Towards a MIP-Cut Metascheme

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Cutting planes (*cuts*) are very popular in the OR community, where they are used to strengthen the Linear Programming (LP) relaxation of Mixed-Integer Programs (MIPs) in the hope of improving the performance of an exact LP-based solver. In particular, an intense research effort has been devoted to the study of families of *general* cuts, whose validity does not require the presence of a specific MIP structure—as opposed to problem-specific cuts such as, e.g., subtour elimination or comb inequalities for the traveling salesman problem.

Among general cuts, Gomory's Mixed-Integer Cuts (GMICs) play a central role both in theory and in practice. These cuts have been introduced by Ralph Gomory about 50 years ago in his seminal paper [1]. Though elegant and computationally cheap, they were soon abandoned because they were considered of little practical use [2]. The situation changed radically more than 30 years later, when Balas, Ceria, Cornuéjols and Natraj [3] found how to take advantage of exactly the same cuts but in an different framework. In our view, this is a good example of the importance of a sound framework for MIP cuts.

Even today, MIP solvers are quite conservative in the use of general cuts, and in particular of GMICs, because of known issues due to the iterative accumulation of the cuts in the optimal LP basis. This leads to numerical instability because of a typically exponential growth of the determinant of the LP basis.

Following our recent joint work with Balas and Zanette [4, 5], in this talk we argue that the known issues with cutting plane methods are largely due to the overall *framework* where the cuts are used, rather than to the cuts themselves. This is because the two main cutting plane modules (the LP solver and the cut generator) form a closed-loop system that is intrinsically prone to instability. Hence a kind of "decoupling filter" needs to be introduced in the loop if one wants to exploit the full power of a given family of cuts.

A main goal of the talk is to refocus part of the current research effort from the definition of new cut families to the way the cuts are actually used. In fact, cutting planes still miss an overall "meta-scheme" to control cut generation and to escape local optima by means of diversification phases—very well in the spirit of Tabu or Variable Neighborhood Search meta-schemes for primal heuristics. The development of sound meta-schemes on top of a basic separation tool is therefore an interesting new avenue for future research, with contributions expected from all the three CP/AI/OR communities. The relax-and-cut framework for GMICs recently proposed in the joint work with Salvagnin [6] can be viewed as a first step in this direction.

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