set of Removed Sets of $\Pi^{lexipref}_{E,IC}$ is the set of preferred stable models of $\Pi_{E,IC}$ according to the lexipref strategy.*

Example 3. We now present the implementation of example 1.

$$\Pi_{E,IC} = \left\{\begin{array}{lll} a \leftarrow not\ a'. & a' \leftarrow not\ a. & b \leftarrow not\ b'. \\ c \leftarrow not\ c'. & c' \leftarrow not\ c. & b' \leftarrow not\ b. \\ r^1_a \leftarrow a'. & r^2_{\neg a \vee b} \leftarrow a, b'. & r^2_{\neg a \vee c} \leftarrow a, c'. \\ r^3_{\neg b} \leftarrow b. & r^3_{\neg c} \leftarrow c. & \end{array}\right\}$$

$$\Pi_E^{lexipref,size} = \left\{\begin{array}{ll} size(1,1) \leftarrow 1\{r^1_a\}1. & size(2,1) \leftarrow 1\{r^2_{\neg a\vee b}, r^2_{\neg a \vee c}\}1. \\ size(2,1) \leftarrow 2\{r^2_{\neg a\vee b}, r^2_{\neg a \vee c}\}2. & size(3,1) \leftarrow 1\{r^3_{\neg b}, r^3_{\neg c}\}1. \\ size(3,2) \leftarrow 2\{r^3_{\neg b}, r^3_{\neg c}\}2. & \end{array}\right\}$$

$$minimize[size(1,1) = 9, size(2,2) = 6, size(2,1) = 3, size(3,2) = 2, size(3,1) = 1].$$

which has the following stable models (with their associated weight):

$$\begin{array}{cc} \{a', b', c', r^1_a, size(1,1)\} & (9) \\ \{a', b', c, r^1_a, r^3_{\neg c}, size(1,1), size(3,1)\} & (10) \\ \{a', b, c', r^1_a, r^3_{\neg b}, size(1,1), size(3,1)\} & (10) \\ \{a', b, c, r^1_a, r^3_{\neg b}, r^3_{\neg c}, size(1,1), size(3,2)\} & (11) \\ \{a, b', c', r^2_{\neg a\vee b}, r^2_{\neg a\vee c}, size(2,2)\} & (15) \\ \{a, b', c, r^2_{\neg a\vee b}, r^3_{\neg c}, size(2,1), size(3,1)\} & (4) \\ \{a, b, c', r^2_{\neg a\vee c}, r^3_{\neg b}, size(2,1), size(3,1)\} & (4) \\ \{a, b, c, r^3_{\neg b}, r^3_{\neg c}, size(3,2)\} & (2) \end{array}$$

The stable model with the minimal weight is $\{a, b, c, r^3_{\neg b}, r^3_{\neg c}, size(3,2)\}$ which corresponds to the Removed Set $\{\neg b, \neg c\}$ of E according to the *lexipref* strategy.

[4] Variables are represented by words starting by an upper-case letter.

6 Conclusion

We provide a framework, based on Removed Sets Fusion, for merging belief bases where preferences are expressed among belief bases. We propose a new prioritized merging operation, $\Delta^{RSF}_{lexipref,IC}$, which is equivalent to the iterated removed sets revision operation $\Delta^{PRSF}_{\beta,IC}$ and we show that this operation satisfies most of the postulates proposed for prioritized merging in [12]. We provide an implemementation of prioritized merging thanks to answer sets programming. We have now to compare PRSF with the already proposed prioritized merging operations stemming from lexicographic preorder [4], [18]. Moreover an experimental study has to be conducted in order to evaluate the behaviour of the $\Delta^{RSF}_{lexipref,IC}$.

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