Chapter 9
Dual Measurement Method (DMM)

9.1 Possibility of Resolving the Secondary Flow

LDA measurements are component measurements. This means that the measured velocity components are always referred to as coordinates in the LDA optical system. For direct measurements of velocity components in the flow field coordinate system, LDA coordinates have to be arranged to agree with the flow field coordinates. In other cases, all interested velocity components in the flow field can be obtained from coordinate transformation, as described in Chap. 6. Correspondingly, LDA users in practical applications always try to make sure either the accurate coincidence between two coordinate systems or an accurate rotation by a given angle. The alignment error has mostly been assumed to be small and hence is often neglected. This is generally allowed in most flow measurements at which the flow field could still be truly represented from the measurements despite the measurement errors. In contrast, however, there are cases at which a small error in LDA alignment to the flow field coordinates could lead to total misinterpretation of the actual flow field. Such a case has been for instance encountered by Zhang and Parkinson (2001, 2002) while trying to measure the very weak but very important secondary flow structure in a high speed water jet of a Pelton turbine (Fig. 9.1) In principle, it is generally impossible to align LDA velocity components u and v to be exactly coincident with velocity components u_x (axial) and u_t (tangential) of the jet flow. An inevitable small alignment error $\tau \neq 0$ simply means that the measured velocity component v additionally comprises a part of the axial velocity component, as given by

$$v = u_t \cos\tau + u_x \sin\tau \tag{9.1}$$

Although the bias angle τ as an error parameter is usually very small, the term $u_x \sin\tau$ in the above equation, however, could be still very large because of high values of the axial velocity component u_x in the high speed jet flow. In the case that this term is comparable to or larger than the term $u_t \cos\tau \approx u_t$, the secondary flow pattern represented by u_t would be sensitively and totally misinterpreted from measurements. For this reason the mentioned secondary flow in a cross section of the high speed jet flow could be measured neither directly nor indirectly through coordinate transformation.

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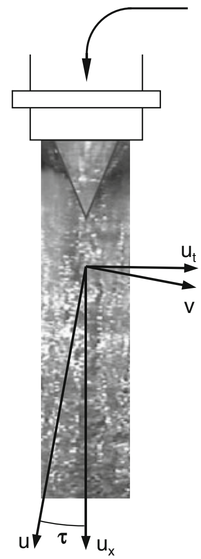

Fig. 9.1 Possible inaccurate LDA alignment to a high speed jet flow and the associated problem of resolving the secondary flow structure in the jet cross section

Such a problem arising from inaccurate LDA alignment could be basically solved, when the alignment error i.e. the bias angle τ could be accurately identified and then the appropriate correction calculation of measurement date is implemented. In the mentioned measurement of the secondary flow structure in the high speed water jet, a method that is known as the Dual Measurement Method (DMM) was developed and applied to accurately identify the geometrical deviation in the LDA alignment to the jet flow. After finding out the alignment error, measurement data could be correctly evaluated. It dealt with a quite interesting flow phenomenon which could be detected by DMM.

The background of the dual measurement method is the two-measurement principle which is sometimes also called two-step method. The most famous example of using this principle is the Michelson-Morley experiment (Hecht 1990) that was constructed, by subsequently rotating the apparatus horizontally for 90°, to be able to measure the difference of light speeds in different directions, provided that this difference would exist. Another example that is directly related to LDA techniques is to exactly check the constant shift frequency generated in one of two laser beams of a laser beam pair, as it will be shown in Chap. 18.

In this chapter, the dual measurement method will be firstly presented in the version of its initial application to the high speed jet flow with complex secondary flow structures (Zhang and Parkinson 2001, 2002). Then the extended form of DMM (Zhang 2005) will be shown that can be applied to other special cases.

The readers would probably be interested in how the laser beams could enter into the jet that has a turbulent and hence opaque surface. The corresponding measurement technique of using an optical wedge can be found in Chap. 18 showing application examples.

9.2 DMM in Basic Form

The dual measurement method was initially developed to accurately resolve the secondary flow structure in the high speed jet flow of a Pelton turbine. The jet was generated by an injector which was connected to a bend, as shown in Fig. 9.2a Because of the bend effect and the resultant change in the flow state, the flow after passing through the bend is characterised by the existence of a secondary flow structure across the pipe section, as shown in Fig. 9.2b based on LDA measurements of tangential velocity components. It deals with a typical secondary flow pattern that clearly demonstrates two identical areas with flow rotations. This flow structure remains while passing through the injector, as it was measured and has been shown in Fig. 9.2c, provisionally without mentioning how this secondary flow structure could be measured. Although it deals with a small scale secondary flow structure, it represents the main reason for the jet instability. One of the most serious disturbances on the jet because of related secondary flows is the generation of a chain of droplets on the jet surface, as indicated in Fig. 9.2a. The measurement of the secondary flow structure in the mentioned high speed water jet represents a highly difficult task and could only be conducted by means of DMM.

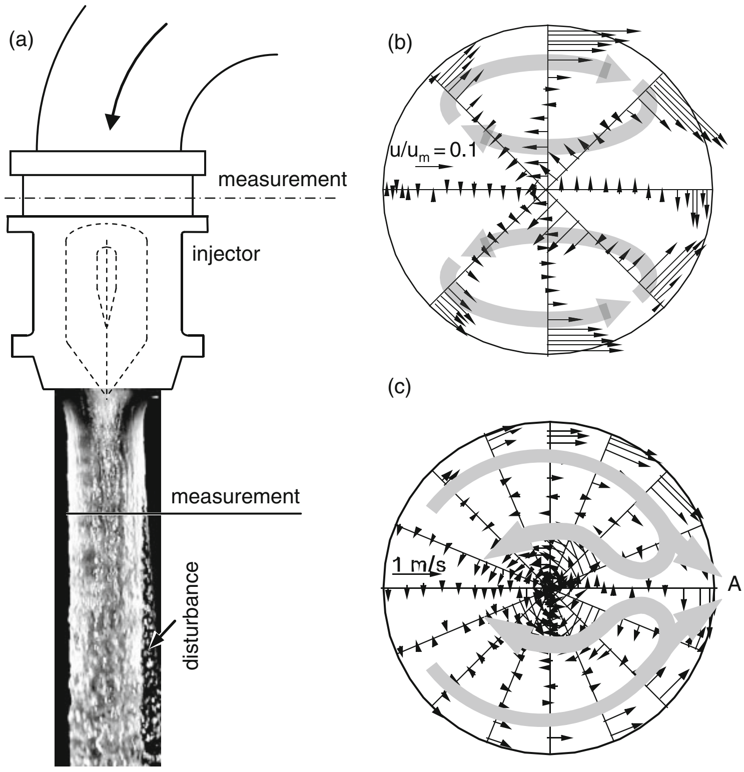

Fig. 9.2 Application example of the Dual Measurement Method (DMM) for accurately resolving the secondary flow in the high speed jet of a Pelton turbine (Zhang 2009)

The jet flow was made accessible for LDA measurements from all directions around the jet axis, as shown in Fig. 9.3a where a support platform for LDA head was installed. The LDA head is mounted on the traversing board of the support device that enables a velocity profile in the jet to be measured. For the measurement technique of getting the laser beams into the jet with turbulent and rough surface the readers are referred to Chap. 18.

The LDA optics was aligned for direct measurements of the tangential velocity component without using coordinate transformation. In assuming an alignment error i.e. a bias angle $\tau \neq 0$ as shown in Fig. 9.3b, the measured velocity component v takes

$$v_0 = u_t \cos\tau + u_x \sin\tau \tag{9.2}$$

In this equation the positive bias angle is defined as it is shown in Fig. 9.3.

The dual measurement method is constructed so as to arrange an additional flow measurement by simply rotating the LDA head around the jet axis by 180° (Fig. 9.3c). While rotating the measurement system the alignment error i.e. the bias angle τ can be assumed to be constant. The measured velocity component v this time at the same point in the jet flow is then given by

$$v_1 = -u_t \cos\tau + u_x \sin\tau \tag{9.3}$$

The second term on the r.h.s. of the above equation is the part arising from the axial velocity component u_x. For exact LDA arrangement with $\tau = 0$ there is

$$v_1 = -v_0 \tag{9.4}$$

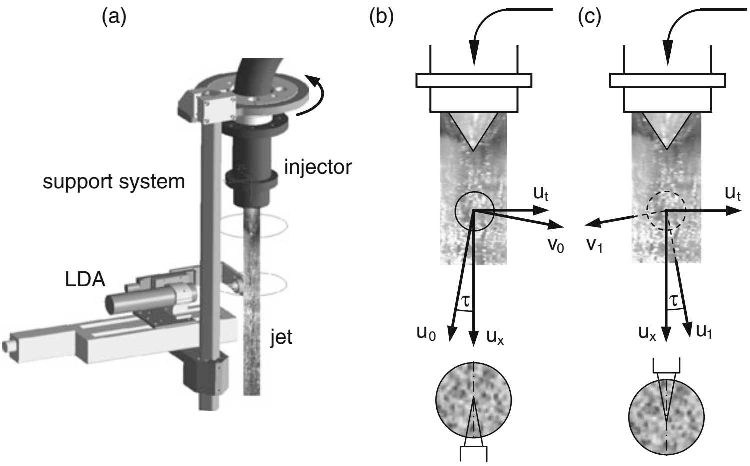

Fig. 9.3 DMM principle for accurately resolving the secondary flow structure in the high speed jet of a Pelton turbine

This equation represents a criterion of the faultless LDA alignment for direct measurement of velocity component u_t. Both values (v_1 and v_0) act as mirrored about $v = 0$. This ideal "mirrored view" of two velocity components v_1 and v_0 will be more or less disturbed by errors with $\tau \neq 0$ in LDA alignment. It can be concluded that any deviation from Eq. (9.4) is quantitatively related to the bias angle τ Since this bias angle τ is actually a geometrical or mechanical arrangement error, it basically causes a systematic error which, according to Eqs. (9.2) and (9.3), takes $u_x \sin\tau$. This systematic error in the velocity measurement is called velocity shift because it acts as an additive quantity in both Eqs. (9.2) and (9.3).

From Eqs. (9.2) and (9.3) one obtains

$$v_{sh} = u_x \sin\tau = \frac{v_0 + v_1}{2} \tag{9.5}$$

as well as for $\tau << 1$

$$u_t = \frac{v_0 - v_1}{2} = v_0 - v_{sh} \tag{9.6}$$

It is now clear that through twice measurements of the same flow the velocity shift as the outcome of the LDA alignment error can be exactly identified. This dual deal of the jet flow measurement leads to direct determination of the tangential velocity component that otherwise could not be accurately measured. Because of this, the applied method is called the Dual Measurement Method (DMM).

With regard to the assumption that the velocity shift v_{sh} is a kind of systematic error involved in the LDA arrangement, it needs only to be determined one time at a fixed point in the flow by means of DMM. It can then be directly applied, according to Eq. (9.6), to correct the measurement results that are achieved at other points in the flow.

In the above mentioned example, the dual measurement method was applied to identify the bias angle τ and the associated velocity shift. The completed two measurements of the same velocity profile across the jet have been shown in Fig. 9.4a Because it deals with the same velocity profile, the "mirrored view" of two-time measurements would have been expected. That is to say that two velocity profiles should have symmetrically lain on both sides of the neutral line with $v = 0$, if the bias angle τ would be zero. The measured deviation of the symmetry line from $v = 0$, as shown in Fig. 9.4a, just corresponds to the velocity shift calculated by Eq. (9.5). In this measurement example, the velocity shift reads at 0.4 m/s. It corresponds to a bias angle of $\tau = 0.92°$ (for $u_x = 25$ m/s). Obviously this is a quite small angle. The associated velocity shift, however, is of the same order as the existing velocity component itself or even higher. With respect to the velocity shift determined from Fig. 9.4a measurement results could be immediately corrected, as shown in Fig. 9.4b. Both measurements after correction then behave as mirrored at $v = 0$, as expected.

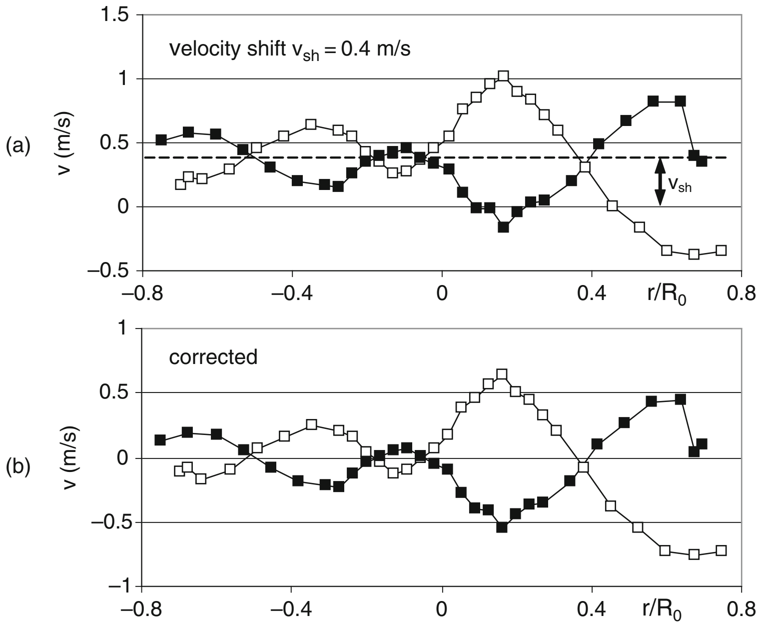

Fig. 9.4 Determination of the velocity shift by means of DMM and correction of the measurement results across the jet flow

With respect to the constant systematic error of $\nu_{sh} = 0.4$ in the installed system the complete secondary flow structure in a jet section has been measured and corrected, as already shown in Fig. 9.2c. Obviously the secondary flow that contains two counter-rotating vortices exhibits the similar flow structure as in the pipe ahead of the injector. At the point A both streams from two flow areas come together. Because of the absence of any rigid boundaries that could guide the jet flow, the fluid tends to escape from the jet. As the consequence a chain of water droplets comes about, as this has often been observed in hydro power plants with Pelton turbines since more than half century (Zhang 2009).

The comparison between Fig. 9.4a and Fig. 9.4b points out that a small bias angle τ could lead to total misinterpretation of the flow. In fact, none of the two measurements in Fig. 9.4a represents the real flow. While the real flow pattern in the jet section involves the swirling flow structures (Fig. 9.2c), each uncorrected measurement (Fig. 9.4a) simply shows a transversal motion of the fluid almost with $\nu > 0$.

Basically it is sufficient to apply the DMM to a single point in the flow to determine the bias angle τ and hence the systematic error in form of the velocity shift ν_{sh} In the presented example as shown in Fig. 9.4, the dual measurement method has been applied to a survey across the jet. This could be well realized by the constructed measurement system according to Fig. 9.3. The constant velocity shift across the jet straightforwardly demonstrates the reliability of DMM. The velocity shift as the systematic error involved in measurements has been thus verified.

It should be emphasized that the velocity shift could only be assumed as a constant systematic error, when the bias angle τ is kept constant while rotating the LDA support system. The consistency of the bias angle τ thus represents the prerequisite for applying the DMM.

9.3 DMM with Coordinate Transformation

In the last section, the direct dual measurement method has been presented. It is known as the direct method because in the applied example the LDA coordinates were initially set to be coincident with the flow coordinates by $u = u_x$, $v = u_y$ and $w = u_z$. Sometimes it is advantageous to rotate the LDA coordinate system by an angle α against the flow system, so that a coordinate transformation matrix $R(\alpha)$ generally exists between two coordinate systems (Chap. 6).

Because the angle of rotation of one coordinate system against another is by no means exact, its real value has to be assumed to involve a small LDA alignment error i.e. bias angle τ (<<1) and hence takes $\alpha + \tau$ according to Fig. 9.5 As for the general case, the velocity component to be measured in the secondary flow is denoted by u_y, instead of u_t that is used for the example in Sect. 9.2.

The alignment of a two-component LDA head according to Fig. 9.5a enables the measurements of velocity components u_0 and v_0. The velocity component u_x and u_y in the flow system are then simply obtained by

$$u_x = u_0 \cos(\alpha + \tau) + v_0 \sin(\alpha + \tau) \tag{9.7}$$

$$u_y = -u_0 \sin(\alpha + \tau) + v_0 \cos(\alpha + \tau) \tag{9.8}$$

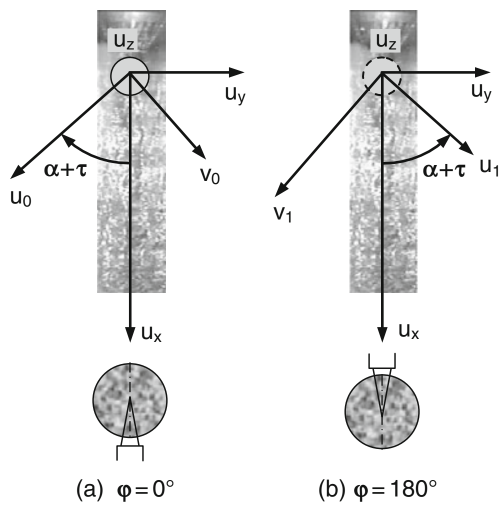

Fig. 9.5 DMM principle for accurately resolving the secondary flow structure in a high speed jet flow

Since the small alignment error τ only causes the negligible relative change in the axial velocity component u_x, so $\tau = 0$ can be applied to Eq. (9.7). Thus u_x acts as a component that is not affected by the alignment error τ. To Eq. (9.8) the approximations $\sin(\alpha + \tau) \approx \sin\alpha + \tau\cos\alpha$ and $\cos(\alpha + \tau) \approx \cos\alpha - \tau\sin\alpha$ because of $\tau << 1$ will be applied. This leads to

$$u_y = (-u_0 \sin\alpha + v_0 \cos\alpha) - u_x \tau \tag{9.9}$$

and further with the velocity shift $v_{sh} = u_x \tau$

$$u_y = (-u_0 \sin\alpha + v_0 \cos\alpha) - v_{sh} \tag{9.10}$$

Like in Eq. (9.5) the velocity shift appears again in the form of $v_{sh} = u_x \sin\tau$ i.e.$v_{sh} \approx u_x \tau$ because of $\tau << 1$. In addition, it has been again confirmed as an error that shifts the measurement results. Basically the velocity component u_y is intended to be determined by the first term on the r.h.s. of Eq. (9.10). This will be only true if $\tau = 0$ is true. The existence of the velocity shift v_{sh} as the consequence of the alignment error τ could significantly influence the determination of velocity component u_y, as already shown in the last section.

In order to examine the velocity shift existing in Eq. (9.10), the dual measurement method is again applied. This means that the LDA set-up needs to be turned around the x-axis by 180°, as shown in Fig. 9.5b. Attention has to be paid so that the bias angle τ as a systematic error has to remain constant.

From Fig. 9.5b and in analogy to Eq. (9.8), the velocity component u_y is directly written as

$$u_y = u_1 \sin(\alpha + \tau) - v_1 \cos(\alpha + \tau) \tag{9.11}$$

Based on similar calculation as that leading to Eq. (9.10) and newly with $u_x = u_1 \cos\alpha + v_1 \sin\alpha$ one obtains from the above equation

$$u_y = (u_1 \sin\alpha - v_1 \cos\alpha) + v_{sh} \tag{9.12}$$

Eq. (9.10) and (9.12) are found as basic equations to resolve both the velocity shift and the velocity component u_y. It yields

$$v_{sh} = u_x \tau = \frac{v_0 + v_1}{2}\cos\alpha - \frac{u_0 + u_1}{2}\sin\alpha \tag{9.13}$$

$$u_y = \frac{u_1 - u_0}{2}\sin\alpha + \frac{v_0 - v_1}{2}\cos\alpha \tag{9.14}$$

For $\alpha = 0$ one obtains Eq. (9.5) and (9.6), respectively.

Basically, the application of DMM in this case with coordinate transformation is completed. Although the velocity component u_y in Eq. (9.14) seems to be directly obtained from the dual measurements without via the calculation of the velocity shift, the velocity shift, however, gives a clear indication about the inaccuracy in the LDA alignment. It will also be used, according to Eq. (9.10), to directly correct measurements at other measurement points in the flow with the same LDA setup. In practical applications, one can estimate the possible velocity shift according to $\nu_{sh} = u_x \tau$ by assuming the possible alignment error τ before carrying out the LDA measurement. When the estimated velocity shift is negligible against the velocity component u_y to be measured, then the dual measurement would not be necessary.

Sometimes it may be convenient to conduct calculations by using the velocity component u_x which is often assumed to be known. For this purpose, two cases in Fig. 9.5 are again considered. The velocity component ν in LDA system can be expressed by

$$\nu_0 = u_x \sin(\alpha + \tau) + u_y \cos(\alpha + \tau) \tag{9.15}$$

$$\nu_1 = u_x \sin(\alpha + \tau) - u_y \cos(\alpha + \tau) \tag{9.16}$$

From these two equations and with respect to $\sin(\alpha + \tau) \approx \sin\alpha + \tau \cos\alpha$ it follows

$$\frac{\nu_0 + \nu_1}{2} = u_x \sin(\alpha + \tau) = u_x \sin\alpha + u_x \tau \cos\alpha \tag{9.17}$$

The velocity shift can thus be resolved as

$$\nu_{sh} = u_x \tau = \frac{\nu_0 + \nu_1}{2\cos\alpha} - u_x \tan\alpha \tag{9.18}$$

It is completely equivalent to Eq. (9.13).

Especially it follows from (9.15) and (9.16) directly

$$u_y = \frac{\nu_0 - \nu_1}{2\cos(\alpha + \tau)} \approx \frac{\nu_0 - \nu_1}{2\cos\alpha} \tag{9.19}$$

This equation is completely equivalent to Eq. (9.14). Further for $\alpha = 0$, Eq. (9.6) is again obtained.

9.4 Extension of DMM

The Dual Measurement Method (DMM) has been constructed based on the "mirrored view" of two measurements. Through the direct comparison between two measurements of the same velocity component, the alignment error in LDA optics and the associated velocity shift can be exactly determined. However, it often comes about that the positioning of the LDA head at 180° for the second measurement is impossible. This will limit the application of DMM in such a way that by positioning the LDA head at another angle the measurement cannot be compared with the first measurement, because it deals with the measurements of two different velocity components. This means that two measurements are generally not sufficient to identify the alignment error and the associated velocity shift. As an extension of DMM, a method of carrying out three arbitrary measurements at a fixed point in the flow from three different azimuth angles φ has been developed and also experimentally validated by Zhang (2005). The concept was that at a fixed point in the flow, any three velocity components measured at three different azimuth angles have to lie in a same plane and thus are related to a unique plane vector, when the LDA optics is aligned to the flow without any alignment error. Otherwise a unique plane vector would not exist, when the LDA system is aligned to the flow system with an error $\tau \neq 0$. Indeed, this concept has been clearly shown in Fig. 9.3. Because of the alignment error $\tau \neq 0$ the two measured velocity components v_0 and v_1 do not lie in the plane perpendicular to the jet axis.

9.4.1 Direct Component Measurements

The flow is given in the flow coordinate system by $\vec{u}_{\text{flow}} = (u_\text{x}, u_\text{y}, u_\text{z})$. Herein u_x represents the component in the main flow direction. As an example, the jet flow with the axial component u_x has been shown in Fig. 9.3 The velocity components related to LDA are involved in the velocity vector $\vec{u}_{\text{LDA}} = (u, v, w)$. Usually a two-component LDA system is applied and the measured velocity components are denoted by u and v, respectively. The theoretically exact coincidence between the LDA and the flow systems is assumed to be given by $u = u_\text{x}$ and $v = u_\text{y}$ The corresponding arrangement of the LDA head to the flow (Fig. 9.6a, b) is denoted as the basic arrangement. Deviations from this basic arrangement are confirmed by

(1) bias angle $\tau \neq 0$. It is unknown and should be identified through measurements. Here τ is like in Fig. 9.3 the angle between velocity components u and u_x;
(2) rotation of the LDA head about the x-axis for further carrying out two measurements at two other azimuth angles (Fig. 9.6c, $\varphi = \varphi_1$ and $\varphi = \varphi_2$).

With respect to these two deviations, specified by angles τ and φ, the general relationship between velocity components in two coordinate systems should be established. The velocity shift is expected to appear again in the form $v_{\text{sh}} = u_\text{x}\tau$. For the reason of mathematical simplicity the LDA alignment with τ and φ as parameters is assumed to be achieved by successively getting φ and τ from the basic arrangement with $u = u_\text{x}$ and $v = u_\text{y}$ (Fig. 9.6a). In the first step, when the LDA head has rotated about the x-axis by φ (Fig. 9.6c), the relationship between velocity components in LDA and flow systems is given by

$$u' = u_\text{x} \tag{9.20}$$

$$v' = u_\text{y} \cos\varphi - u_\text{z} \sin\varphi \tag{9.21}$$

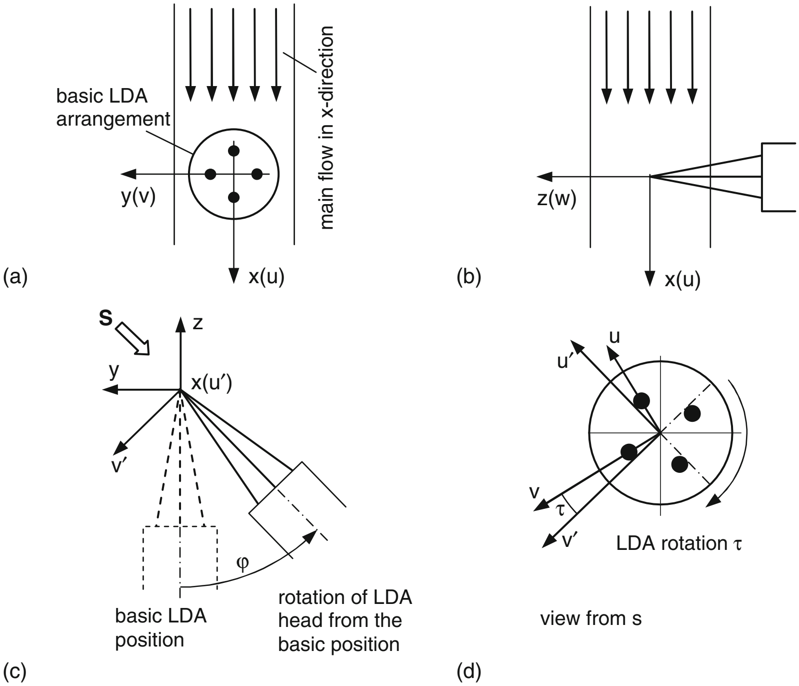

Fig. 9.6 Explanation of extended Dual Measurement Method

or in the form of matrix

$$\begin{bmatrix} u' \\ v' \end{bmatrix} = R' \begin{bmatrix} u_x \\ u_y \\ u_z \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\varphi & -\sin\varphi \end{bmatrix} \begin{bmatrix} u_x \\ u_y \\ u_z \end{bmatrix} \tag{9.22}$$

The LDA head is subsequently turned about its axis by τ (Fig. 9.6d). Corresponding relationship between velocity components is given by

$$\begin{bmatrix} u \\ v \end{bmatrix} = R'' \begin{bmatrix} u' \\ v' \end{bmatrix} = \begin{bmatrix} \cos\tau & -\sin\tau \\ \sin\tau & \cos\tau \end{bmatrix} \begin{bmatrix} u' \\ v' \end{bmatrix} \tag{9.23}$$

From Eqs. (9.22) and (9.23), the general relationship between velocity components in two coordinate systems with τ and φ as parameters is obtained as

$$\begin{bmatrix} u \\ v \end{bmatrix} = R \begin{bmatrix} u_x \\ u_y \\ u_z \end{bmatrix} = \begin{bmatrix} \cos\tau & -\cos\varphi\sin\tau & \sin\varphi\sin\tau \\ \sin\tau & \cos\varphi\cos\tau & -\sin\varphi\cos\tau \end{bmatrix} \begin{bmatrix} u_x \\ u_y \\ u_z \end{bmatrix} \tag{9.24}$$

with $R = R''R'$

Equation (9.24) represents the background for both the basic DMM, as described in Sect. 9.2, and the extended DMM. In fact, for $\varphi_0 = 0$ and $\varphi_1 = 180°$ both v_0 and v_1 are obtained which are equal to Eqs. (9.2) and (9.3), respectively. Correspondingly Eqs. (9.5) and (9.6) will also be obtained.

As mentioned at the beginning of this section, three single measurements at a fixed point in the flow would be necessary to determine the velocity shift, if the positioning of the LDA head at $\varphi_1 = 180°$ is ineffectual. This can be achieved for instance by positioning the measurement volume on the x-axis ($y = 0$, $z = 0$) and the LDA optics to three azimuth angles $\varphi = 0$, $\varphi_1$and φ_2. The following relationships between velocity components from measurements and in the actual flow field can be obtained from Eq. (9.24):

$$v_0 = u_x \sin\tau + u_y \cos\tau \tag{9.25}$$

$$v_1 = u_x \sin\tau + u_y \cos\varphi_1 \cos\tau - u_z \sin\varphi_1 \cos\tau \tag{9.26}$$

$$v_2 = u_x \sin\tau + u_y \cos\varphi_2 \cos\tau - u_z \sin\varphi_2 \cos\tau \tag{9.27}$$

These three velocity components represent those which are measured by LDA method at a fixed point in the flow from three different azimuth directions. They will be used to determine the LDA alignment error and the associated velocity shift, in order to finally accurately determine the velocity component u_y.

By eliminating $u_z \cos\tau$ in Eqs. (9.26) and (9.27) one obtains

$$v_1 \sin\varphi_2 - v_2 \sin\varphi_1 = u_x \sin\tau(\sin\varphi_2 - \sin\varphi_1) + u_y \cos\tau \sin(\varphi_2 - \varphi_1) \tag{9.28}$$

The term $u_y \cos\tau$ in this equation will be replaced by that from Eq. (9.25). The velocity shift is finally resolved as

$$v_{sh} = u_x \sin\tau = \frac{(v_1 \sin\varphi_2 - v_2 \sin\varphi_1) - v_0 \sin(\varphi_2 - \varphi_1)}{(\sin\varphi_2 - \sin\varphi_1) - \sin(\varphi_2 - \varphi_1)} \tag{9.29}$$

The actual velocity component u_y then results from Eq. (9.25) with $\cos\tau \approx 1$

$$u_y = v_0 - \sin\tau \cdot u_x = v_0 - v_{sh} \tag{9.30}$$

Eq. (9.29) represents the extended DMM in determining the velocity shift by three single measurements. As long as the velocity shift is obtained through this way, it can be applied to Eq. (9.30) to determine the velocity component u_y, which is simply the same as that in Eq. (9.6) and represents the velocity component in the secondary flow. The prerequisite of the applicability of the method is to maintain the bias angle τ as the constant systematic error in the measurement system. This can be ensured for instance by an appropriate mechanical system like that is shown in Fig. 9.3a.

For recognising the LDA arrangement error and the associated velocity shift in such an application, it is recommended that one locates the LDA measurement volume on the x-axis. This enables the LDA users to merely rotate the LDA support system, without having to realign the measurement volume.

The reliability of Eq. (9.29) could be successively verified by making use of the measurement results which have been shown in Fig. 9.2c. As shown, the flow on the jet axis was measured eight times. According to Eq. (9.29) any combination of three initial measurements is sufficient for calculating the velocity shift as a systematic error. Corresponding calculations (altogether 56 combinations) showed the satisfactory consistency of the velocity shift with a maximum uncertainty of about 14% around $\nu_{\mathrm{sh}} = 0.4$ m/s.

In addition, Eq. (9.29) also points out that for the purpose of determining the velocity shift only measurements of the ν-component are required. In other words, the one-component LDA instrument can be applied.

Finally, the velocity component u_{z} at the measuring point can also be derived. From Eq. (9.26) for instance this velocity component is calculated as

$$u_{\mathrm{z}} = \frac{u_{\mathrm{x}} \sin\tau + u_{\mathrm{y}} \cos\varphi_1 \cos\tau - \nu_1}{\sin\varphi_1 \cos\tau} \tag{9.31}$$

With respect to $\nu_{\mathrm{sh}} = u_{\mathrm{x}} \sin\tau$ and u_{y} from Eq. (9.30) as well as $\tau << 1$ leading to $\cos\tau \approx 1$, the above equation becomes

$$u_{\mathrm{z}} = \frac{(\nu_0 - \nu_{\mathrm{sh}}) \cos\varphi_1 - (\nu_1 - \nu_{\mathrm{sh}})}{\sin\varphi_1} \tag{9.32}$$

Correspondingly this velocity component can also be calculated from Eq. (9.27) as

$$u_{\mathrm{z}} = \frac{(\nu_0 - \nu_{\mathrm{sh}}) \cos\varphi_2 - (\nu_2 - \nu_{\mathrm{sh}})}{\sin\varphi_2} \tag{9.33}$$

Combining Eq. (9.32) and (9.33) to eliminate ν_0 yields

$$u_{\mathrm{z}} = \frac{(\nu_1 - \nu_{\mathrm{sh}}) \cos\varphi_2 - (\nu_2 - \nu_{\mathrm{sh}}) \cos\varphi_1}{\sin(\varphi_2 - \varphi_1)} \tag{9.34}$$

These last three equations are fully equivalent.

9.4.2 Method of Using Coordinate Transformation

In Sect. 9.2, DMM was applied to the case at which the LDA coordinate system is rotated against the flow system by the angle $\alpha + \tau$ (see Fig. 9.5) inclusive the unknown alignment error τ. Based on the "mirrored view" of two velocities and the direct comparison between them, the velocity shift as a systematic error can be identified. Restriction of the "mirrored view" may be encountered if the positioning of the LDA head at $\varphi = 180°$ is impossible. Like in the case treated in the last section, three measurements at a single point in the flow are required in order to determine the velocity shift that is resulted from the alignment error. Clearly, the LDA head with rotation by an angle $\alpha + \tau$ (Fig. 9.5a) has to be positioned at $\varphi = 0$, $\varphi_1$and φ_2, successively. To establish the relationship between velocity components in LDA and flow field systems, Eq. (9.24) that is available for $\alpha = 0$ can be made of use. Because the angle α always appears in the form $\alpha + \tau$, Eq. (9.24) is directly taken over for the present use by substituting the bias angle τ through $\alpha + \tau$. In accordance with Eqs. (9.25) to (9.27) the following relationships between measurements and the actual flow are obtained:

$$v_0 = u_\mathrm{x} \sin(\alpha + \tau) + u_\mathrm{y} \cos(\alpha + \tau) \tag{9.35}$$

$$v_1 = u_\mathrm{x} \sin(\alpha + \tau) + u_\mathrm{y} \cos\varphi_1 \cos(\alpha + \tau) - u_\mathrm{z} \sin\varphi_1 \cos(\alpha + \tau) \tag{9.36}$$

$$v_2 = u_\mathrm{x} \sin(\alpha + \tau) + u_\mathrm{y} \cos\varphi_2 \cos(\alpha + \tau) - u_\mathrm{z} \sin\varphi_2 \cos(\alpha + \tau) \tag{9.37}$$

They are velocity components which are measured by LDA method at a fixed point in the flow from three different azimuth directions.

Equations (9.36) and (9.37) are combined to eliminate $u_\mathrm{z}\cos(\alpha+\tau)$. This results in, similar to Eq. (9.28)

$$v_1 \sin\varphi_2 - v_2 \sin\varphi_1 = u_\mathrm{x} \sin(\alpha + \tau)(\sin\varphi_2 - \sin\varphi_1) + u_\mathrm{y} \cos(\alpha + \tau) \sin(\varphi_2 - \varphi_1) \tag{9.38}$$

The term $u_\mathrm{y}\cos(\alpha + \tau)$ in this equation will be replaced by that from Eq. (9.35), leading to

$$u_\mathrm{x} \sin(\alpha + \tau) = \frac{v_1 \sin\varphi_2 - v_2 \sin\varphi_1 - v_0 \sin(\varphi_2 - \varphi_1)}{\sin\varphi_2 - \sin\varphi_1 - \sin(\varphi_2 - \varphi_1)} \tag{9.39}$$

With respect to $\tau << 1$ and thus $\sin(\alpha + \tau) \approx \sin\alpha + \tau\cos\alpha$ the velocity shift is finally resolved as

$$v_\mathrm{sh} = u_\mathrm{x}\tau = \frac{1}{\cos\alpha} \cdot \frac{(v_1 \sin\varphi_2 - v_2 \sin\varphi_1) - v_0 \sin(\varphi_2 - \varphi_1)}{(\sin\varphi_2 - \sin\varphi_1) - \sin(\varphi_2 - \varphi_1)} - u_\mathrm{x}\tan\alpha \tag{9.40}$$

For $\alpha = 0$ this equation is simplified to Eq. (9.29). Further for the positioning of the LDA head at $\varphi_1 = 180°$ the above equation becomes the same as Eq. (9.5) which represents the basic form of DMM.

After the velocity shift has been determined through Eq. (9.40), the velocity component u_y can then be determined from Eq. (9.35). With respect to $\tau << 1$ and thus $\sin(\alpha + \tau) \approx \sin\alpha + \tau\cos\alpha$ and $\cos(\alpha + \tau) \approx \cos\alpha$ one obtains

$$u_{\mathrm{y}} = \frac{1}{\cos\alpha}(v_0 - u_{\mathrm{x}} \sin\alpha) - v_{\mathrm{sh}} \tag{9.41}$$

For $\alpha = 0$ this equation is simplified to Eq. (9.30).

Because the velocity component u_{x} appears in both Eqs. (9.40) and (9.41), the two-component LDA system is required for the current case with $\alpha \neq 0$.