Interpretation of Dual Peak Time Signal Measured in Network Systems

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Abstract. During a computer network activity there are many events which occur in certain moments of time with a given likelihood. The statistical aspect of this can be described by a probability density function of a random variable connected with the event time. The function in many cases has a specific feature which is not covered by any well known probability distributions. It has two maxima, in particular when time signal in network systems is measured. It is suggested that the function is a convolution of other ones. However, in order to simplify the probability modeling, a special distribution named *if* can be defined, especially for discrete variables. Interpretation of dual peak.

Keywords: distribution, pdf, pmf, if, multi-maximal, time, ping, peek.

1 Introduction

In this article the authors generally examine the time of execution of various processes in computer systems. When measuring these times, two phenomena were found during further analysis of the results. The time of execution of a given process is not the same in consecutive trials. It can be said that this is a 'distribution' of these times. The second phenomenon found by the authors is that in many cases the time of execution is not concentrated around one value but there are two peaks in these distributions. This is the effect of various execution paths of programs due to the execution of conditional statements. Distribution with this feature is called by authors an *if* distribution.

Such distribution can be used to better describe a computer networking system, which software is naturally based on caching or conditional program statements. Additionally, some examples of a simple sequence of command statements were found, e.g. writing the data to a system log or performing a ping transaction.

2 Industry Standard, Statistical Evaluation of Measurement Data

The industrial standard used for verification of server systems is a benchmark. It is the way of system verification by one synthetic factor. Such a measurement

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method allows for comparison systems but has many imperfections. The first one is standardized system workload prepared to collect a stable result. It is expected that if the system A achieved a better result than the system B, the relation of the introduced measure will be kept during every next repetition of the test. However, if the server is exposed to real workload within a real computer network, one cannot exclude that the system B will be more efficient in the particular real application, in spite of a clear test indication pointing to the server A. One tries to resolve this defect of the tests based on a synthetic factor calculated from the mean value by extending the family of simulations. This, in the authors' opinion, complicates the problem of performance estimation instead of helping. After performing the research there are many results for the A and B systems, which are not correlated with the system's architecture until it is better recognized.

Many questions regarding server's performance [1,2] remain without answers. Based for instance on the test result, one is unable to claim how the modification of the B system, let us say the server's CPU exchange, will influence the network system performance. In time-limited systems the maximal measurement data values are used for analysis. Typically to estimate the performance, the measurements of the mean value of system parameters are used. Because of that, the measurement error is averaged out and the method becomes resistant to any measurement errors. The mean value is representative for the measurement data when the system is of a stationary type, but from the measurement practice point of view it turns out that it is not an assumption, which is simple to fulfill.

Let us assume for a given network system that there is a necessity of measure the response time R. A basic example of such a measurement using a ping command is presented in Fig. 1. The ping utility (iputils software packet) provides

```
ping -c 5 192.168.16.11
PING 192.168.16.11 (192.168.16.11) 56(84) bytes of data.
64 bytes from 192.168.16.11: icmp_seq=1 ttl=64 time=1.38 ms
64 bytes from 192.168.16.11: icmp_seq=2 ttl=64 time=0.241 ms
64 bytes from 192.168.16.11: icmp_seq=3 ttl=64 time=0.257 ms
64 bytes from 192.168.16.11: icmp_seq=4 ttl=64 time=0.265 ms
64 bytes from 192.168.16.11: icmp_seq=5 ttl=64 time=0.253 ms
--- 192.168.16.11 ping statistics ---
5 packets transmitted, 5 received, 0% packet loss, time 4001ms
rtt min/avg/max/mdev = 0.241/0.480/1.388/0.454 ms
```

Fig. 1. Example of performance measurement using a ping command

the following basic statistical values (last line of output results in Fig. 1): `min` is the minimal value of the RTT time ($x_{\min}$), `avg` is the mean value (x), `max` is the maximal value ($x_{\max}$), as described in equations:

$$x_{\min} = \min(x_i) \tag{1}$$

$$\bar{x} = \frac{1}{n} \sum_{n}^{i=1} x_i \tag{2}$$

$$x_{\max} = \max(x_i) \tag{3}$$

where x_i are measured values of time. The values comes from basic statistical processing. Let us note that `mdev` (Fig. 1) is not the standard deviation (4) but is the mean deviation (5). The standard deviation σ is defined as the square root of the variance σ^2.

$$\sigma = \sqrt{\frac{1}{n-1} \sum_{i=1}^{n} (x_i - \bar{x})^2} \tag{4}$$

The mean deviation (also called mean absolute deviation) is the mean of the absolute deviations of a set of data which contains mean values of the data. For a sample of the size n, the mean deviation is defined as [1]:

$$md = \frac{1}{n} \sum_{n}^{i=1} |x_i - \bar{x}| \ . \tag{5}$$

3 System Log as a Source Data

A typical method of collecting data about server's performance is a log analysis. In network systems usually a log file is a source of data to analyze the behavior of various processes. A network server log is treated as a measurement device for data acquisition of the execution time of a chosen operation. In the research [3] the authors have an existing networking system, which writes necessary parameters to the system data log. The authors want to acquire the execution time of data processed by a selected operation in the existing system. Let us define the system log as:

$$\begin{matrix} t_1 - o_1 \\ t_2 - o_2 \\ \vdots \\ t_L - o_L \end{matrix} \tag{6}$$

Let us define the difference between two time stamps in the following way:

$$\Delta_l = t_1 - t_{l-1} \tag{7}$$

A log is a pair of a time-stamp and operation name that was executed by the system. In the observation time T, the L number of log lines was collected in the order from 1 to L. Each operation can be classified as one set of operation type. The number of operation types from 1 to K is finite in the system. The

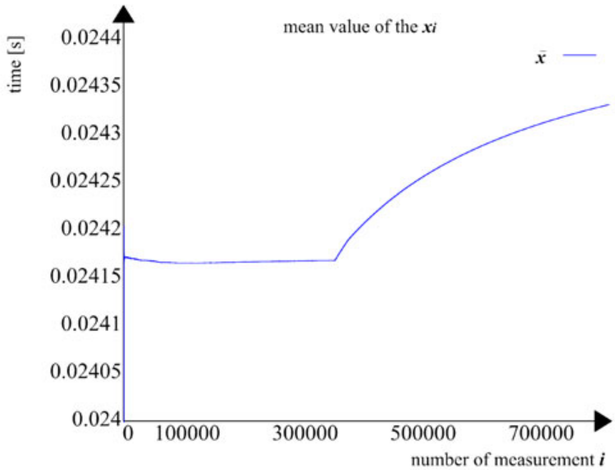

Fig. 2. Change of the mean value of the execution time x_i during the selected measurement process

assumption is that the log is well-detailed and descriptive, so the log entry of a given type obtains service only from one resource.

During the research conducted by the authors [4], measurement errors important for the analysis, entered by log records, were noticed on the network server. It turns out (Fig. 2, 3) that the server's activity, without any commands except writing timestamps to the log file, seems to be non-stationary. In a set up range (from $i = 0$ to about 360 000) the mean value varies and after that it should enter a steady state. Except of this, it can be noticed in the pictures 3 and 4 that the mean value jumps when iteration i is about 360 000 and 380 000 during the experiment. The second jump is better visible in Fig. 3. Because of the fact that floating of the mean value in the measurement process can be noticed (Fig. 2), it is necessary to better investigate the non-stationary nature of this process. During the analysis of the time of execution of processes (time-stamps) from the log file (Fig. 3) the problem of distribution of the characteristic *if* often occurs. Let us notice that for the description of the stationary process x_s a histogram was used, which indicates two maxima. Following the observation, a decomposition of the observed process can be done into the stationary process x_s as well as a shift of the process by the constant value A, which is changing during the experiment (Fig. 4). This is a way which describes the system work more precisely. The time signal $x(t)$ can be written as a multiplication of the function of the stationary process x_s and the constant value.

$$x(s) = A(t)x_s(t) \ . \tag{8}$$

By gathering data of the server's performance from the log file one can not take into consideration the writing procedure to the log. However, in this case all the data used during the analysis will be burdened with an error.

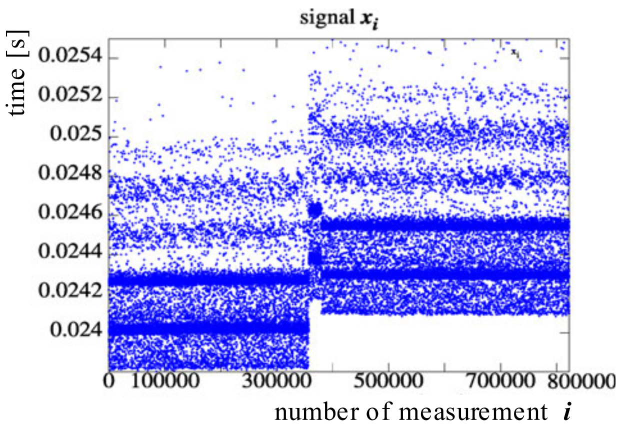

Fig. 3. Data value of the execution time x_i during measurement process

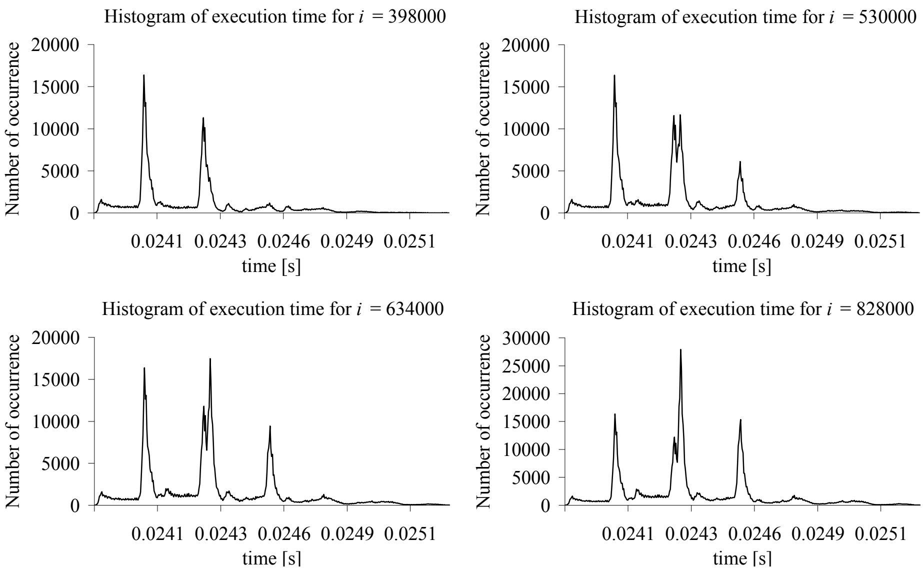

Fig. 4. Histogram of PrintToDebugLog execution time x_i for selected number of samples i

4 Applying Probability Mass Function for Computer Measurement

Let us extend the numerical characteristic that is computed from a sample of observations. Beside these basic statistical parameters (1)–(5), it is possible to

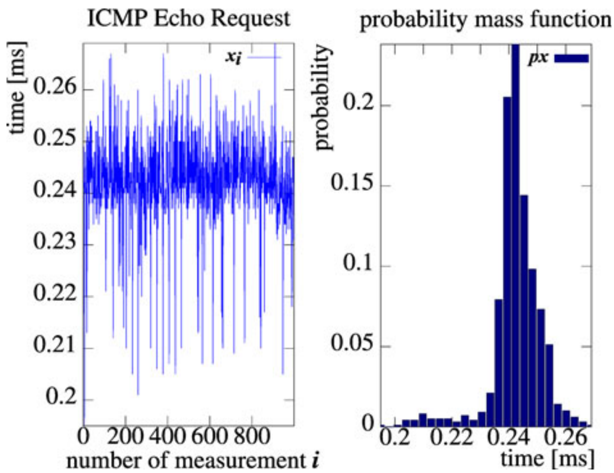

Fig. 5. ICMP Echo_Request measurement time series and its histogram

collect data and build histograms [5]. Examples of a measurement time series and its histogram are shown in Fig. 5. If a continuous $x(t)$ function exists, the probability density function can be defined as:

$$p_x \lfloor a \leq x(t) \leq b \rfloor = \int_a^b x(t)dt \ . \tag{9}$$

Sometimes the distribution function is used [6] to designate the probability density function (pdf). However, this term can be considered as the probability distribution function, or as the cumulative distribution function, or it may be the probability mass function rather than the density. So it is necessary to be aware of the meaning of this name. The probability mass function (pmf) specifies the probability that a discrete random variable is exactly equal to some value [7]. The pmf and the pdf are not the same. The values of the pdf which are defined only for continuous random variables do not represent the probabilities. Alternatively, the integral of the pdf over a range of possible values can be used, and thanks to that the probability of a random variable falling within that range can be shown.

5 *if* Distribution Basic Definition Form

Let us assume the existence of a statistically independent conditional statement x and operation y (Fig. 6). Operation y is responsible for writing data to a disk. If the buffer, filled with the data, is full, $x^{(2)}$ operation writes the data to the disk, else $x^{(1)}$ operation writes the data to the buffer in RAM. The times of execution

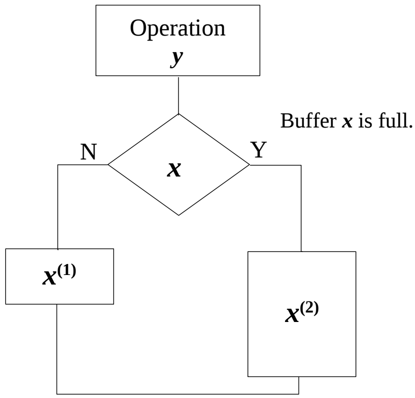

Fig. 6. Algorithm of program execution with a conditional statement x

of operations $x^{(1)}$ and $x^{(2)}$ are different. The time in the measurement process is defined only for discrete values of the independent variable, so the time is an independent variable. The signal belonging to the discrete time domain is called a discrete-time signal [8]. The independent variable of a mathematical object (sequence) that represents a discrete-time signal belongs to the set of integer numbers.

In a computer performance analysis [9,10], a discrete-time signal can be seen as generated by the system and defined in the discrete time domain, rather than seen as discrete-time signals obtained by sampling continuous-time signals. A discrete-time signal is mathematically represented (10) by a sequence of values x_i:

$$x(t) \rightarrow x_i, i \leq \infty \ . \tag{10}$$

The i-th sample of the sequence is the i-th value x_i in the sequence. The independent variable i belongs to the set of integer numbers, so x_i is mathematically defined only for integer values of i.

Let us define a probability mass function for a conditional statement x (Fig. 6). The value of the measurement $x^{(1)}(t)$ is the time of execution of an $x^{(1)}$ operation and it is a discrete signal $x_i^{(1)}$. Analogously, $x^{(2)}(t)$ is a discrete signal $x_i^{(2)}$. For a basic *if* distribution form, let us assume that all the results of the measurement are the same. This means that:

$$x_i^{(1)} = const \tag{11}$$

$$x_i^{(2)} = const \ . \tag{12}$$

Probability of the execution of the $x^{(1)}$ path can be denoted as $px^{(1)}$. The measurement value signal $x_i^{(2)}$ in the path $x^{(2)}$ has the probability $1 - px^{(1)}$. For $x^{(1)}$, $x^{(2)}$, the *if* distribution is defined by $px^{(1)}$, $px^{(2)}$, which are the probabilities of the execution of operations $x^{(1)}$, $x^{(2)}$ respectively (13).

$$\text{for measurement } x_i^{(1)}, x_i^{(2)} \textit{ if} \text{ distribution is defined by} \\ px^{(1)},\ px^{(2)} \text{ where } px^{(2)} = 1 - px^{(1)} \tag{13}$$

The case when the formula (13) follows the conditions (11), (12), is a basic form of the *if* distribution. In other words, when the basic form signals $x_i^{(1)}$ and $x_i^{(2)}$ follow the formulas (14), (15), the basic form of the *if* probability mass function has two peaks, as shown in Fig. 7.

$$x_i^{(1)} = t_1 \tag{14}$$

$$x_i^{(2)} = t_2\ . \tag{15}$$

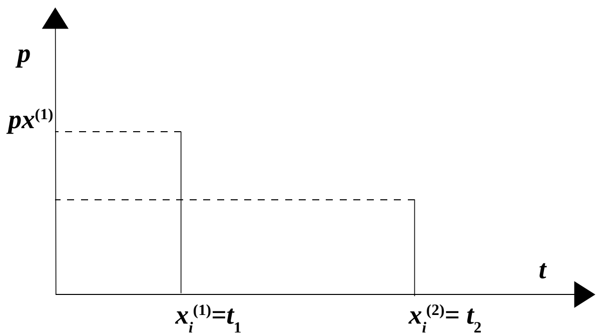

Fig. 7. Basic form of *if* probability mass function

In the example (Fig. 6) the operation y and the conditional statement x are executed together. Let z be a program executing both x and y operation. If a measurable result of the execution of $x(t)$ and $y(t)$ exists as independent discrete functions x_i and y_i, what will the z execution time be? If the z execution time is presented as the probability mass function pz, it will be a convolution [8] of the probability mass function px and py (16):

$$pz_j = px * py = \sum_{j=1}^{i} px_j py_{i-j} \tag{16}$$

where: px, py, pz are time probability mass functions (Fig. 5) as results of the execution x, y, z, px_i, py_i, pz_i are probabilities for the given interval i of px, py, pz. $x^{(1)}$ and $x^{(2)}$ are disjointed events and are dependent. However, if operation x consists of $x^{(1)}$, $x^{(2)}$ then the event is complete event. The x cumulative distribution function (CDF) is equal 1. The convolution pmf function $px * py$ are defined if x, y are independent. The example of the convolution is shown in Fig. 8.

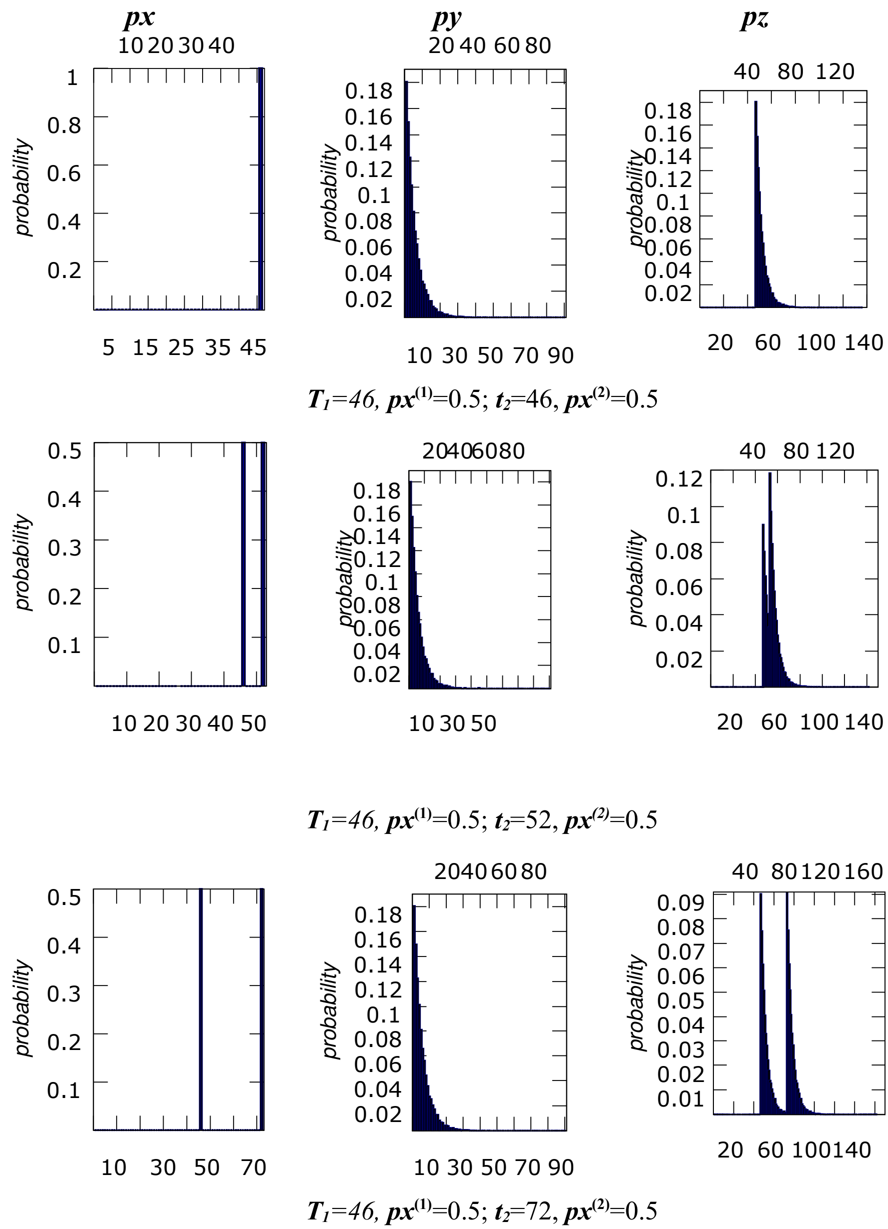

Fig. 8. Example of the convolution of probability mass functions *if* and exponential distribution

6 Extended Definition of the *if* Distribution

The $px^{(1)}$, $px^{(2)}$ are time probability mass functions of measurements that are results of the execution of $x^{(1)}$, $x^{(2)}$. In general $px^{(1)}$, $px^{(2)}$ can receive any probability mass function distribution.

$$\text{for measurements } x_i^{(1)}, x_i^{(2)}$$
$$\textit{if} \text{ distribution is defined by } px^{(1)}, px^{(2)} \text{ where}$$
$$\sum_{i=1}^{m} px_i^{(2)} = 1 - \sum_{i=1}^{j} px_i^{(1)} \quad . \tag{17}$$

The $px_i^{(1)}$, $px_i^{(2)}$ are probabilities for a given interval i of $px^{(1)}$, $px^{(2)}$.

In case the number pt_1 of the $x^{(1)}$ operation is known, the probability of the execution of the $x^{(1)}$ operation can be calculated. Then, for defining the px function it is possible to use pt_1 probability and a given g distribution instead of $px^{(1)}$.

For a signal $x_i^{(1)}$ it is possible to find a function g, which, after multiplication by pt_1, gives $px^{(1)}$ (17). It is not necessary to use a convolution because pt_1 is a single constant value.

$$px^{(1)} = pt_1 \cdot g \quad . \tag{18}$$

It is obvious that:

$$pt_2 = 1 - pt_1 \quad . \tag{19}$$

Analogously, for $x_i^{(2)}$ it is possible to find a function h, which multiplicated by pt_2, gives $px^{(2)}$ (20).

$$px^{(2)} = pt_2 \cdot h \quad . \tag{20}$$

7 No Software-Based Process with *if* Distribution

The dual peak function in this article was defined as one related to the execution of a conditional statement – this is why its name is the *if* distribution. However, it must be noted, that the physical interpretation of the process by means of the *if* function is not only confined to systems based on the programming execution of conditional statements.

As a practical example, let us analyze the result of the measurement of round trip time on the empty network from one server and two workstations (Fig. 9). For this measurement experiment three hardware identical computers with the same CentOs (Red Hat) operating system were prepared. From one computer (server) the ‘ping’ command was executed and the round-trip time (RTT) result was logged to two other computers (workstations).

The real estimation of $x^{(1)}$ and $x^{(2)}$ can be changing over the time and is depended on various parameters. The values depend on load of a network. In the example we want to show that characteristic distribution (named *if* is not only connected with execution of conditional command). In the measurements

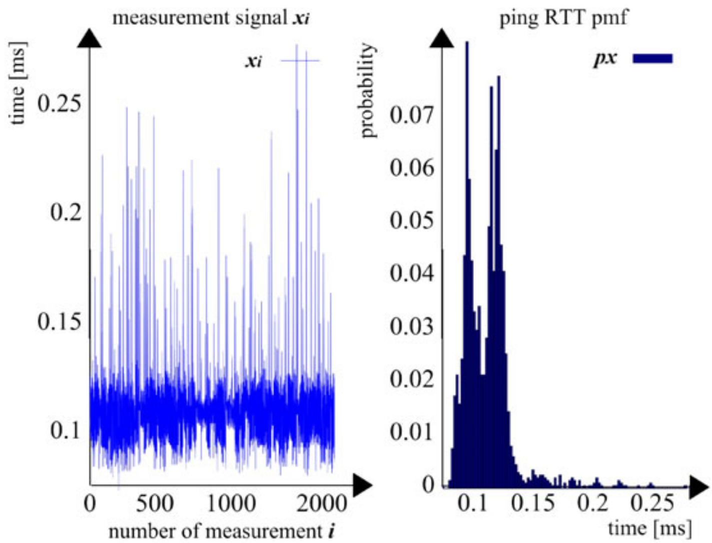

Fig. 9. Example of the measurement of round-trip time in a network with one server and two workstations

with normal load, we expected the same results for both stations. However, the difference was constant for all experiments. In this case the diference between $x^{(1)}$ and $x^{(2)}$ does not describe the process but rather depicts its variation area.

8 Conclusions

The performed research of network systems show that the measurement values of system activity, which before the experiment appeared to be simple and have the character of a constant value, have a much more complex statistical description. Writing to the server's log file was selected as the analysis base, because of set of data registered as the real workload and replies of the server. The process measurements were performed on the network server. The probability mass function was used instead of the mean value. The mean value was changing during the experiment and its usage was less accurate than the probability mass function. A description by the probability mass function is more accurate in case of the experiments made, because the process which was non-stationary can be described, in some cases, by a stationary process and a constant factor value, changes of which are easy to detect. The analysis of the stationary element x_s leads to a description which is defined as a conditional statement. So a simplified and complex process definition was introduced and it was named the *if* distribution. A statistical description by means of the *if* distribution can be used not only for systems implemented with conditional statements in programs, but also in many hardware based cases in the analysis of networks' behavior.

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