The Use of Fuzzy Numbers in Practical Project Planning and Control

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Abstract. The paper proposes how to use fuzzy numbers in project planning and control in such a way that it would have a chance to be used in practice. The method is destined for all the projects, but especially for those where in the initial phase the knowledge about the project is very incomplete and is made stepwise more precise during the project execution, also for those in which initial assumptions about the project execution are due to later changes.

Keywords: fuzzy number, earned value, project scheduling, project control.

1 Introduction

It is generally agreed upon the fact that each project is by its very nature and definition to some degree unique, and that today the project environment is so unstable, that project planning is biased by a large degree of uncertainty. Thus, it is essential to take this into account both in project planning and control, and to do this in a way that comprises all the projects, also those where the degree of uncertainty is very high, like some IT as well as research and development projects. It is generally accepted that fuzzy numbers are a good tool to express uncertainty, limited knowledge. Thus, their use in project management has been frequently proposed in the literature, however, it seems that the practical implementation of those proposals is almost non-existent. In this paper we want to propose such an application of fuzzy numbers to project management which is on one hand simple enough to be actually used, and on the other hand is comprehensive enough to be

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useful for most project, also those innovative, very unique, with a high degree of risk and uncertainty

2 State of the Art of Project Control Versus Project Planning

We start by presenting what has already been done in the project management research which can be taken as the basis of our method. We will present both some elements of the use of fuzzy numbers in project management and the crisp approaches, which we want to modify applying fuzzy numbers to them.

2.1 Project Planning

We will not talk here about project planning based on crisp numbers, because we claim that such planning is in may projects simply impossible or deceiving. Many projects (examples will be presented later on) are in the planning phase unknown to such a degree, that each crisp estimate of duration and cost is not realistic and does not allow a good project management, especially risk management. In our opinion at least some projects require taking in the planning phase non-crisp information into account.

Fuzzy numbers have been extensively applied to the project planning (e.g. [2,11]). Most proposals are based on fuzzy optimization, where the objective is to minimize the project execution time, and the activities duration are given in the form of fuzzy numbers. The problem is that if the activities duration times are fuzzy, the notions like critical path, activities floats etc. are not unequivocally defined, and new notions like "the possibility degree to which this or that project path will be critical" have to be introduced, which seem to have little chance to be applied in practice. In this paper we propose not to consider them and to restrict ourselves to this part of the application of fuzzy optimization to project planning which is the simplest possible and yet useful from the practical point of view. What has to be retained of the vast spectrum of proposals of the application to fuzzy numbers to project planning is, in the author's opinion, the simplest and most appealing practical possibility: the possibility to model project activities planned duration and planned cost in the form of fuzzy numbers and to obtain all the spectrum of possible planned project durations and cost values.

Let n be thus the number of project activities. Let us assume that all the constraints imposed on the activities (like precedence relation, resource limits etc.) are known and are taken into account whenever the optimal makespan of the project is determined, let us denote the set of constraints known in the moment of planning as $\Re_0$. Let $\tilde{D}_i \ (i = 1,...,n)$ be the fuzzy planned duration of each activity and $\tilde{C}_i \ (i = 1,...,n)$ the fuzzy planned cost. To facilitate the application, we propose to use only trapezoidal fuzzy numbers, represented unequivocally by four parameters. Thus, each fuzzy number $\tilde{A}$ will be unequivocally represented

by four numbers $\left(a^1, a^2, a^3, a^4\right)$, where $a^1 \le a^2 \le a^3 \le a^4$ such that each interval $P^t = \left[a^2 - t\left(a^2 - a^1\right), a^3 + t\left(a^4 - a3\right)\right]$ represents possible values of the magnitude represented by $\tilde{A}$ for the uncertainty level $t \in [0,1]$. If we assume the highest level of uncertainty, i.e. t=1, we have $P^1 = \left[a^1, a^4\right]$, thus we admit a large spectrum of possible values, one of each the magnitude represented by $\tilde{A}$ will eventually take. If we assume the lowest uncertainty value, i.e. t=0, we take into account the narrow interval (which may be reduced to one single point) $P^0 = \left[a^1, a^4\right]$ as the set of possible values of the magnitude represented by $\tilde{A}$.
Addition of trapezoidal fuzzy numbers can be defined as

$$\tilde{A} + \tilde{B} = \left(a^1, a^2, a^3, a^4\right) + \left(b^1, b^2, b^3, b^4\right) = \left(a^1 + b^1, a^2 + b^2, a^3 + b^3, a^4 + b^4\right)$$

With such representation, we can do calculation which would be easy to implement in the practice and will lead to information important to project managers. We can calculate the planned project fuzzy cost simply as $P\tilde{C} = \sum_{i=1}^{n} \tilde{C}_i$ and also the planned project duration. As far as the latter is concerned, the summation of $\tilde{D}_i$ $(i = 1,...,n)$ is done over those activities which belong to the critical path (or one of critical paths) and the set of activities forming the critical path may change according to the crisp values of activities duration. That is why the easiest approach from the practical point of view is to choose a view values of the uncertainty level $t \in [0,1]$, let us say an increasing sequence $T = \{t_1 = 0, t_2, ..., t_{m-1}, t_m = 1\}$, and for each j=1,…,m calculate the optimistic and pessimistic project duration ($OPD(t_j)$ and $PPD(t_j)$, respectively for the given uncertainty level, using optimisation methods for determining the shortest project makespan under the given constraints. $OPD(t_j)$ will be calculated by assuming for the project activities indexed by i=1,…,n crisp durations $d_i^2 - t_j\left(d_i^2 - d_i^1\right)$, and $PPD(t_j)$ with crisp durations $d_i^3 + t_j\left(d_i^4 - d_i^3\right)$, taking into account $\Re_0$. The set of couples $\left\{\left(OPD(t_j), PPD(t_j)\right)\right\}_{j=1}^{m}$ gives the project manager an approximation of possible project durations. We will called this set of couples fuzzy project duration $P\tilde{D}$. Of course, in special cases where the critical path will be the same independently of which crisp values from the ranges predicted in the activities fuzzy duration values occur (an example will be presented further on), we can do a simple summation of the corresponding trapezoidal fuzzy numbers and get a “normal” trapezoidal fuzzy number as $P\tilde{D}$.

2.2 Project Control

The problem of project control is treated in literature almost exclusively for crisp planned duration and cost values. The use of fuzzy numbers in project control, described in [6] and [8], refers to crisp planned values and fuzzy numbers are used to judge to which degree the actual values can be treated as planned or as “good” o “bad”. The only project control approach for fuzzy activities duration values, presented in [5], is very general, it has to be made more specific to order to have a chance of being applied in practice.

The method of controlling a project which has been planned (in terms of activities duration and cost) in terms of fuzzy numbers, which will be presented in this paper, will inevitably be based on the crisp approach to project control, thus a short critical review of this approach will be presented.

The basic approach for the control of project cost, but also time and scope, is called Earned Value Method. The method is presented, in its recently modified versions, among other in [1,7,9]. Its idea is as follows: We have a project with *n* activities and a set of constraints $\Re_0$ imposed on the activities, for each activity $i=1,...n$ there is defined the corresponding work to be done W_i expressed in some physical units like meters, tons or hours, the planned cost of each unit UC_i and the planned duration of the activity D_i. Of course, the ratio W_i/D_i is the planned work efficiency for the *i*-th activity (it will be denoted as E_i), and the planned cost of the activity, denoted as C_i, will be equal to $W_i \cdot UC_i$. W_i ($i=1,...n$) is treated as fixed, changes in this parameter mean changes in the scope of the project and is explicitly not taken into account in the Earned Value Method. Applying the normal optimisation methods which allow to find a schedule, taking into account $\Re_0$, which gives the shortest possible project duration, we know the project planned duration *PD*, and adding up the C_i of all the activities we know the project planned cost *PC*. Now, the idea of the method is that we do not treat *PD* and *PC* as fixed values. On the contrary, we acknowledge that in the planning phase we will almost certainly be wrong in their estimation and that we will be the better in estimating those two values the more the project is advanced. This is because of various reasons, linked to the newness and uncertainty inherent in each project, to human factors, communication problems, changing environment etc.

This is why once the project realisation is started, at fixed time intervals during the project execution, let us number them $l=1,...,L$, where *L* is unknown till the actual end of the project and is the number of the last control moment, the planned duration *PD* and the planned cost *PC* of the whole project will be reestimated, recalculated. Let us use PD_0 and PC_0 as alternative symbols for *PD* and *PC*, they express the result of time and cost estimation in the planning phase. PD_l and PC_l, $l=1,...,L$, will be the same magnitudes reestimated al the consecutive control moments, when our knowledge about the project is usually higher than at the beginning, let it be only because of the fact that the project has already be executed to some extent which always gives some insight into the project and its environment. Normally, the greater *l*, the closer the values PD_l and PC_l should be to the actual project duration and cost, *AD* and *AC* respectively, known only once the project is finished - although there may be cases where because of some unexpected event near the end of the project this will not be true.

As far as the consecutive values of PD_l and PC_l, $l = 1,...,L$, are concerned, we can write the following obvious formulae:

$$PD_l = ADB_l + RD_l \tag{1}$$

$$PC_l = ACB_l + RC_l \tag{2}$$

where ADB_l stands for actual duration of the project before the control moment *l*, thus in fact for the time that has elapsed from the actual beginning of the project till moment *l*, RD_l stands for the remaining duration, thus the time after moment *l* which we think, according to our knowledge at moment *l*, is needed to terminate the project, ACB_l is the actual cost of the project incurred before moment *l*, and RC_l is the remaining cost, thus the cost that we think, again according to our knowledge at moment *l*, is still needed to be incurred after moment *l* in order to terminate the project.

Formulae (1) and (2) help to structure the problem, but do not facilitate the solution: although ADB_l and ACB_l at control moment l are known exactly, RD_l and RC_l are not and must be the subject of replanning and reestimation on the basis of the knowledge we possess at control moment *l*.

The various approaches to estimate RD_l and RC_l proposed in the literature ([7,9]) have the following common feature: they are based on the project "behaviour" up to moment *l*, trying to express this behaviour, the history of the past project realisation, in the form of synthetic indicators referring to the whole project, not to individual activities or even group of activities. Now we will present the idea of those indicators.

The Earned Value indicators whose aim is to show how the project has been performed up to control moment *l*, are calculated on the basis of the following values:

- BCWS(*l*) (Budgeted Cost of Work Scheduled up to moment *l*), equal to $\sum_{i=1}^{n} pp_i(l) \cdot W_i \cdot CU_i$, where $pp_i(l)$ is the portion of work W_i (expressed as positive fraction not greater than 1) that was planned to be finished before control moment *l*;

- BCWP(l) (Budgeted Cost of Work Performed up to moment l), equal to $\sum_{i=1}^{n} ap_i(l) \cdot W_i \cdot CU_i$, where $ap_i(l)$ is the portion of work W_i that has actually been done before control moment l;
- ACWP(l) (Actual Cost of Work Performed up to moment l) – the actual cost incurred in the project till the control moment l, which is taken from actual accounting documents but can be expressed in an analogous way: $\sum_{i=1}^{n} ap_i(l) \cdot W_i \cdot ACU_i$, where ACU_i is the average actual cost of performing one unit of W_i before control moment l.

Of course, we can also calculate BCWS(l,i), BCWP(l,i), ACWP(l,i) for individual activities (the corresponding formulae are then identical to the above listed ones, but with the summation symbol), but usually, for simplicity and because of lack of time synthetic values for the whole project are calculated. It should be mentioned that BCWP(l) is called earned value in control moment l and is interpreted in two ways:

- The cost we should have incurred according to plan for the work actually performed so far in the project.
- The amount of work that has been actually performed so far in the project (measured not in physical units, but in planned unitary cost, which is not an obvious measurement of work performed and is used to facilitate things, but causes some problems, which will be illustrated later on).

At the same time BCWS(l) is interpreted as the amount of work that according to plan should have been accomplished up to moment l. Again, the amount of work is measured here in monetary units, in planned unitary cost, which may be a problematic matter.

Lipke [7] introduced the notion of earned schedule at control moment l, ES(l), equal to the moment corresponding to s-th control point[1], when BCWS(s)=BCWP(l), thus to the moment when we should or should have accomplished the work we have accomplished up to moment l. Then, according to our notation, ES(l)= ADB_s.

Then the history of the project up to control moment l is estimated, using the following indicators:

- Cost Performance Index in control moment l: $CPI(l) = \frac{BCWP(l)}{ACWP(l)}$
- Schedule Performance Index in control moment l: $SPI(l) = \frac{BCWP(l)}{BCWS(l)}$

[1] For simplicity reason we assume that such an s from the set of control points exists, otherwise some insignificant modifications would have been necessary, because the moment we are interested in might fall between two control points.

- Modified Schedule Performance Index in control moment *l*: $MSPI(l)=\frac{ES(l)}{ADB_l}$.

The interpretation of the indices is generally such that if they are smaller than 1, something is wrong with the project, if they are greater than 1, something is good with the project, value 1 means that something is as it was planned. This something is project cost in case of *CPI(l)* and the pace of project work accomplishment in case of *SPI(l)* and *MSPI(l).*

As far as formulae (1) and (2) are concerned, they are used by applying for RD_l and

RC_l the following general formulae

$$RD_l = (PD - ADB_l) \cdot DI_l \tag{3}$$

$$RC_l = (PC - ACB_l) \cdot CI_l \tag{4}$$

where DI_l and CI_l are, respectively, some duration/cost indicators equal to or based on *SPI(l)* and *MSPI(l)* (in case of DI_l) and *CPI(l)* and *SPI(l)* (in case of CI_l). The underlying philosophy is as follows: for formula (4), if so far for the work accomplished we paid twice as much as we had planned (which is expressed by *CPI(l)*), we can assume that we will pay twice as much as we have planned for the rest of the project (in this case CI_l would be equal to *CPI(l)*) , and maybe if we additionally have some delay in the amount of work accomplished, expressed by *SPI(l)*, this will have some (unfortunately, unspecified more clearly) influence on the total cots of the project (that is why sometimes CI_l is based both on *CPI(l)* and *SPI(l)*). Similarly for the project duration (formula (3)): if so far we have accomplished half of the work that has been planned (which will be shown by *SPI(l)*), or we have done so far the work which should have been ready two months ago (expressed by *MSPI(l)*), we can assume that the same degree of lateness will apply to the rest of the project.

The problem with such a philosophy is that it works only sometimes. This is said clearly in the corresponding papers, especially in [9], where the authors summarize various variants of formula (4). They equip each variant with an assumption of the type: "if the rest of the project follows the *SPI (CPI, SPI*CPI* etc.*)* pattern". Well, how should we know and what this means? There may be projects where we can say that we will probably continue to work at the same pace and at the same cost as before, but there is certainly a large class of project where such a statement would be completely impossible, and in most project at least some of yet non-started activities will certainly behave in a manner not at all connected to the behaviour of the already accomplished ones. What is more, the indicators *SPI(l), CPI(l)* and *MSPI(l)* may be deceiving, for at least four reasons:

- They are based on the information about the amount of work expressed in planned cost, thus 2 physical units planned to be made by higher paid workers mean more work than then the same 2 units planned to be made by lower paid workers, whereas in both cases the actual amount of work is the same. If we simply change the order of the two activities versus the planned order, we will get the information that we have done more (or less) work than planned, which will not be true;
- They try to combine the information concerning all the activities into one indicator, if one activity is very late (or much more expensive) versus the plan, but another one very early (or much cheaper) versus the plan, we may get the information that everything has been going on exactly as planned, because the two variances may compensate each other. The worst thing will happen if the activity with problems (late, more expensive), hidden by the situation of other activities, is the one which will influence the behaviour of the future activities (e.g. because some of the non-started activities will be performed by the same subcontractor). In such a case the information delivered by the Earned Value Method will be completely deceiving;
- They are based entirely on the project history and do not take into account any new pieces of information about the project future which where not available in the planning phase (like new prices, a different subcontractor than the one which was planned etc.)
- The Earned Value Method is considered in a complete detachment from the reactive scheduling problem – i.e. rescheduling in reaction to what has happened in the project so far, treated in many papers (e.g.[10]). It is as if rescheduling was considered as something to be avoided, because it is difficult? Not possible in practice? In our opinion the project management supporting systems offer the possibility to find a new optimal or sub-optimal schedule in each control model l (with PD_l as the objective function, whose possible values will be of interest to us) taking into account the available information about the project history up to control moment l, and each project control without reactive scheduling does not make much sense. To justify this statement, let us start with the following example:

Example 1. The project consists of three activities. They have the following characteristics: $D_1 = D_2 = 1; D_3 = 2$, $W_1 = W_2 = 10; W_3 = 20$, $CU_1 = CU_2 = CU_3 = 1$. It has been planned that the 3. activity will be executed in the 1. and 2. time unit, activity 1 in the first time unit and activity 2 in the 2. time unit, thus PD=2. The first control moment (l=1) is the end of the first time unit. It is then stated that in the first time unit we have executed and finished the 1. and 2. activity, activity 3 has not been started, the unitary cost has been as planned. It is easy to check than we have BCWS(1)=BCWP(1)=ACWP(1)=20, all the indices discussed above will be equal to 1, formula (1) will give $PD_1 = PD_0 = 2$, and in reality simple rescheduling (determining the optimal project duration under the new conditions) would show that the shortest possible project duration according to the knowledge in the 1st control moment is 3 (the 3. activity has still to be executed entirely and its planned duration is 2 time units).

Another example justifying the statement comes from [4], where it is described in detail. The project goal is to introduce a new product into the market. The project comprises the following activities: specifying the product on the basis of the market's functional requirements, designing the product, building the prototype, testing the prototype, manufacturing the product, marketing the product to potential customers. Hardie [4] gives a good justification of the fact that the initial schedule, made at the planning moment, is bound to be changed in this type of project, and what is more, not just changed because the duration and cost of each activity may change, but also the scope (the work to be done) is very difficult to estimate at the beginning. And what is even more, Hardie [4] justifies that some activities may have to be repeated, and in the planning phase we even cannot say which of them, when, how often and to which extent. If in the testing phase an error in the design is detected, the designing activity has to be repeated, the same may happen if once the product is on the market, the customers are not satisfied with its functions, and also in the manufacturing phase, if some technical problems are detected – then the product will have to be redesigned and nobody is able to tell exactly in the planning phase whether it will be necessary and how much time and cost this will require. A very similar situation exists in most IT projects, where it is only during the project execution that the final functionality of the project product takes shape, in cooperation with the customer.

2.3 Complex Project Management – Planning and Control

In our opinion the Earned Value Method in its classical (presented in this section) form, combined with planning based on crisp activities cost and duration planned values and detached from reactive scheduling, may be useful only for a rather narrow class of projects, it what is more, it may sometimes be difficult to decide whether our project belongs to this class. For such projects as described in the previous section, we claim that:

- Planning using crisp values does not make sense, because we know that almost no value in the planning phase is known exactly, that almost each project parameter in the planning phase might be much more truthfully described in the form of fuzzy numbers. Using crisp planning values together with the classical form of Earned Value Method may to situation like the following one:

Example 2. The 1. activity has the planned cost equal to the trapezoidal fuzzy number (9,10,10,11), the 2. activity – to the trapezoidal fuzzy number (1,10,10,20). Thus, the knowledge about the cost of the 2. activity is much less precise than it is for the 1. activity. If we use crisp planning, we take for both activities the planned cost 10. The activities should be executed one after another. If the execution of the 1. activity costs in reality 13 and the control moment falls after its termination, we will – according to the Earned Value principles – conclude that also the second activity will cost 13 (as the first one was 30% more expensive in the crisp approach, the same is assumed for the foll-woingone). In the crisp approach this will give the information: cost overrun in case of both activities. In fact, a cost negative variance with respect to the plan has occurred only for the 1. activity, because value 13 falls outside the interval of our prevision for this activity. In case of activity 2, on the other hand, value 13 does not mean a negative cost variance – it was known that such a value might occur for this activity. Treating in the control moment after the termination of the 1. activity value 13 as an important information about the future, i.e. about the second activity, may be very deceiving, because first of all 13 is not a new information, we knew before the cost of the 2. activity might be 13, and what is more, we may lose from our sight another piece of information, possibly essential: if we were completely wrong in estimating the cost of the 1. activity, we may have been wrong in estimation the cost of the 2. activity, which might also fall outs the range predicted by us and be equal e.g. to 22. Everything depends of course on the specific situation, but it seems that project planning and control based on crisp estimations $C_1 = C_2 = 10, PC_0 = 20$ and $PC_1 = 26$ are of limited use in view of the actual information available in the planning phase: $\tilde{C}_1 = (9,10,10,11), \tilde{C}_2 = (1,10,10,20), P\tilde{C}_0 = (10,20,20,31)$.

- The method of project control should also be based on non-crisp values, thus we should calculate in each control moment l $P\tilde{D}_l$ and $P\tilde{C}_l$, equal respectively to $(pd_l^1, pd_l^2, pd_l^3, pd_l^4)$ and $(pc_l^1, pc_l^2, pc_l^3, pc_l^4)$, although in most cases he "degree of fuzziness", i.e. the differences $pd_l^4 - pd_l^1$ and $pc_l^4 - pc_l^1$, should get smaller the greater *l*. However, sometimes it will not be the case: sometimes, like in the product design, manufacturing and marketing project described in the previous section, we may only later during the project execution discover how little we know about a certain activity. In such a case, the $P\tilde{D}_l$ and $P\tilde{C}_l$ should reflect the present state of knowledge, even if it is "more fuzzy" than in previous control moments. The project manager should be in each stage of the project aware of the state of knowledge about the future of the project and the degree of its crispness, which is essential for a good project risk management.
- The method of project control should take into account not only the knowledge about the past of the project, but also each piece of new information about its future which is available (such an approach is presented in [3], however, it is based on crisp values). The information about the past should of course be used as well, but it should not be blindly aggregated without looking at dependencies between

activities. Also, the amount of work accomplished should not be expressed in monetary units. Additionally, the method should use reactive scheduling in order to determine in each moment the optimal fuzzy planned makespan of the project.

A proposal of a method fulfilling these conditions will be presented in the next section.

3 Fuzzy Project Planning and Control Method

In the method proposed we assume the following approach:

In the planning phase we propose to estimate the cost and duration of each activity as trapezoidal fuzzy numbers, respectively as $\tilde{C}_i \ (i=1,...,n)$ and $\tilde{D}_i \ (i=1,...,n)$. We treat *n*, $\tilde{D}_i \ (i=1,...,n)$, $\tilde{C}_i \ (i=1,...,n)$ as synonyms of n_0, $\tilde{D}_{i,0} \ (i=1,...,n_0)$, $\tilde{C}_{i,0} \ (i=1,...,n_0)$, as these parameters will be reevaluated in the subsequent control moment (in each control moment l=1,2..,L we will new estimations $\tilde{D}_{i,l} \ (i=1,...,n_l)$, $\tilde{C}_{i,l} \ (i=1,...,n_l)$, except for the activities for which actual values will already be known. The number of activities may change in the course of the project, let it be only for the above mentioned possibility of having to repeat an activity already executed earlier. Of course, we also define the constraints according to the knowledge in the planning phase, denoted as $\Re_0$, which may also change in the course of the project. Additionally, in the planning phase we define additional dependencies of activities, which may be of the following types:

- Activities executed by the same subcontractor or the same group of people or which will use the same type of material resource
- Activities which probably will have a similar length, but this length cannot be determined exactly yet (e.g. time necessary to get an official permission from a state office or to get the ordered parts from a company with which we have not cooperated so far)
- Activities linked by a learning process, i.e. once one of the activities is executed, the following one will be executed in a shorter time and maybe at a lower cost, because the project team has gained some experience

Also other groups of activities can be defined, between which there are some dependencies in the sense that once one of them is executed and we know its actual duration and actual cost, we can better estimate the duration and cost of another activity still to be executed.

In the planning phase we estimate $P\tilde{D}$ and $P\tilde{C}$ defined in the previous section, which will be alternatively denoted as $P\tilde{D}_0$ and $P\tilde{C}_0$. Of course, these values have to be compared with the available project budget and with requirement as to the project completion time, to evaluate the risk of not meeting the dead line and/or the budget requirements.

Then at control moments l=1,2,..,L, where L is unknown until the project is finished (we only have chose the intervals between each control moment, like one month or one week, it depends on the degree of uncertainty and risk linked to the project) , we will re-estimate the project planned cost and duration, calculating $P\tilde{D}_l$ and $P\tilde{C}_l$. We will do it using the following algorithm (for each l=1,2,..,L):

1. For the activities which have been finished (the set of their indices will be denoted as $I_{F,l}$) we will find out their actual duration and actual cost, AD_i and AC_i respectively.
2. For the activities which have been started but not finished (the set of their indices will be denoted as $I_{S,l}$) find out how much they have taken so far ($AD_{i,l}$) and how much they have cost co far ($AC_{i,l}$), and also ask the persons responsible for each of these activities to
    a. Either estimate the percentage of work that has been accomplished at those activities $ap_{i,l}$. In this case the position of $AD_{i,l}$ and $AC_{i,l}$ with respect to $ap_{i,l}\tilde{D}_{i,l-1}$, $ap_{i,l}\tilde{C}_{i,l-1}$ will be presented to the decision maker and the position of $AD_{i,l}/ap_{i,l}$ and $AC_{i,l}/ap_{i,l}$ with respect to $\tilde{D}_{i,l-1}$, $\tilde{C}_{i,l-1}$ For example, the trapezoidal fuzzy number $ap_{i,l}\tilde{D}_{i,l-1}$ represents the planned duration of the accomplishment of $ap_{i,l}$ of the whole work to be done in the i-th activity. The actual time of executing this work ($AD_{i,l}$), as well as both values divided by $ap_{i,l}$ ($\tilde{D}_{i,l-1}$ and $AD_{i,l}/ap_{i,l}$), referring to the total planned time of the activity estimated before and the same time estimated in moment l on the basis of the time that has elapsed executing this activity so far, should give the user and indication as to a new estimate $\tilde{D}_{i,l}$ It may based on $AD_{i,l}/ap_{i,l}$ (i.e. it may a fuzzy number $\left(d_{i,l}^1, d_{i,l}^2, d_{i,l}^3, d_{i,l}^4\right)$ such that $AD_{i,l}/ap_{i,l}$ belongs to the interval $\left(d_{i,l}^1, d_{i,l}^4\right)$ or even $\left(d_{i,l}^2, d_{i,l}^3\right)$), or it may based on a "combination" of $\tilde{D}_{i,l-1}$ and $AD_{i,l}/ap_{i,l}$

($\tilde{D}_{i,l-1}$ shifted to the direction of $AD_{i,l}/ap_{i,l}$ or maybe made "slimmer" in the sense that if e.g $AD_{i,l}/p_{l,i} > d^3_{i,l-1}$, than values $d^1_{i,l}, d^2_{i,l}$ will be higher than $d^1_{i,l-1}, d^2_{i,l-1}$. A similar kind of reasoning might be conducted for $\tilde{C}_{i,l-1}$ and $AC_{i,l}/p_{l,i}$. We do no think any more precise indication should be given, as the user should be open not only to the information about the past of the activity, abut also to any other information concerning its future;

b. Or give a direct estimation of $\tilde{D}_{i,l}$, $\tilde{C}_{i,l}$, without using the percentage of work accomplished so far, which is meaningless or difficult to measure in some cases.

3. For all the yet unstarted activities find those which were identified as related in some way to those already finished ones or those started. Show the decision maker the actual values for the related activities (AD_i and AC_i for $i \in I_{F,l}$ and $ap_{i,l}$ (if available) $AD_{i,l}$, $AC_{i,l}$ for $i \in I_{S,l}$) and ask to give the new estimations $\tilde{D}_{i,l}$, $\tilde{C}_{i,l}$ for those $i \in \{1,2,...,n_l\} \setminus (I_{F,l} \cup I_{S,l})$ for which the information about the finished or started activities was important. It may happen that some $\tilde{D}_{i,l}$ and $\tilde{C}_{i,l}$ will become crisp or "almost crisp" in this moment, even if they concern the not yet started activities. This will be the case of activities very similar to those already finished.
4. Identify all new information about resources, subcontractors, prices, scope concerning the future of the project, update the $\tilde{D}_{i,l}$ and $\tilde{C}_{i,l}$ ($i \in \{1,2,...,n_l\} \setminus (I_{F,l} \cup I_{S,l})$) concerned.
5. Identify all the unplanned activities which will have to be additionally executed, also those which will have to be repeated for some reasons, estimate $\tilde{D}_{i,l}$ and $\tilde{C}_{i,l}$ for them, update n_l and $\Re_l$ (taking into account the constraints or rather relations that have actually occurred (especially, the order in which activities have been accomplished so far) and any new information about them concerning the yet unstarted activities);

With the updated information we will calculate $P\tilde{C}_l = \sum_{i=1}^{n_l} \tilde{C}_{i,l}$ and $P\tilde{D}_l = \{(OPD_l(t_j), PPD_l(t_j))\}_{j=1}^{m}$, where $OPD_l(t_j)$, will be calculated on the basis crisp durations $d^2_{i,l} - t_j(d^2_{i,l} - d^1_{i,l})$ ($i = 1,...,n_l$) and $PPD_l(t_j)$ with crisp durations $d_{i,l}^3 + t_j\left(d_{i,l}^4 - d_{i,l}^3\right)$, using of course rescheduling, taking into account $\Re_l$. In those cases where the estimates will not be fuzzy any more but crisp, we treat the crisp number identically as a fuzzy triangular one with all the four parameters equal. The we will compare the fuzzy estimates $P\tilde{C}_l$ and $P\tilde{D}_l$ with the available budget and the deadline, taking if necessary some steps to prevent in time problems with budget or time overrun which we are able to predict now, in the control moment *l*, but, let us emphasize it, which will come into light only once the project is finished. The estimates $P\tilde{C}_l$ and $P\tilde{D}_l$ are previsions about the future whose role is – if they are for some reasons unacceptable - to give us time to react before they (or rather some unsatisfactory crisp values from their domain) actually do occur.

4 Example

Let us consider the following example, based on [4]. Let the project consist of five activities:

- Activity 1: identifying market needs
- Activity 2: designing a new product
- Activity 3: testing the prototype
- Activity 4: manufacturing the first series of the product
- Activity 5: Marketing and selling the manufactured product

The goal of the project is to successfully introduce the product into the market, thus sell at least 90% of he first series of the product, within two years from the project start. $\Re_0$ is such that all the five activities should be executed in a sequence – the *i*th activity after the (*i*-1)th is finished (*i*=2,..,5). The initial duration estimates are as follows (in months): $D_{1,0} = (1,2,2,3), D_{2,0} = (2,4,5,6), D_{3,0} = (4,6,6,7), D_{4,0} = (3,5,6,7), D_{5,0} = (1,2,2,3)$, we will not consider cost in our example. As here we have only one path in the project network, and this will be the critical path, we can calculate $P\tilde{D}$, in this special case, directly as the sum of all the fuzzy durations. We have thus $P\tilde{D}$=(11,19,21,26). We see that the deadline two years is in danger – it is possible that, the project will last up to 26 months, but let us assume that the overrun and the degree of its possibility seems for the moment acceptable.

Let us now suppose the first control moment (*l*=1) is at the end of the 3. time unit. It stated that:

- Activity 1 has not been finished, we think it will still need about 1 month (not less that 0,75 month, but not more than 1,5 month) to be finished. It has already been executed for 3 months ($AD_{1,1} = 3$)

- Although Activity 1 still goes on, we have already started Activity 2 one month ago ($AD_{2,1} = 1$) and we think we have done 10% of the work
- The other activities are not started
- We know that marketing and selling will be overtaken by a subcontractor, who will do the marketing campaign (he says it will take 2 months and the corresponding contract has been signed) and buy from us the whole first series, taking care of selling it on the market, although offering us a slightly lower price than we expected on the market. However, he requires us, and we admit it might be useful, to do a redesigning of the product after its testing, as it seems that testing is bound to indicate some problems which will have to be improved in the design of the project.

The above information will lead to the following reestimations:

- We will now have 6 activities in the project ($n_1 = 6$), Activity 6 – redesigning the product – should be executed after Activity 3 is finished and before Activity 4 is started. This defines the new set of constraints $\Re_1$, together with the fact that Activity 2 has actually overlapped Activity 1, they are both being executed in the control moment 1.
- $\tilde{D}_{1,1} = 3 + (0.75,1,1,1.5) = (3.75,4,4,4.5)$
- We have $0{,}1 \cdot D_{2,0} = 0.1 \cdot (2,4,5,6) = (0.2,0.4,0.5,0.6)$ - this is how much time we thought executing 10% of Activity 2 would take. Actually it has take one month, which is outside our previsions. Calculating $AD_{1,1} / 0.1$ we get 10 months, which gives us the time the activity would be finished if we worked at the same pace as before. Confronting our prevision $\tilde{D}_{2,0} = (2,4,5,6)$ with the estimate 10 and taking into account some information from the team executing the activity, we give the following new estimate of its duration: $\tilde{D}_{2,1} = (7,8,8,9)$.
- The recently added Activity 6 will be carried out by the same team as Activity 2 and between the two activities there is also the link of learning: redesigning the product will probably be easier and shorter that designing it from the beginning. Thus, basing ourselves on the information we get from the team and on $\tilde{D}_{2,1} = (7,8,8,9)$, we estimate $\tilde{D}_{6,1} = (1,2,2,3)$
- The information from the subcontractor allows us to “defuzzyfy” the estimate of Activity 5 ($\tilde{D}_{5,1} = (2,2,2,2)$), under the assumption that the subcontractor is reliable and keeps to the contracts he has signed. But if it is so, we do not have to worry about selling our product, as it will him who will buy it and it will be his problem to sell it. The information about

the lower price would influence our cost estimate, which are not taken into account in the illustrative example.

- The other estimates ($D_{3,1} = (4,6,6,7), D_{4,1} = (3,5,6,7)$) are taken form the previous stage, as no new information concerning those two activities is available.

From the control moment on Activity 1 and Activity 2 will executed simultaneously, but as the estimated remaining time of Activity 1 is $(0.75,1,1,1.5)$ and the estimated remaining time of Activity 2 is (6,7,7,8), it is clear that Activity 1 will not be on the critical path for any value from the range (0.75,1.5). Thus, we calculate PD_1 summing up the estimates for all the other activities (including Activity 6) and the time that has elapsed to the control moment, and we get PD_1=(19,25,26,30). Thus, the deadline of 24 months is in high danger. However, we have to remember that we are only after the 3. month of the execution of the project, we still have 21 months till the deadline and we have time, knowing already now that there is a high risk of not keeping the deadline, to undertake adequate steps, negotiating, seeking for additional resources etc.

5 Conclusions

We have presented a method of planning and controlling projects characterised by a high degree of uncertainty, innovativeness and due to much changes during the project execution, which cannot be foreseen in the planning phase. The method requires the users to think while estimating project parameters in terms of trapezoidal fuzzy numbers, which in fact means only giving four parameters: an optimistic one, a pessimistic one and one or two medium ones, which may also be equal to each other. It seems that such an approach would be acceptable in practice. The approach requires in each control moment not an automatic generation of numbers which do not take into account the really important information about the project history and its future, but a deeper insight into the development of the project, the influence of its environment and the interdependencies between various project elements (activities, resources etc.). This will mean a stronger effort in planning and controlling projects, but will give as a reward a more reliable information about the project and the risk connected to it, and, what is the most important thing, this information will usually be available early enough to take adequate actions.

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