Research on a Novel Ant Colony Optimization Algorithm

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Abstract. In this paper, an adaptive optimization system is established. In order to improve the global ability of basic ant colony algorithm, a novel ant colony algorithm which is based on adaptively adjusting pheromone decay parameter has been proposed， and it has been proved that for a sufficiently large number of iterations, the probability of finding the global best solution tends to 1. The simulations for TSP problem show that the improved ant colony algorithm can find better routes than basic ant colony algorithm.

1 Introduction

Ant colony algorithm is a new heuristic optimization algorithm initially proposed by Italian scholar Dorigo M, etc in 1991. It adopts positive feedback theory, speeds up evolution course, and it is a parallel algorithm in essence that the individual can cooperate with each other and find better routes by exchanging and transmitting information continuously. Ant colony algorithm is the hot and front research topic in the field of heuristic optimization algorithm both domestic and abroad. Now it has been successfully applied in solving TSP problem, frequency distribution problem in communication, controlling parameter optimization problem and image treatment problem, etc. But at present most scholars only put out algorithm method and application method with no theoretical analysis of algorithm convergence. Until recent years some scholars gave proof of some algorithm convergence. Up to now the most important document about convergence proof is [7-10]. Documents [8, 9] give the graph-based proof of ant colony algorithm convergence based on the theory of describing the pheromone strength on the best routes as Markow stochastic process of discrete time. Document 10 constructs a branch Ant colony algorithm in accordance with stochastic process which proves the algorithm convergence based on the ant quantity, route quantity and birth and death rate by the way of stochastic process. This paper proposes a novel Ant colony algorithm algorithm which is based on adaptively adjusting pheromone decay parameter and proves its convergence by fully considering the effect of the pheromone decay parameter to the algorithm convergence.

2 Basic ANT COLONY ALGORITHM Summarize

Ant colony algorithm is initially applied in solving TSP problem. Let’s explain ant colony system by taking TSP problem as example. Suppose: Set c is composed of D

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cities, m ants, d_{ij} (i,j=1,2,…, D) indicating the distance between City i and City j, l_{ij} (i,j=1,2,…, D) indicating the line between two cities, and $\tau_{ij}(t)$ indicating the remained pheromone strength on route between City i and City j on t hour to simulate the actual ant secretion. Ant k (k=1, 2, … , m) decides its direction according to the information quantity on each route when it moves. And at the same time take notes of the cities Ant k has passed in taboo table $tabu_k$(k=1,2,….,n). The set adjusts dynamically in accordance with evolution process. The ant counts mode shift probability in searching process according to the information quantity and heuristic information on the route. P^k_{ij} indicates the mode shift probability that Ant k shifting from City i to City j on t hour, expressed in formula (1).

$$p_{ij}^{k}(t)=\begin{cases}\dfrac{[\tau_{ij}(t)]^{\alpha}\cdot[\eta_{ij}(t)]^{\beta}}{\sum\limits_{s\subset allowed_k}[\tau_{is}(t)]^{\alpha}\cdot[\eta_{is}(t)]^{\beta}}, & If\ j\in allowed_k \\ 0, & else\end{cases} \tag{1}$$

In this formula, $allowed_k$={c-$tabu_k$} indicates the cities the ants are allowed to choose in next step. α is information heuristic parameter indicating the relative importance of the locus and reflecting the role of the information accumulated in the process of moving. The bigger the value becomes, the more possible the ant chooses the route other ants have passed and the stronger the cooperation among the ants is. β is the expectation heuristic parameter indicating the relative importance of visibility and reflecting the valued degree of the heuristic information in the process of choosing routes. The bigger the value becomes, the closer the mode shift probability is to the greedy rule. η_{ij} (t) is the heuristic function. It is expressed in formula (2).

$$\eta_{ij}(t)=\frac{1}{d_{ij}} \tag{2}$$

In order to avoid the remained information covering the heuristic information, renew the remained information after each ant finished one step or passed all D cities. It is expressed in formula (3) and (4).

$$\tau_{ij}(t+D)=(1-\rho)\cdot\tau_{ij}(t)+\Delta\tau_{ij}(t) \tag{3}$$

$$\Delta\tau_{ij}(t)=\sum_{k=1}^{m}\Delta\tau_{ij}^{k}(t) \tag{4}$$

$\Delta\tau_{ij}(t)$ indicates the pheromone the ants left on the route between City i and City j in this cycle. The algorithm depends on the counting model. In the most used Ant-Cycle model, i.e., formula (5)

$$\Delta\tau_{ij}(t)=\begin{cases}\dfrac{Q}{L_k}, ant\ k\quad by\ city\ to\quad city\quad j \\ 0,\qquad else\end{cases} \tag{5}$$

Q is the constant indicating the pheromone strength. It affects the convergence speed of algorithm to some degree. L_k is the total length of the route the ant passed in this cycle. The parameter α,β,ρ in Basic ant colony algorithm, Q may decide its optimum group by experiment. Stopping factors may use fixed evolution algebra or it may stop counting when the evolution tendency is not obvious.

3 The Adoptive ANT COLONY ALGORITHM

Because of the existence of pheromone decay parameter ρ, if ρ is bigger, the information quantity on the unsearched route will reduce to 0 and lower the global searching ability of the algorithm. Although reducing the value of ρ will enhance the global searching ability of algorithm to some degree, the algorithm convergence speed will be lowered. This paper changes the value of ρ by adaptive adjusting according to actual situation. First, give ρ an initial value within (0, 1).

Definition 1
When the better routes remain unchanged after N (N is a constant) times cycle, the better route is doubted to be in partial minimum value.

When the better route is doubted to be in partial minimum, ρ will adjust adaptively by adopting formula (6).

$$\rho(t) = \begin{cases} \xi \cdot \rho(t-1), if & \xi \cdot \rho(t-1) \geq \rho_{\min} \\ \rho_{\min}, & else \end{cases} \tag{6}$$

In formula (6), ζ(0,1) is decay binding modulus, ρ_{min} is the minimum value of ρ which can prevent ρ lowering the convergence speed when ρ is too small. In addition, compared with basic ANT COLONY ALGORITHM, this algorithm renews the pheromone globally according to the better solution of iterations instead of the partial renewal of pheromone. In this way, the improved algorithm enhances the convergence speed.

Adaptive ant colony algorithm process is as following:

Step 1: Parameter Initialization. Order time t=0, cycle number NC=0. Set the maximum cycle number NC_{max}, randomly distribute m ants to D cities, place the starting city in taboo table $tabu_k$, set the value of α,β,ρ, $\tau_{ij}=\tau_0$, i,j∈R.

Step 2: NC=NC+1;

Step 3: Ant individually chooses City i according to the probabilities counted by mode shift formula (1) and go forward, $j \in \{C - tabu_k\}$

Step 4: Amend taboo table indicator, i.e. remove the ant in a new city after choosing the city and remove the chosen city in the taboo table of the ant individual.

Step 5: If the cities in Set c have not been passed through, i.e. k<m, then k=k+1 and skip to Step3.

Step 6: After m ants finished one cycle, count the route distance L with formula (7) and retain the shortest route L_{min}, order the better route in the first cycle as L_g.

$$L = \sum_{i=1}^{n-1} d(c_{tabu_{k(i)}}, c_{tabu_{k(i+1)}}) \tag{7}$$

Step 7: Global renewal of pheromone. Renew the pheromone globally according to formula (3),(4) and (8) after the ants passed all cities. If $\tau_{ij}(t+1) \le \tau_{min}$, then order $\tau_{ij}(t+1) = \tau_{min}$.

$$\Delta \tau_{ij}(t) = \begin{cases} \dfrac{Q}{L_{\min}}, \\ 0, \end{cases} \tag{8}$$

Step 8: When the better route is doubted to be in partial minimum value, renew the pheromone decay parameter with formula (6).

Step 9: Compared L_{min} with L_g got in this cycle, if $L_{min} < L_g$, order $L_g = L_{min}$ and replace the better route table.

Step 10: If the cycle number $NC \ge NC_{max}$, then the cycle finished and get the better route length L_g and better route table. Otherwise clear away taboo table and skip to Step 2.

4 The Convergence Analysis of New Algorithm

The directions of the principle 1: With regarding to $\forall \tau_{ij}$, there is always having $\tau_{\min} \le \tau_{ij} \le \tau_{\max}$ .Among that, $\tau_{\max} \in [\frac{1}{\rho_1} g(s^*), \frac{1}{\rho_{\min}} g(s^*)]$.

Proving: Suppose: The optimum route of some on behalf of the ant looking for food is S0, aggregation A={ ρ_1, ρ_2, …, ρ_k } Among that, $\rho_1 \ge \rho_2 \ge \dots \rho_k \ge \rho_{\min}$,

(1) The optimum disambiguation does not appear to be like being caught in minimal value of part in t fall generation.

a. If $l_{ij} \notin S_0$,there is $\tau(1) = (1-\rho_1)\tau_0$, $\tau(2) = (1-\rho_1)^2 \tau_0, \dots, \tau(n) = (1-\rho)^t \tau_0$.

Think that $t \to \infty$, $\tau(t) \to 0$, it will be forced to arrive at $\tau_{\min}$.

b. If $l_{ij} \in S_0$, $\tau(1) = (1-\rho_1)\tau_0 + \rho_1 \Delta\tau_{ij}$, $g(s^*) = \max(\rho_i \Delta \tau_{ij})$, replace the day after tomorrow by that t falls time, there is $\tau(t) = (1-\rho_1)^n \tau_0 + \sum_{i=0}^{t-1} (1-\rho_1)^i g(s^*)$.

There will be $(1-\rho) \subset (0,1)$ because $\rho \subset (0,1)$, so that, $\tau_{\max} = \frac{1}{\rho_1} g(s^*)$.

(2) There is k times to be like being caught in minimal value of part time giving a disambiguation preferential treatment most in the t among time of iteration, so ,be based on formula (6), the value of the ρ needs to be adjusted. We write ρ_1 is initial value, ρ_2, ρ_3, …, ρ_n is adjusted value, The corresponding iteration number of times adjusting front is respectively $t_1, t_2, t_3, \dots, t_{k-1}$. $t_k = t, t_0 = 0$.

a. If $l_{ij} \notin S_0$,

$\tau(t) = \prod_{i=1}^{k} (1-\rho_i)^{t_i - t_{i-1}} \tau(0) \le (1-\rho_{\min})^{t_k} \tau(0) = (1-\rho_{\min})^t \tau(0)$ When $t \to \infty$, $\tau(t) \to 0$, it will be forced to arrive at $\tau_{\min}$.

b. If $l_{ij} \in S_0$, $M_1 = \sum_{i=0}^{t_1-1}(1-\rho_1)^i$, $M_2 = (1-\rho_2)^{t_2-t_1} \cdot M_1 + \sum_{i=0}^{t_2-t_1-1}(1-\rho_2)^i$

……

$M_k = (1-\rho_k)^{t_k-t_{k-1}} \cdot M_{k-1} + \sum_{i=0}^{t_k-t_{k-1}-1}(1-\rho_k)^i \cdot$

There is $\tau_{ij\max} = \prod_{i=1}^{k}(1-\rho_i)^{t_i-t_{i-1}} \cdot \tau_0 + M_k g(s^*)$, because of the

$\rho_1 \ge \rho_i \ge \rho_{\min}$ (i=2,3, …, k). So that,

$$\tau_{ij\max} \ge (1-\rho_1)^{t_k}\tau_0 + \sum_{i=0}^{t_{k-1}}(1-\rho_1)^i g(s^*) .$$

$$\tau_{ij\max} \le (1-\rho_{\min})^{t_k}\tau_0 + \sum_{i=0}^{t_{k-1}}(1-\rho_{\min})^i g(s^*) \cdot$$

If $t \to \infty$, Method according to above $\frac{g(s^*)}{\rho_1} \le \tau_{ij\max} \le \frac{g(s^*)}{\rho_{\min}}$.

Synthetically (1)(2), for $\forall \tau_{ij}$, the $\tau_{\min} \le \tau_{ij} \le \tau_{\max}$ is right, among that, $\tau_{\max} \subset [\frac{1}{\rho_1}g(s^*), \frac{1}{\rho_{\min}}g(s^*)]$.[the proving is over]

Theorem 1: Supposing of the P0 (n) is t time of iteration inner algorithm discovering the probability untying S0 giving preferential treatment most first，then with regarding to arbitrarily small $\varepsilon > 0$ and big sufficient iteration number of times t, the $P_0(t) \ge 1-\varepsilon$ is right.

Proving: Because of information amounts τ_{ij} is restricted between $\tau_{\min}$ and $\tau_{\max}$, In the process that ant structure solves, therefore, the feasibility changing probability chooses state as Pmin > 0,what is more, there is

$$P_{\min} \ge \hat{P}_{\min} = \frac{\tau_{\min}^{\alpha}\eta_{\min}^{\beta}}{(D-1)\tau_{\max}^{\alpha}\eta_{\max}^{\beta} + \tau_{\min}^{\alpha}\eta_{\min}^{\beta}} .$$

The letter of D expresses the number of cities in the formula. There will be allowed probability comes into being of the $\hat{P} \ge \hat{P}_{\min}^{\,n} > 0$ with regarding to S', in the list, n is stand for max. Therefore, $P_0(t) = 1-(1-\hat{P})^t$ .Think that, when t is sufficient big, with regarding to arbitrarily small $\varepsilon > 0$,there is $P_0(t) \ge 1-\varepsilon$. [the proving is over]

Theorem 1. gets the probability explaining that the algorithm can be used during the period of the $t \to \infty$,which giving a solution preferential treatment most.

Theorem 2. Set up t * being S0 iteration number of times discover the optimum solution first, $\forall$ (i,j)$\in$ S_0, $\forall$ (k,l)$\in$ L $\wedge$ (k,l)$\notin$ S_0,the there is $\tau_{ij}(t) > \tau_{kl}(t)$ for any t0 when t>t*++t0,among that, $t_0 = \frac{1-\rho_1}{\rho_1}$.

Proving: Consider the very worst condition, order t^* replaces the day after tomorrow w time again and again, there is $\tau_{ij}(t^*) = \tau_{\min}$, $\tau_{kl}(t) = \tau_{\max}$, We in case that t * is at any time corresponding $\rho_1^{'}$, $t^* + t^{'}$ o'clock carve corresponding $\rho_k^{'}$, ($\rho_1 \geq \rho_1^{'} \geq \rho_2^{'} \geq .. \rho_k^{'} \geq \rho_{\min}$), Within the [t^* , $t^* + t^{'}$], the optimum disambiguation seems going forward being caught in minimal value of part takes that t0 parts for plain iteration number of times as $t_1^{'}, t_2^{'}, \cdots t_{k-1}^{'}$, we write it as $t_k^{'} = t^* + t^{'}$, In the light of the method ,we can prove that

$$\tau_{ij}(t^* + t^{'}) = \prod_{i=1}^{k} (1-\rho_i^{'})^{t_i^{'} - t_{i-1}^{'}} \tau_{\min} + M_{t^{'}} g(s^*) \quad > \quad M_{t^{'}} g(s^*) \geq t^{'}(1-\rho_1)^{t^{'}-1} g(s^*)$$

$\tau_{kl}(t^* + t^{'}) = \max\{\tau_{\min}, (1-\rho_{\min})^{t^{'}} \cdot \tau_{\max}\}$, $\tau_{ij} \in S_0$, Usually, the overall situation renews ascending, τ_{ij} monotonous ascending, the reason why having $\tau_{ij} > \tau_{\min}$ to be founded according to information. Therefore, if we want to testify $\forall t, \tau_{ij}(t) > \tau_{kl}(t)$ is right, we as lonely as testifying

$$t^{'}(1-\rho_1)^{t^{'}-1} g(s^*) \quad > \quad (1-\rho_{\min})^{t^{'}} \tau_{\max}$$

$$t^{'} > \frac{\tau_{\max} \cdot (\frac{1-\rho_{\min}}{1-\rho_1})^{t^{'}-1} \cdot (1-\rho_{\min})}{g(s^*)} \geq \frac{\frac{g(s^*)}{\rho_1} \cdot (\frac{1-\rho_1}{1-\rho_1})^{t^{'}-1} \cdot (1-\rho_1)}{g(s^*)} = \frac{1-\rho_1}{\rho_1} = t_0 .$$ [the proving is over]

5 Simulation and Discussion

In order to test the efficiency of the improved algorithm, we conduct simulation study to Oliver 30, Ei150 in TSP problem and get the simulation result after 20 times

Table 1. Calculate Result

	Algorithm	Optimum solution	Average solution
Oliver30	Improved ACO	425.29	433.25
	Basic ACO	432.56	440.87
Eil51	Improved ACO	428.39	436.54
	Basic ACO	434.85	440.75

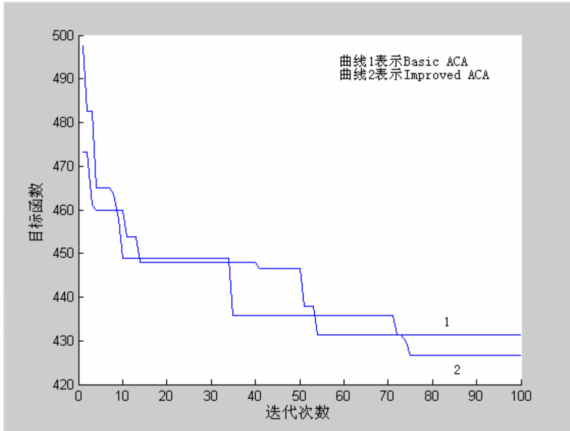

Fig. 1. Oliver 30 the better route evolution curve

operation. The simulation results of different parameter α are in Table 1, its evolution curve in Fig. 1.

By Table 1, we could see that the improved ant colony algorithm has advantage than Basic ant colony algorithm in finding better routes, figure 1 explain this point more clearly. In figure 1, the basic ant colony algorithm will be in partial minimum value in about 55 iterations in solving Oliver30 problem and affect finding the better routes. The improved ant colony algorithm adaptively adjusts the pheromone decay parameter and skips the partial minimum value although its value remains unchanged in 36-71 generations.

6 Conclusions

The pheromone of the basic ant colony algorithm renews through constant decay parameter and this will often cause the algorithm falling in the partial better routes in searching process. This paper enhances the algorithm globalization by adaptively adjusting pheromone decay parameter and avoids falling into the partial minimum value when the ants are searching too concentrated. The theorem in this paper proves the ant colony algorithm convergence and points out that this algorithm can find the global best solution tends to 1 for a sufficiently large number of iterations. Based on the simulation result of TSP problem, the algorithm globalization has been improved and the improved algorithm is effective.

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