Chapter 8
Zero Correlation Method (ZCM)

8.1 Shear Stress Measurements with Non-coincident LDA

The most significant turbulence parameters of a turbulent flow are specified by corresponding Reynolds stresses that are given in the matrix form at Eq. (2.11). The knowledge about the turbulent shear stresses for instance τ_{uv} basically demands, according to Eq. (5.7), coincident measurements of two velocity components u and v. For such measurements a two-component LDA-system has usually to be applied and optically arranged for direct measurements of u and v (Chap. 4). This is well available because most LDA systems are designed so. The different case, however, will be encountered in measurements of all three velocity components. In such a case, the third i.e. on-axis velocity component has usually to be separately measured by off-axis alignment of the LDA head. This can be achieved either directly by the method according to Fig. 8.1a, or indirectly according to Fig. 8.1b. In the case of indirect measurements, the on-axis velocity component is calculated by transforming the velocity components u_{x} and u_{φ}

$$u_{\mathrm{y}} = \frac{u_{\varphi} - u_{\mathrm{x}} \cos\varphi}{\sin\varphi} \tag{8.1}$$

This equation has already been given by Eq. (6.40).

Because it deals here with non-coincident measurements of velocity components u_{x} and u_{y}, the turbulent shear stress $\overline{u'_{\mathrm{x}}u'_{\mathrm{y}}}$ could not be directly obtained by regarding Eq. (5.7). For indirectly obtaining any turbulent shear stress, the Reynolds stress matrix shown in Eq. (2.11) is considered again. Because of equalities $\tau_{\mathrm{xy}} = \tau_{\mathrm{yx}}, \tau_{\mathrm{xz}} = \tau_{\mathrm{zx}}$ and $\tau_{\mathrm{yz}} = \tau_{\mathrm{zy}}$ there are indeed altogether six independent Reynolds turbulent stresses. According to the statement given by Durst et al. (1981) and for the most general case of non-coincident LDA measurements, six single measurements are needed for resolving all six Reynolds stresses in a three-dimensional space. In the case of considering the flow in a two-dimensional plane, then three single measurements are necessary. The corresponding measurement method has already been shown in Sect. 6.2 with respect to Fig. 6.7.

Measurements of complete turbulent stresses in a two-dimensional plane within a flow field, however, can be simplified by making use of a highly reasonable

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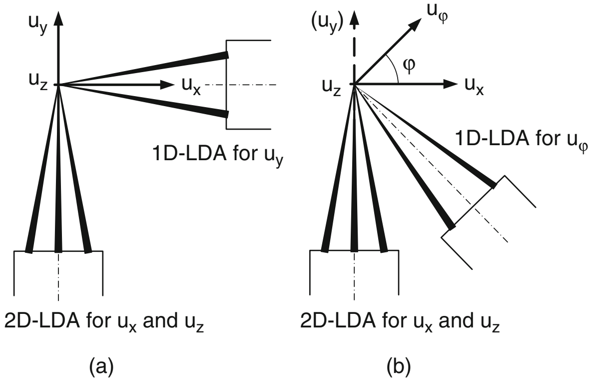

Fig. 8.1 Separate measurement of the on-axis velocity component u_y by relocating the LDA head; The velocity component u_z is perpendicular to the drawing plane (**a**) direct measurement of the velocity component u_y; (**b**) indirect measurement of the velocity component u_y

assumption to the turbulent flow. The so-called Zero Correlation Method (ZCM) (Zhang and Eisele 1998a, Zhang 1999) enables the complete stress components in a two-dimensional plane to be determined by just carrying out two single measurements. The method thus does not provide any more expenses, because two measurements are necessary anyway for obtaining both the magnitude and the direction of the mean velocity. This method will be presented in below.

8.2 Basics of ZCM

The significant attribute of a turbulent flow is the stochastic velocity fluctuations. In dealing with a stationary turbulent flow, flow fluctuations basically comprise the fluctuations of both the magnitude and the direction of the velocity vector, as illustrated in Fig. 8.2 for a turbulent flow which was measured by the LDA method. Because of the stochastic feature of flow fluctuations, the fluctuation of the velocity direction in the $x - y$ plane can be considered to symmetrically lie on both sides of the mean velocity vector that has its direction at $\overline{\varphi}$. For further consideration, the velocity component along the mean velocity vector is denoted by u_1.

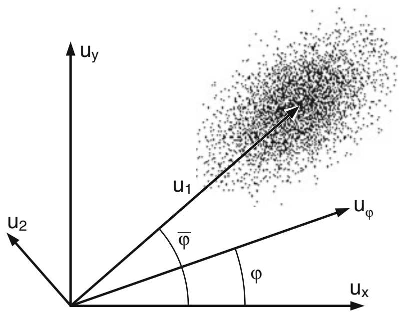

Fig. 8.2 Turbulent flow and velocity fluctuations measured by LDA

In its perpendicular, the velocity component is u_2. Clearly there is $\overline{u}_2 = 0$. The symmetrical distribution of velocity fluctuations around the mean velocity vector is mathematically expressed by the zero correlation between flow fluctuations in velocity components u_1 and, u_2 as given by

$$C = \overline{u'_1 u'_2} = 0 \tag{8.2}$$

From velocity components u_1 and u_2 the velocity components u_x and u_y can be obtained from the orthogonal coordinate transformation that has already been outlined in Sect. 6.1. The same transformation is also applicable to the flow fluctuations. In effect, Eqs. (6.6) and (6.7) can be directly applied to represent the fluctuation velocities in the present case as

$$u'_\mathrm{x} = u'_1 \cos\overline{\varphi} - u'_2 \sin\overline{\varphi} \tag{8.3}$$

$$u'_\mathrm{y} = u'_1 \sin\overline{\varphi} + u'_2 \cos\overline{\varphi} \tag{8.4}$$

In these equations, the mean flow direction is denoted by angle $\overline{\varphi}$ that is calculated by

$$\tan\overline{\varphi} = \frac{\overline{u}_\mathrm{y}}{\overline{u}_\mathrm{x}} \tag{8.5}$$

For later convenience this equation is also represented as

$$\tan 2\overline{\varphi} = \frac{2\tan\overline{\varphi}}{1-\tan^2\overline{\varphi}} = \frac{2\overline{u}_\mathrm{x}\overline{u}_\mathrm{y}}{\overline{u}_\mathrm{x}^2 - \overline{u}_\mathrm{y}^2} \tag{8.6}$$

The statistical turbulence properties including the Reynolds normal and shear stresses can be calculated from the velocity fluctuations u'_x and u'_y given above. With respect to the zero correlation condition given by Eq. (8.2), the following relationships are obtained

$$\overline{u'^2_\mathrm{x}} = \overline{u'^2_1}\cos^2\overline{\varphi} + \overline{u'^2_2}\sin^2\overline{\varphi} \tag{8.7}$$

$$\overline{u'^2_\mathrm{y}} = \overline{u'^2_1}\sin^2\overline{\varphi} + \overline{u'^2_2}\cos^2\overline{\varphi} \tag{8.8}$$

$$\overline{u'_\mathrm{x} u'_\mathrm{y}} = \frac{1}{2}\left(\overline{u'^2_1} - \overline{u'^2_2}\right)\sin 2\overline{\varphi} \tag{8.9}$$

Also to be mentioned is that these three equations can also be directly obtained from Eqs. (6.9), (6.10), and (6.11). The velocity components u and v there need to be considered to be the components on the main flow direction $\overline{\varphi}$ and its perpendicular, respectively. Because of predefined zero correlation condition by Eq. (8.2) $\tau_\mathrm{uv} = 0$ should be applied.

Based on similar calculations, the turbulence properties related to any other components u_φ, according to Fig. 8.2, are given by

$$\overline{u_\varphi'^2} = \overline{u_1'^2}\cos^2(\varphi - \overline{\varphi}) + \overline{u_2'^2}\sin^2(\varphi - \overline{\varphi}) \tag{8.10}$$

$$\overline{u_\varphi' u_{\varphi+90^\circ}'} = -\frac{1}{2}\left(\overline{u_1'^2} - \overline{u_2'^2}\right)\sin 2(\varphi - \overline{\varphi}) \tag{8.11}$$

Because both normal stresses $\overline{u_1'^2}$ and $\overline{u_2'^2}$ are positive, the normal stress $\overline{u_\varphi'^2}$ is positive in all directions. In fact, $\overline{u_1'^2}$ and $\overline{u_2'^2}$ represent two principal normal stresses and $\overline{u_\varphi'^2}$ lies between them. From Eqs. (8.7) and (8.8) two principal normal stresses can be resolved as

$$\overline{u_1'^2} = \frac{1}{2}\left(\overline{u_x'^2} + \overline{u_y'^2}\right) + \frac{1}{2\cos 2\overline{\varphi}}\left(\overline{u_x'^2} - \overline{u_y'^2}\right) \tag{8.12}$$

$$\overline{u_2'^2} = \frac{1}{2}\left(\overline{u_x'^2} + \overline{u_y'^2}\right) - \frac{1}{2\cos 2\overline{\varphi}}\left(\overline{u_x'^2} - \overline{u_y'^2}\right) \tag{8.13}$$

Eq. (8.12) is indeed equal to Eq. (6.18) that represents the principal normal stress at the angle φ_m in the $x - y$ plane, see also Fig. 6.3. Obviously the zero correlation condition given by Eq. (8.2) assumes the angle φ_m for the principal normal stress to be equal to the main flow direction.

The above two equations are subsequently inserted into Eqs. (8.10) and (8.11), respectively. The following expressions are then obtained

$$\overline{u_\varphi'^2} = \frac{1}{2}\left(\overline{u_x'^2} + \overline{u_y'^2}\right) + \frac{\cos 2(\varphi - \overline{\varphi})}{2\cos 2\overline{\varphi}}(\overline{u_x'^2} - \overline{u_y'^2}) \tag{8.14}$$

$$\overline{u_\varphi' u_{\varphi+90^\circ}'} = -\frac{\sin 2(\varphi - \overline{\varphi})}{2\cos 2\overline{\varphi}}\left(\overline{u_x'^2} - \overline{u_y'^2}\right) \tag{8.15}$$

These last two equations indicate that the complete Reynolds stresses in the $x - y$ plane can be well resolved from two independent i.e. non-coincident measurements ($\overline{u_x'^2}$ and $\overline{u_y'^2}$). In comparison with the method of three measurements, as presented in Sect. 6.2 with respect to Fig. 6.7, the method presented here clearly shows advantages in the simplification of turbulence measurements. This accessibility is simply based on the assumption of zero correlation condition $\overline{u_1' u_2'} = 0$ as specified in Eq. (8.2). For this reason the method shown above is called Zero Correlation Method (ZCM).

The turbulent shear stress $\overline{u_x' u_y'}$ is obtained by setting $\varphi = 0$ in Eq. (8.15), which results in

$$\overline{u_x' u_y'} = \frac{1}{2}\tan 2\overline{\varphi}\left(\overline{u_x'^2} - \overline{u_y'^2}\right) \tag{8.16}$$

and because of Eq. (8.6)

$$\overline{u'_x u'_y} = \frac{\bar{u}_x \bar{u}_y}{\bar{u}_x^2 - \bar{u}_y^2} \left(\overline{u'^2_x} - \overline{u'^2_y} \right) \tag{8.17}$$

Eq. (8.16) is comparable to Eq. (6.15). It can be directly applied to Fig. 8.1a.

8.3 Extension of ZCM

8.3.1 Non-orthogonal Velocity Components

The Zero Correlation Method (ZCM) introduced in the last section applies to the orthogonal velocity components like u_x and u_y in a two-dimensional plane, as shown in Fig. 8.1a. Because LDA arrangement like that in Fig. 8.1b is also often available, the ZCM must be modified to extend its applications.

From Eq. (8.14) the turbulent normal stress $\overline{u'^2_y}$ is resolved as

$$\overline{u'^2_y} = \frac{\cos\varphi \cos(2\overline{\varphi} - \varphi)\overline{u'^2_x} - \cos 2\overline{\varphi}\,\overline{u'^2_\varphi}}{\sin\varphi \sin(2\overline{\varphi} - \varphi)} \tag{8.18}$$

This equation signifies that from measurements of two non-orthogonal turbulence components $\overline{u'^2_x}$ and $\overline{u'^2_\varphi}$ in a two-dimensional plane the turbulence component $\overline{u'^2_y}$ in the same plane can be determined as well. One has again the case of orthogonal normal stresses. All results achieved in Sect. 8.2, especially Eq. (8.16), can be applied. The mean flow angle $\overline{\varphi}$ is calculated by Eq. (8.5), where the mean velocity component $\bar{u}_y$ is obtained from Eq. (8.1).

8.3.2 Three-Dimensional Flow Turbulence

Calculations presented above are performed for turbulence properties in a two-dimensional plane. For purposes of applying the ZCM to three-dimensional turbulent flows in the Cartesian coordinate system, Eq. (8.17) is taken into account as the reference for other components. The completeness of turbulent stresses can be expressed in a matrix as given by

$$\overline{u'_i u'_j} = \begin{vmatrix} \overline{u'^2_x} & \frac{\bar{u}_x \bar{u}_y}{\bar{u}_x^2 - \bar{u}_y^2} \left(\overline{u'^2_x} - \overline{u'^2_y} \right) & \frac{\bar{u}_x \bar{u}_z}{\bar{u}_x^2 - \bar{u}_z^2} \left(\overline{u'^2_x} - \overline{u'^2_z} \right) \\ \frac{\bar{u}_x \bar{u}_y}{\bar{u}_x^2 - \bar{u}_y^2} \left(\overline{u'^2_x} - \overline{u'^2_y} \right) & \overline{u'^2_y} & \frac{\bar{u}_y \bar{u}_z}{\bar{u}_y^2 - \bar{u}_z^2} \left(\overline{u'^2_y} - \overline{u'^2_z} \right) \\ \frac{\bar{u}_x \bar{u}_z}{\bar{u}_x^2 - \bar{u}_z^2} \left(\overline{u'^2_x} - \overline{u'^2_z} \right) & \frac{\bar{u}_y \bar{u}_z}{\bar{u}_y^2 - \bar{u}_z^2} \left(\overline{u'^2_y} - \overline{u'^2_z} \right) & \overline{u'^2_z} \end{vmatrix} \tag{8.19}$$

Obviously three non-coincident measurements, from which both the mean velocities ($\overline{u}_\mathrm{x}, \overline{u}_\mathrm{y}$ and $\overline{u}_\mathrm{z}$) and the turbulence components ($\overline{u_\mathrm{x}'^2}, \overline{u_\mathrm{y}'^2}$ and $\overline{u_\mathrm{z}'^2}$) at a fixed point in the flow are obtainable, are sufficient to complete the Reynolds stress matrix with nine elements. The advantage of the ZCM has been thus again demonstrated.

8.4 Restriction and Validation of ZCM

The zero correlation method is indeed an approximation method that is introduced to simplify the turbulence measurements. Respective application restrictions and accuracies must be considered. From Eqs. (8.14) and (8.15), it is evident that the main flow angle $\overline{\varphi}$ in the used coordinate system should not be equal or very close to 45°. This limitation could also be confirmed from Eq. (8.16) when measurements of u_x and u_y should be accomplished. It is therefore recommended to arrange the appropriate $x-y$ coordinate system for measurements. At best the x-axis is set to closely agree with the main flow direction.

The extent of errors resulting from the ZCM depends on the homogeneity of turbulence in the respective flow to be measured. In actual fact, the method does work accurately as long as the zero correlation condition according to Eq. (8.2) is highly satisfied. This is always the case when the local flow does not sensitively affected by the rigid surface or boundaries in the flow. An experimental verification of the method was conducted once through measurements in a turbulent channel flow (Zhang and Eisele 1998a, Zhang 1999). In this validation measurement, a two-component coincident LDA system was applied, so that turbulent stresses $\overline{u_\mathrm{x}'^2}, \overline{u_\mathrm{y}'^2}$ and $\overline{u_\mathrm{x}'u_\mathrm{y}'}$ were directly obtained. Based on such measurements the validation of the ZCM could be well accomplished by following calculations:

(1) From the coincident measurements of velocity components u_x and u_y other velocity components u_φ and $u_{\varphi+90°}$ for each given angle φ can be calculated via coordinate transformation according to Eq. (6.4) as

$$u_\varphi = u_\mathrm{x}\cos\varphi + u_\mathrm{y}\sin\varphi \tag{8.20}$$

$$u_{\varphi+90°} = -u_\mathrm{x}\sin\varphi + u_\mathrm{y}\cos\varphi \tag{8.21}$$

Subsequently the covariance i.e. the correlation between velocity fluctuations u'_φ and $u'_{\varphi+90°}$ is calculated by

$$\overline{u'_\varphi u'_{\varphi+90°}} = \sum_{\mathrm{i}=1}^{\mathrm{N}} \left(u_\varphi - \overline{u}_\varphi\right)\left(u_{\varphi+90°} - \overline{u}_{\varphi+90°}\right) \tag{8.22}$$

This covariance indeed represents the turbulent shear stress and is a function of angle φ.

(2) On the other side the covariance $\overline{u'_{\varphi}u'_{\varphi+90^{\circ}}}$ for each given angle φ can also be directly calculated from two normal stresses $\overline{u'^2_x}$ and $\overline{u'^2_y}$ by using Eq. (8.15). Because of the use of this equation as a result of the ZCM, the calculation results must be considered to be approximate.
(3) Calculation results from calculations in (1) and (2), respectively, will be compared.

This validation procedure was applied to the validation measurements (Zhang and Eisele 1998a, Zhang 1999). Figure 8.3a shows the comparison of respective calculations in function of each given angle φ. Obviously the ZCM provides satisfactory measurement results.

The same validation can be completed by concerning the turbulence component $\overline{u'^2_{\varphi}}$. On one side, this component can be calculated from velocity data that are obtained by Eq. (8.20), leading to accurate values of variance. On the other side, it can also be calculated directly from $\overline{u'^2_x}$ and $\overline{u'^2_y}$ by means of Eq. (8.14), leading to approximated values. The comparison between two calculations in relying on the mentioned validation measurement is shown in Fig. 8.3b. The same satisfactory results were obtained.

Strictly, the zero correlation condition according to Eq. (8.2) is not satisfied if applied to the turbulent boundary layer. With respect to the main flow direction $u_1 = u_x$ and to the perpendicular $u_2 = u_y$, the covariance $\overline{u'_1u'_2}$ i.e. $\overline{u'_xu'_y}$ in the turbulent boundary layer is indeed a function of the distance to the wall surface and therefore does not disappear as it is assumed in the ZCM. The error arising from

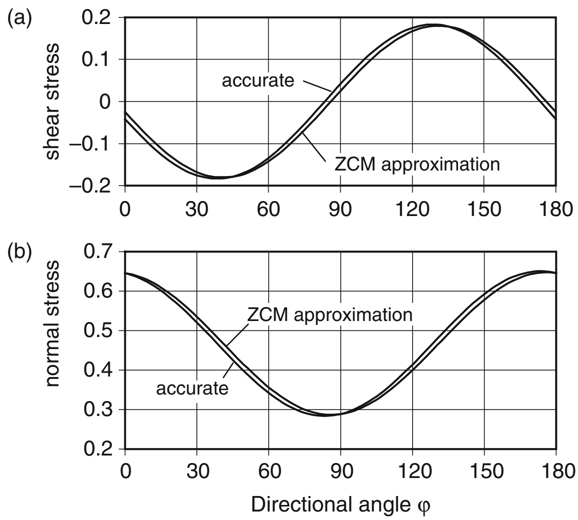

Fig. 8.3 Experimental validations of the ZCM by comparing the accurate turbulent stress in directional distribution with that calculated by the ZCM (Zhang and Eisele 1998a)

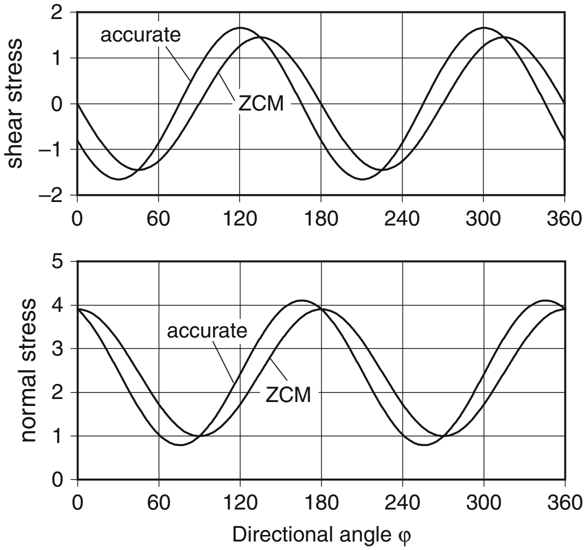

Fig. 8.4 Comparison of accurate turbulent stress in directional distribution with that calculated by ZCM for the flow in the turbulent boundary layer at $y^+ = 90$ (Zhang and Zhang 2002)

the assumption $\overline{u'_x u'_y} = 0$, when applied to the turbulent boundary layer, has been analyzed by Zhang and Zhang (2002). With known turbulent stresses $\overline{u'^2_x}, \overline{u'^2_y}$ and $\overline{u'_x u'_y}$ at a certain wall distance y^+ the directional distributions of the normal and shear stresses can be calculated by Eqs. (6.13) and (6.14), respectively. On the other hand, these distributions can also be calculated by assuming $\overline{u'_x u'_y} = 0$ which in effect simulates the application of ZCM. Figure 8.4 represents the comparison of respective calculations to the turbulent state at $y^+ = 90$ in the turbulent boundary layer. With certain reservation the ZCM in this application provides quite satisfactory result. This obviously arises from the fact that the amplitudes of both the normal and shear stresses in their trigonometric functions are mainly determined by the normal stresses $\overline{u'^2_x}$ and $\overline{u'^2_y}$.The assumption of $\overline{u'_x u'_y} = 0$ in the ZCM merely results in a shift in the directional distribution of respective turbulent stresses. In practical applications, it is indeed a matter of the application requirement, whether the resultant error could be accepted or not.