PSO Heuristics Algorithm for Portfolio Optimization*

Yun Chen and Hanhong Zhu

Department of Public Economics & Administration
Shanghai University of Finance and Economics (SUFE)
200433 Shanghai, China
zhh@mail.shufe.edu.cn

Abstract. One of the most studied problems in the financial is the intractability of the portfolios. Some practical formulations of the problem include various kinds of nonlinear constraints and objectives and can be efficiently solved by approximate algorithms. In this paper, we present a meta-heuristic algorithm named Particle Swarm Optimization (PSO) to the construction of optimal risky portfolios for financial investments. The PSO algorithm is tested on two portfolio optimization models and a comparative study with Genetic Algorithm has been implemented. The PSO model demonstrates high computational efficiency in constructing optimal risky portfolios. Preliminary results show that the approach is very promising and achieves results comparable or superior with the state of the art solvers.

Keywords: Swarm Intelligence (SI), Particle Swarm Optimization (PSO), Portfolio Management (PM), Sharp Ratio (SR), Efficient Frontier (EF).

1 Introduction

Portfolio management is one of the most studied topics in finance. The problem is concerned with managing the portfolio of assets that minimizes the risk objectives subjected to the constraint for guaranteeing a given level of returns. In this paper, we deal with the so-called Mean-Variance portfolio selection, which is formulated in the similar way done by Markowitz [1]. His theory has revolutionized the way people think about portfolio of assets, and has gained widespread acceptance as a practical tool for portfolio optimization. But in some cases, the characteristics of the problem, such as its size, real-world requirements [2], very limited computation time, and limited precision in estimating instance parameters, may make analytical methods not particularly suitable for tackling large instances of the constrained Mean-Variance Model. Therefore researchers and practitioners have to resort to heuristic techniques, in which we can found state-of-the-art solvers for the problem.

There are some literatures of solving PO problem using heuristic methods, these methods consist of Genetic Algorithms (GA), tabu search and Simulated Annealing (SA) [3], local search and quadratic programming procedure [4], Ant Colony Optimization (ACO) [5], neural network model [6] and others.

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This study presents the PSO algorithm to solve the portfolio optimization problem. PSO is a population based stochastic optimization technique developed in 1995 [7]. Asset allocation in the selected assets is optimized using a PSO based on Markowitz's theory. The rest of the paper is organized as follows. Section 2 describes models for portfolio optimization. In section 3, Back ground of PSO and previous work are summarized. PSO model for optimal portfolio is also discussed. In order to test the efficiency of the proposed PSO solver, a simulation and comparative study with GA heuristics is performed in Section 4. Final conclusions and future research are drawn in Section 5.

2 Models for Portfolio Optimization (PO)

One of the fundamental principles of financial investment is diversification where investors diversify their investments into different types of assets. Portfolio diversification minimizes investors' exposure to risks, and maximizes returns on portfolios. It can be referred to as a multi-objective optimization problem.

There are many methods to solve the multi-objectives optimization problems. One basic method is to transfer the multi-objective optimization problems into a single-objective optimization problem. We can divide these methods into two different types: (1). We can select one important objective function as the objective function to optimize. The rest of objective function can be defined as constraints conditions. (2). We construction an evaluation function which is used as the only objective function. This paper used the second method.

Markowitz Mean-Variance Model

The Markowitz Mean-Variance model [1] for security selection of risky portfolio construction is described as:

$$\text{Min} \sum_{i=1}^{N} \sum_{j=1}^{N} w_i w_j \sigma_{ij} \tag{1}$$

$$\text{Subject to } \sum_{i=1}^{N} w_i r_i = \mathrm{R}^*, \tag{2}$$

$$\sum_{i=1}^{N} w_i = 1, \tag{3}$$

$$0 \le w_i \le 1 \quad i = 1, \cdots, N. \tag{4}$$

where N is the number of different assets, σ_{ij} is the covariance between returns of assets i and j, w_i is the weight of each stock in the portfolio, r_i is the mean return of stock i and R^* is the desired mean return of the portfolio. We also use the second method to model portfolio optimization problem as the following two models:

2.1 Efficient Frontier Model

We can find the different objective function values by varying desired mean return R^*, so a new named risk aversion parameter $\lambda \in [0,1]$ has been introduced, the sensitivity of the investor to the risk increase as λ increasing from zero to unity. With the λ, the model can be described as:

$$\text{Min}\lambda\left[\sum_{i=1}^{N}\sum_{j=1}^{N} w_i w_j \sigma_{ij}\right]-(1-\lambda)\left[\sum_{i=1}^{N} w_i r_i\right]. \tag{5}$$

$$\text{Subject to}\sum_{i=1}^{N} w_i = 1, \tag{6}$$

$$0 \le w_i \le 1 \;\; i = 1,\cdots,N. \tag{7}$$

In the model included parameter λ, we can draw a continuous curve that is called an efficient frontier according the Markowitz theory[1], the curve composed of mean return and variance according different λ, and every point on an efficient frontier curve indicates an optimum, and this indicates the portfolio optimization problem is a multi-objective optimization.

2.2 Sharpe Ratio Model

Instead of focusing on the mean variance efficient frontier, we seek to optimize the portfolio Sharpe Ratio (SR) [8]. The Sharpe ratio is quite simple and it is a risk-adjusted measure of return that is often used to evaluate the performance of a portfolio. It is described as the following equation:

$$SR = \frac{R_p - R_f}{StdDev(p)} \tag{8}$$

where *p* is the portfolio, R_p is the mean return of the portfolio *p*, R_f is the test available rate of return of a risk-free security. StdDev(p) is the standard deviation of R_p. Adjusting the portfolio weights w_i, we can maximizing the portfolio Sharpe Ratio in effect balancing the trade-off between maximizing the expected return and at the same time minimizing the risk. In this study, the PSO optimization can find the most valuable portfolio with good stock combinations.

3 PSO Algorithm for Portfolio Optimization

3.1 Particle Swarm Optimization

Particle Swarm Optimization (PSO) is a population based stochastic optimization technique developed by Kennedy and Eberhart in 1995 [7], inspired by social behavior of bird flocking. It belongs to Swarm Intelligence (SI), which originates from the study of natural creatures living in a group. Each individual possess little or no wisdom, but by interacting with each other or the surrounding environment, they can perform very complex tasks as a group.

PSO could be explained well in an imagined scenario: a group of birds are flying in an area to look for food, and there's only one piece of food in this area. the easiest way to find the food is to follow the one who is closest to the food.

The basic concept of PSO lies in accelerating each particle toward its *pbest* which was achieved so far by that particle, and the *gbest* which is the best value obtained so far by any particle in the neighborhood of the particle, with a random weighted acceleration at each time step.

Each particle tries to modify its position using the following information:

- The current positions ($\vec{X}(t)$),
- The current velocities ($\vec{V}(t)$),
- The distance between the *pbest* and the current position ($\vec{P_i} - \vec{X}(t)$),
- The distance between the *gbest* and the current position ($\vec{P_G} - \vec{X}(t)$).

In this paper, we will apply PSO algorithm to solve the portfolio optimization problem.

3.2 Fitness function

Fitness function is a critical factor in the PSO method. Every particle in the PSO's population has a fitness value, and it moves in solution space with respect to its previous position where it has met the best fitness value. In this paper, the sharpe ratio (according equation 8) and the effect frontier model (according equation 5) will be used as objective functions and which are defined as:

$$f_P = \frac{\sum_{i=1}^{N} w_i r_i - R_f}{\sum_{i=1}^{N} \sum_{j=1}^{N} w_i w_j \sigma_{ij}} \tag{9}$$

$$f_P = -\left(\lambda\left[\sum_{i=1}^{N}\sum_{j=1}^{N} w_i w_j \sigma_{ij}\right] - (1-\lambda)\left[\sum_{i=1}^{N} w_i r_i\right]\right) \tag{10}$$

Where f_p is the fitness value of particle *p*. At every step, a particle's personal best position and the best neighbor in the swarm are updated if an improvement in any of the best fitness values is observed.

3.3 Particles Movement

In the algorithm of PSO, each solution is called a "particle", and every particle has its position, velocity, and fitness value. At each iteration, every particle moves towards its personal best position and towards the best particle of the swarm found so far. The velocity changes according to formulation (11):

$$\vec{v}_i(t+1) = w\vec{v}_i(t) + c_1 r_1[\vec{p}_i(t) - \vec{x}_i(t)] + c_2 r_2[\vec{p}_g(t) - \vec{x}_i(t)] \tag{11}$$

where *t* is the iteration sequence of the particle *i*, c_1 and c_2 are positive constant parameters called acceleration coefficients which are responsible for controlling the maximum step size, r_1 and r_2 are random numbers between (0, 1), w is a constant. and $\vec{v}_i(t+1)$ is particle *i*'s velocity at iteration *t* + 1. $\vec{v}_i(t)$ is particle *i*'s velocity at iteration t. $\vec{x}_i(t)$ is particle *i*'s position at iteration t. $\vec{p}_i(t)$ is the historical individual best position of the swarm. Finally, the new position of particle *i*, $\vec{x}_i(t+1)$, is calculated as shown in (12)

$$\vec{x}_i(t+1) = \vec{x}_i(t) + \vec{v}_i(t+1) \tag{12}$$

The details about PSO algorithm can be referred to the paper[9]. To improve the performance of PSO, the parameter can be adjusted. For example, the constant *w* can be replaced by formulation (13), and also the constant c_1 and c_2 by function (14) and (15). But in the experiment, the result of large scale portfolio (i.e. 40 stocks) didn't significantly improved.

$$w = w_{\max} - \frac{w_{\max} - w_{\min}}{iter_{\max}} \times iter \tag{13}$$

$$c_1 = c_{1\max} - \frac{c_{1\max} - c_{1\min}}{iter_{\max}} \times iter \tag{14}$$

$$c_2 = c_{2\max} - \frac{c_{2\max} - c_{2\min}}{iter_{\max}} \times iter \tag{15}$$

4 Experiments and Discussion

The PSO experiments for the portfolio optimization has been performed on two restricted risky portfolio of 8 stocks and 15 stocks based on two portfolio optimization models. Table 1 and table 2 have shown the results from PSO algorithm and genetic algorithm (GA) about the two portfolios. All stocks are selected from the Shanghai Stock Exchange 50 Index (the SSE 50 Index). Individual stock's historical daily returns are selected from 1 January 2009 to 3 April 2009.

In order to evaluate the performance of PSO algorithm, we compare PSO with another heuristic Algorithm, named GA. In the experiments, PSO algorithm has been developed using Matlab as software development tool. GA has been developed using the software named GeneHunter [11].

Base on the efficient frontier model, the results of the optimal risky portfolios developed by PSO and GA algorithms for the two portfolios are shown in the Table 1. In the portfolio of 8 stocks and 15 stocks, The fitness values obtained by PSO are all better than those of the GA algorithms.

Base on the Sharpe Ratio model, the results of the optimal risky portfolios developed by PSO and GA algorithms for the two portfolios are shown in the Table 2. In the portfolio of 8 and 15 stocks, when the risk free is 0 and 0.3%, the Sharpe Ratio value obtained by PSO are all better than those of the GA algorithms.

Taking the sets of optimal portfolios obtained with PSO and GA algorithms, we trace out their efficient frontiers in Figure 1 (a)-(d). In situation of the different portfolio optimization models and different portfolios, the PSO efficient frontier is always upon those of the GA algorithm. It means that we can get higher mean return under the same risk, or lower risk under the same mean return from the PSO portfolio than GA portfolios. So PSO portfolio is the best solution.

At the same time, we take the sets of optimal portfolios obtained with PSO on Sharpe Ratio (SR) model and Efficient Frontier (EF) model, we trace out their efficient frontiers in figure 2.(a) and (b). In the portfolio of 8 stocks, when the standard deviation is lower (about 0.03), the efficient frontier based on the SR model is upon those of the EF model. But when the standard deviation increasing, the efficient frontier based on the EF model will be upon those of the SR model. But in the portfolio of

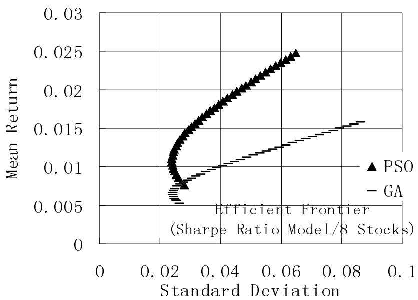
Table 1. Two portfolios' results of PSO and GA algorithms based on the efficient frontier model

λ	PSO/GA	StdDev		ER		Fitness Value	
		8	15	8	15	8	15
1	PSO	0.05%	0.09%	0.57%	0.98%	-0.05%	0.98%
	GA	0.06%	0.07%	0.74%	0.81%	-0.06%	0.81%
0.8	PSO	0.08%	0.09%	0.90%	0.97%	0.12%	0.76%
	GA	0.05%	0.07%	0.74%	0.84%	0.11%	0.66%
0.6	PSO	0.11%	0.10%	0.97%	0.97%	0.32%	0.54%
	GA	0.07%	0.07%	0.74%	0.84%	0.25%	0.48%
0.4	PSO	0.10%	0.09%	0.96%	0.97%	0.53%	0.34%
	GA	0.05%	0.07%	0.74%	0.83%	0.42%	0.29%
0.2	PSO	0.10%	0.08%	0.94%	0.95%	0.74%	0.13%
	GA	0.06%	0.06%	0.74%	0.82%	0.58%	0.11%
0	PSO	0.11%	0.04%	0.96%	0.50%	0.96%	-0.04%
	GA	0.06%	0.05%	0.74%	0.64%	0.74%	-0.05%

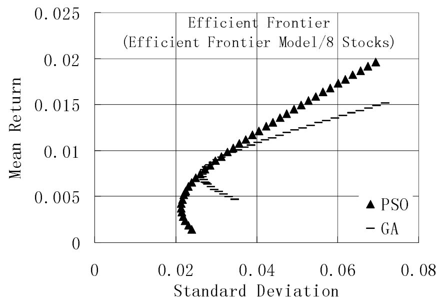
Table 2. Two portfolios' results of PSO and GA algorithms based on the sharpe ratio model

Stocks	RiskFree	PSO/GA	ER	StdDev	Sharpe Ratio
8	0	GA	0.66%	2.42%	27.27%
		PSO	0.86%	2.61%	32.75%
	0.3%	GA	0.53%	2.63%	12.42%
		PSO	0.72%	2.90%	17.83%
15	0	GA	0.69%	2.69%	25.63%
		PSO	0.89%	2.58%	34.50%
	0.3%	GA	0.57%	2.47%	18.96%
		PSO	0.84%	2.76%	26.73%

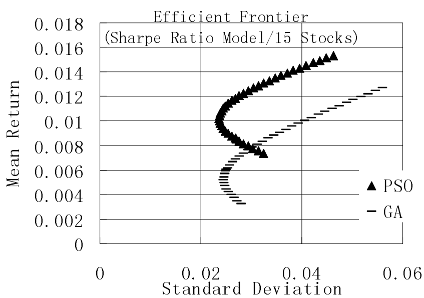
Data has been download from the web: http://finance.yahoo.com. The Risk Free has been selected subjectively. ER is 'expected return', StdDev is 'standard deviation', λ is the risk aversion parameter in the efficient frontier model.



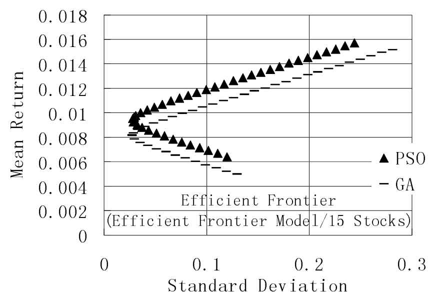
(a) 8 stock, Sharpe Ratio Model



(b) 8 Stock, Efficient Frontier Model

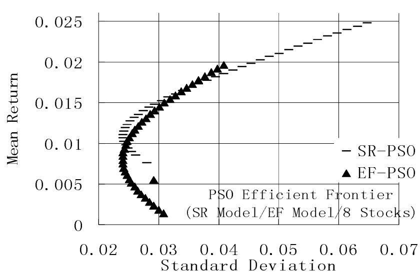


(c) 15Stock, Sharpe Ratio Model

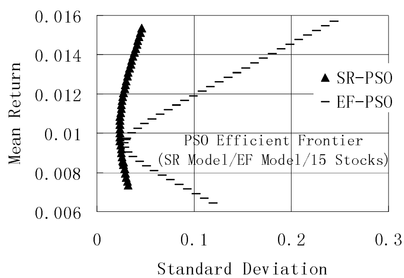


(d) 15Stock, Sharpe Ratio Model

Fig. 1. The Efficient frontier of the portfolio gotten from PSO and GA heuristics



(a) the portfolio of 8 stocks



(b) the portfolio of 15 stocks

Fig. 2. The efficient frontier of the portfolio gotten from PSO on SR model and EF model

15 stocks, the efficient frontier obtained based on SR model is always upon those of EF model. So we can come to conclude that portfolio optimization model is very important for the portfolio optimization.

To sum up, from the experiments on different portfolios and different portfolio optimization models, we can conclude that the PSO approach is better than the GA algorithm in this case. PSO algorithm clearly shows the efficiency and effectiveness of solving high-dimensional constrained optimization problems.

5 Conclusion

The paper focuses on solving the portfolio optimization problem in finance investment management. A meta-heuristic Particle Swarm Optimization method has been developed to optimize investment portfolios, in which the objective functions and constraints are based on the Markowitz model, the Sharp Ratio (SR) model and the Efficient Frontier (EF) model. In order to make a valid comparison with other methods, different test problems were solved and the results obtained when compared with the results of Genetic Algorithms (GA) demonstrated the superiority of the PSO algorithm. At the same time, the portfolio optimization model is a key factor for the portfolio management. Future research may be conducted to further investigate the application of some derived models or hybrid models of PSO to other investment strategy problems.

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