Algorithmic Decision of Syllogisms*

Bora İ. Kumova and Hüseyin Çakır

İzmir Institute of Technology, Department of Computer Engineering, 35430 Turkey
{borakumova,huseyincakir}@iyte.edu.tr

Abstract. A syllogism, also known as a rule of inference, is a formal logical scheme used to draw a conclusion from a set of premises. In a categorical syllogisms, every premise and conclusion is given in form a of quantified relationship between two objects. The syllogistic system consists of systematically combined premises and conclusions to so called figures and moods. The syllogistic system is a theory for reasoning, developed by Aristotle, who is known as one of the most important contributors of the western thought and logic. Since Aristotle, philosophers and sociologists have successfully modelled human thought and reasoning with syllogistic structures. However, a major lack was that the mathematical properties of the whole syllogistic system could not be fully revealed by now. To be able to calculate any syllogistic property exactly, by using a single algorithm, could indeed facilitate modelling possibly any sort of consistent, inconsistent or approximate human reasoning. In this paper we present such an algorithm.

Keywords: Syllogistic reasoning, fallacies, automated reasoning, approximate reasoning, human-machine interaction.

1 Introduction

The first studies on syllogisms were pursued in the field of right thinking by the philosopher Aristotle [1]. His syllogisms provide patterns for argument structures that always yield conclusions, for given premises. Some syllogisms are always valid for given valid premises, in certain environments. Most of the syllogisms however, are always invalid, even for valid premises and whatever environment is given. This suggests that structurally valid syllogisms may yield invalid conclusions in different environments.

Given two relationships between the quantified objects P, M and S, a syllogism allows deducing a quantified transitive object relationship between S and P. Depending on alternative placements of the objects within the premises, 4 basic types of syllogistic figures are possible. Aristotle had specified the first three figures. The 4. figure was discovered in the middle age. In the middle of the 19^{th} century, experimental studies about validating invalid syllogisms were pursued. For instance, Reduction of a syllogism, by changing an imperfect mood into a perfect one [13]. Conversion of a mood, by transposing the terms, and thus drawing another proposition from it of the same quality [11], [10].

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Although shortly thereafter syllogism were superseded by propositional logic [7], they are still matter of research. For instance philosophical studies have confirmed that syllogistic reasoning does model human reasoning with quantified object relationships [2]. For instance in psychology, studies have compared five experimental studies that used the full set of 256 syllogisms [4], [12] about different subjects. Two settings about choosing from a list of possible conclusions for given two premisses [5], [6], two settings about specifying possible conclusions for given premisses [8], and one setting about decide whether a given argument was valid or not [9]. It has been found that the results of these experiments were very similar and that differences in design appear to have had little effect on how human evaluate syllogisms [4]. These empirically obtained truth values for the 256 moods are mostly close to their mathematical truth ratios that we calculate with our algorithmic approach.

Although the truth values of all 256 moods have been analysed empirically, mostly only logically correct syllogisms are used for reasoning or modus ponens and modus tollens, which are generalisations of syllogisms [14]. Uncertain application environments, such as human-machine interaction, require adaptation capabilities and approximate reasoning [16] to be able to reason with various sorts of uncertainties. For instance, we know that human may reason purposefully fallacious, aiming at deception or trickery. Doing so, a speaker may intent to encourage a listener to agree or disagree with the speaker's opinions. For instance, an argument may appeal to patriotism, family or may exploit an intellectual weakness of the listener. We are motivated by the idea for constructing a fuzzy syllogistic system of possibilistic arguments for calculating the truth ratios of illogical arguments and approximately reason with them.

This paper presents an algorithm for deciding syllogistic cases, for algorithmically calculating syllogistic reasoning and an application to automated reasoning. Firstly, categorical syllogisms are discussed briefly. Thereafter an arithmetic representation for syllogistic cases is presented, followed by an approach for algorithmically deciding syllogisms and a possible application for recognising fallacies and reasoning with them.

2 Categorical Syllogisms

A categorical syllogism can be defined as a logical argument that is composed of two logical propositions for deducing a logical conclusion, where the propositions and the conclusion each consist of a quantified relationship between two objects.

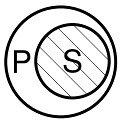
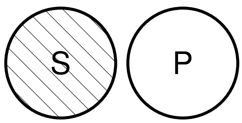
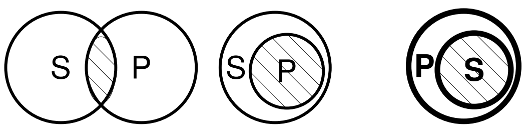
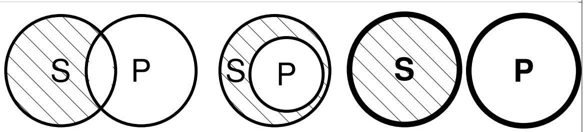
2.1 Syllogistic Propositions

A syllogistic proposition or synonymously categorical proposition specifies a quantified relationship between two objects. We shall denote such relationships with the operator ψ. Four different types are distinguished $\psi \in \{A, E, I, O\}$ (Table 1):

- A is universal affirmative: All S are P
- E is universal negative: All S are not P
- I is particular affirmative: Some S are P
- O is particular negative: Some S are not P

One can observe that the proposition I has three cases (a), (b), (c) and O has (a), (b), (c). The cases I (c) and O (c) are controversial in the literature. Some do not consider them as valid [3] and some do [15]. Since case I (c) is equivalent to proposition A, A becomes a special case of I. Similarly, since case O (c) is equivalent to proposition E, E becomes a special case of O. At this point we need to note however that exactly these cases complement the homomorphic mapping between syllogistic cases and the set-theoretic relationships of three sets. This is discussed below.

Table 1. Syllogistic propositions consist of quantified object relationships

Operator ψ	Proposition Φ	Set-Theoretic Representation of Logical Cases
A	All S are P	
E	All S are not P	
I	Some S are P	 (a) (b) **(c)**
O	Some S are not P	 (a) (b) **(c)**

2.2 Syllogistic Figures

A syllogism consists of the three propositions major premise, minor premise and conclusion. The first proposition consist of a quantified relationship between the objects M and P, the second proposition of S and M, the conclusion of S and P (Table 2).

Since the proposition operator ψ may have 4 values, 64 syllogistic moods are possible for every figure and 256 moods for all 4 figures in total. For instance, AAA-1 constitutes the mood MAP, SAM - SAP in figure 1. The mnemonic name of this mood is Barbara, which comes from syllogistic studies in medieval schools. Mnemonic names were given to each of the in total 24 valid moods, out of the 256, for easier memorising them [3].

Table 2. Syllogistic figures

Figure Name	I	II	III	IV
Major Premise Minor Premise ——— Conclusion	$M\psi P$ $S\psi M$ ——— $S\psi P$	$P\psi M$ $S\psi M$ ——— $S\psi P$	$M\psi P$ $M\psi S$ ——— $S\psi P$	$P\psi M$ $M\psi S$ ——— $S\psi P$

We shall denote a propositional statement with Φ_i, in order to distinguish between possibly equal propositional operators of the three statements of a particular mood, where $i \in \{1, 2, 3\}$.

A further consequence of including the above mentioned cases I (c) and O (c) in our algorithmic approach is that the number of valid moods increases with AAO-4 from 24 to 25. Since no mnemonic name was given to this mood in the literature by now, name it herewith "anasoy".

3 Algorithmic Representation

In the following our approach for algorithmically deciding any given syllogistic mood is presented. Algorithmically analysing all 2624 truth cases of the 256 moods enables us to calculate all mathematical truth values of all moods, sort the moods according their truth values and define a fuzzy syllogistic system of possibilistic arguments.

3.1 Set-Theoretical Analysis

For three symmetrically intersecting sets there are in total 7 possible sub-sets in a Venn diagram (Fig 1). If symmetric set relationships are relaxed and the three sets are named, for instance with the syllogistic terms P, M and S, then 41 set relationships are possible. These 41 relationships are distinct, but re-occur in the 256 moods as basic syllogistic cases. The 7 sub-sets in case of symmetric relationships and the 41 distinct set relationships in case of relaxed symmetry are fundamental for the design of an algorithmic decision of syllogistic moods.

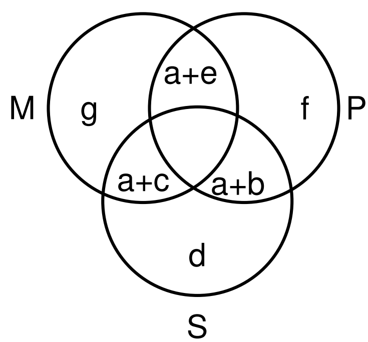


Fig. 1. Mapping the sub-sets of the symmetrically intersecting sets P, M and S onto arithmetic relations

We have pointed out earlier that, including the cases I (c) and O (c) of the syllogistic propositions I and O, is required by the algorithm to calculate correctly. Without these cases, the algorithm presented below, cannot decide some cases of some moods or cannot find valid moods at all. For instance, as valid moods in figure I, only AAA, AAI, AII and EAE can be found by the algorithm, although EAO and EIO are also true. If the algorithm considers the cases I (c) and O (c), then all 6 valid moods of figure I are found. The reason for that is that the syllogistic propositions are basically a symmetric sub-set of the in total 12 distinct set relationships between two named sets. Therefore the cases I (c) and O (c) are required to complement the symmetric relationships between the syllogistic propositions.

3.2 Arithmetic Representation

Based on theses 7 sub-sets, we define 9 distinct relationships between the three sets P, M and S (Table 3). These 9 relationships are mapped homomorphically onto the 9 arithmetic relations, denoted with $\delta1$, ..., $\delta9$. For instance P∩M is mapped onto $\delta1$=a+e and P-M is mapped onto $\delta4$=f+b. These relationships can be verified visually in the Venn diagram (Fig 1).

One can observe that the symmetric relationship between the three sets (Fig 1) is preserved in the homomorphically mapped arithmetic relations (Table 3).

Table 3. Homomorphism between the 9 basic syllogistic cases and 9 arithmetic relations

Sub-Set Number	$\delta1$	$\delta2$	$\delta3$	$\delta4$	$\delta5$	$\delta6$	$\delta7$	$\delta8$	$\delta9$
Arithmetic Relation	a+e	a+c	a+b	f+b	f+e	g+c	g+e	d+b	d+c
Syllogistic Case	P∩M	M∩S	S∩P	P-M	P-S	M-P	M-S	S-M	S-P

The above homomorphism represents the essential data structure of the algorithm for deciding syllogistic moods.

3.3 Algorithmic Decision

The pseudo code of the algorithm for determining the true and false cases of a given moods is based on selecting the possible set relationships for that mood, out of all 41 possible set relationships.

```
DETERMINE mood
   READ figure number {1,2,3,4}
   READ with 3 proposition ids {A,E,I,O}
GENERATE 41 possible set combinations with 9
   relationships into an array
setCombi[41,9]={{1,1,1,1,1,1,1,1,1}, ...,
   {0,1,0,0,1,1,1,1,1}}
VALIDATE every proposition with either validateAllAre,
   validateAllAreNot, validateSomeAreNot or
   validateSomeAre
```

```
DISPLAY valid and invalid cases of the mood
VALIDATE mood
validateAllAre(x,y) //all M are P
   if(x=='M' && y=='P')
  CHECK the sets suitable for this mood in setCombi
 if δ1=1 and δ2=0 then add this situation as valid
   if(setCombi[i][0]==1 && setCombi[i][1]==0)
//similar for validateAllAreNot(), validateSomeAre(),
   validateSomeAreNot()
```

3.4 Statistics about the Syllogistic System

The introduced algorithm enables revealing various interesting statistics about the structural properties of the syllogistic system. Some of them are presented now.

Since our objective is to utilise the full set of all 256 moods as a fuzzy syllogistic system of possibilistic arguments, we have first calculated the truth values for every mood in form of a truth ration between its true and false cases, so that the truth ratio becomes a real number, normalised within [0, 1]. Thereafter we have sorted all moods in ascending order of their truth ratio (Fig 2). Note the symmetric distribution of the moods according their truth values. 25 moods have a ratio of 0 (false) and 25 have ratio 1 (true). 100 moods have a ratio between 0 and 0.5 and 100 have between 05 and 1. 6 moods have a ratio of exactly 0.5.

Every mood has 0 to 21 true and 0 to 21 false cases, which is a real sub-set of the 41 distinct cases. The total number of true or false cases varies from one mood to another, from 1 to 24 cases. For instance, mood AAA-1 has only 1 true and 0 false cases, whereas mood OIA-1 has 3 true and 21 false cases. Hence the truth ratio of AAA-1 is 1 and that of OIA- is 3/21=1/7. The algorithm calculates 2624 syllogistic cases in total, since all cases of the 256 moods map the 41 distinct cases multiple times. Interesting is also that for any given figure the total number of all true cases is equal to all false cases, ie 328 true and 328 false cases. Thus we get for all 4 syllogistic figures the total number of 4 x 2 x 328 = 2624 cases. More statistical details will be discussed in a separate work.

3.5 Fuzzy Syllogistic System

Based on the structural properties of the syllogistic system, we elaborate now a fuzzified syllogistic system.

One can see (Fig 2) that every syllogistic case is now associated with an exact truth ration. We utilise the symmetric distribution of the truth ratios, for defining the membership function FuzzySyllogisticMood(x) with a possibility distribution that is similarly symmetric (Fig 2). The linguistic variables were adopted from a meta membership function for a possibilistic distribution of the concept likelihood [17]. The complete list with the names of all 256 moods is appended (Table A1).

As we have mentioned earlier, the algorithmically calculated truth ratios of the 256 moods (Fig 2) mostly comply with those empirically obtained truth ratios in psychological studies [4]. Hence the suggested possibilistic interpretation should reflect an approximately correct model of the syllogistic system.

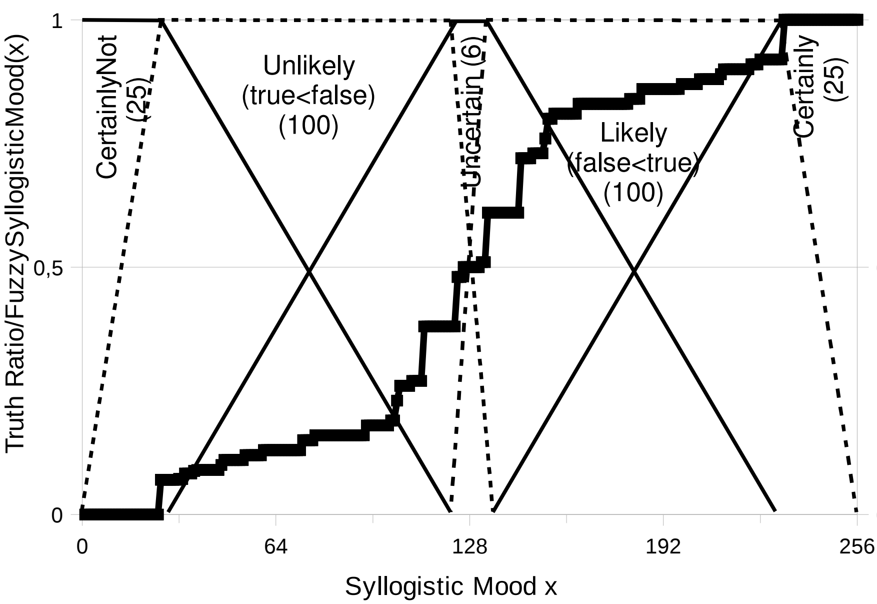


Fig. 2. 256 syllogistic moods sorted in ascending order of their truth ratio true/false, if number of truth cases of a mood is true<false and false/true ratio, if false<true. Definition of the possibility distribution FuzzySyllogisticMood(x) with the linguistic variables CertainlyNot, Unlikely, Uncertain, Likely, Certainly and their cardinalities 25, 100, 6, 100, 25, respectively.

4 Recognising Fallacies and Fuzzy Syllogistic Reasoning

In logic a fallacy is a misconception resulting from incorrect reasoning in argumentation. 7 syllogistic fallacies are known in the literature:

- Equivocation fallacy or fallacy of necessity: Unwarranted necessity is placed in the conclusion, by ignoring other possible solutions.
- Fallacy of undistributed middle: Middle term must be distributed in at least one premiss.
- Illicit major/minor: No term can be distributed in the conclusion, which is not distributed in the premisses.
- Fallacy of exclusive premisses: Two negative premisses.
- Affirmative conclusion from negative premiss: Positive conclusion, but at least one negative premiss.
- E xistential fallacy: Two universal premisses, but particular conclusion.

These fallacies comply exactly with the 7 rules for eliminating invalid moods, which were discovered already by Aristotle [1].

Our objective is to use the whole set of 256 syllogistic moods as one system of possibilistic arguments for recognising fallacies and reasoning with them. For that purpose, we specify the following steps:

- Calculate all truth cases and truth ratio of a given mood.
- Try to recognise fallacies by
  - \+ identifying false or true possibilities: reduction of A to I or E to O, respectively
  - \+ generalising true or false possibilities: generalisation of I to A or O to E.
- Try to map the initial mood to a mood with a truth ratio closer to 1.
- Approximately reason with the truth ratios.

We will now discuss these steps experimentally on the following example (Fig 3). Firstly, we calculate the 3 true (Fig 4) and 3 false (Fig 5) cases of mood AIA-1 and its truth ratio of 0.5. Secondly, we identify following fallacies:

- $\neg\Phi_1$(A): Simply not all stories in The Child's Magic Horn are sad. The truth is that only some stories in The Child's Magic Horn are sad Φ_1(I).
- $\neg\Phi_3$(A): Not all stories I cry at are stories in The Child's Magic Horn, because I will possibly cry at some other stories as well. The truth is that only some of all the stories I cry at are stories in The Child's Magic Horn Φ_3 (I).

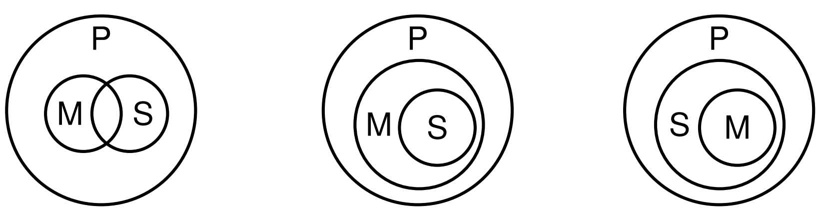
Thirdly, based on the identified fallacies and reductions to Φ_1 (I) and Φ_3 (I), we can easily calculate the mood III-1 to be "more true" for the given sample propositions. In dead, mood III-1 has with 4 false/19 true cases = 0.73, a better truth ratio.

In the last step, we may use the truth ration of the mood for fuzzy syllogistic reasoning as a model for approximate reasoning with quantified propositions.

P:	Sad	A: all M are P
M:	Stories in The Child's Magic Horn	I: some S are M
S:	Tales I cry at	
		A: all S are P

Φ_1(A): All "Stories in The Child's Magic Horn" are "Sad"
Φ_2(I): Some "Tales I cry at" are "Stories in The Child's Magic Horn"
Φ_3(A): All "Tales I cry at" are "Stories in The Child's Magic Horn"

Fig. 3. Sample syllogistic inference with the mood AIA1



(a) (b) (c)

Fig. 4. True syllogistic cases of the mood AIA1

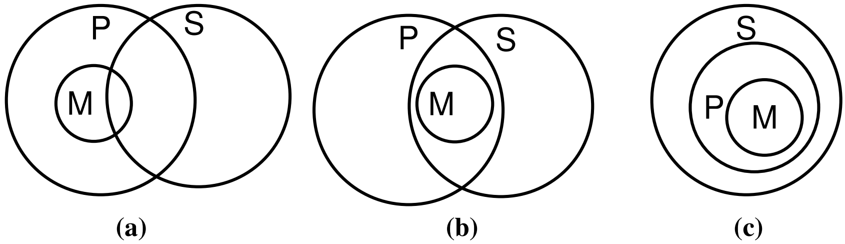


Fig. 5. False syllogistic cases of the mood AIA1

5 Conclusion

We have presented an algorithmic approach for analysing the syllogistic system. The algorithm facilitates structurally analysing the syllogistic moods and reveals interesting statistics about the truth cases of the moods. First experimental results show that the syllogistic system is inherently symmetric.

With the membership function FuzzySyllogisticMood(x) we have proposed a fuzzy syllogistic system of possibilistic arguments and its possible application for recognising fallacies and fuzzy syllogistic reasoning.

Future work shall aim at systematically revealing all significant statistical properties of the syllogistic system, by using the algorithm. We believe that this approach may prove a practical approach for reasoning with inductively learned knowledge, where P, M, S object relationships can be learned inductively and the "most true" mood can be calculated automatically for those relationships. That shall be our future work, alon with examples including recognising intentional or unintentional fallacies, with the objective to facilitate automated human-machine interaction.

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Appendix A: Truth Degree of Syllogistic Moods

The table (Table A1) shows the 256 moods in 5 categories with truth ratio normalised in [0,1]. False, undecided and true moods are not sorted. Unlikely and Likely moods are sorted in ascending order of their truth ratio. The table also shows the possibility distribution of the membership function FuzzySyllogisticMood(x), with x ∈{CertainlyNot, Unlikely, Uncertain, Likely, Certainly}, defined over the truth ratios of the moods.

Table A1. Possibility distribution FuzzySyllogisticMood(x) over the Syllogistic moods in increasing order of truth ratio of the moods

Linguistic Variables	Sum	Moods
CertainlyNot; false; ratio=0	25	AAE-1, AAO-1, AIE-1, EAA-1, EAI-1, EIA-1, AEA-2, AEI-2, AOA-2, EAA-2, EAI-2, EIA-2, AAE-3, AIE-3, EAA-3, EIA-3, IAE-3, OAA-3, AAA-4, AAE-4, AEA-4, AEI-4, EAA-4, EIA-4, IAE-4
Unlikely; rather false; 0<ratio<0.5	100	EIE-1, IEE-1, EIE-2, IEE-2, EIE-3, IEE-3, EIE-4, IEE-4, AOE-2, OAA-2, OAE-2, AOA-1, IAA-1, OAE-1, OEE-1, IAA-2, EOE-3, OEE-3, AOE-4, EOE-4, OOE-3, AEA-1, AEE-1, AAA-3, AEA-3, AEE-3, EAE-3, EAE-4, EOE-1, EOE-2, OEA-2, OEE-2, OEA-4, OEE-4, OIE-1, OOE-1, OOA-4, OOE-4, IOA-3, IOE-3, OIE-3, IOA-4, IOE-4, IEA-1, IEA-2, IEA-3, IEA-4, IIA-1, IIA-2, IIA-3, IIA-4, IAE-1, OAA-1, OEA-1, AIE-2, IAE-2, OEA-3, AIE-4, AAA-2, AAE-2, EAA-1, EEE-1, EEA-2, EEE-2, EEA -3, EEE-3, EEA-4, EEE-4, IOA-1, IOE-1, IOA-2, IOE-2, OIA-2, OIE-2, OIA-4, OIE-4, OOA-2, OOE-2, OOA-3, IIE-1, IIE-2, IIE-3, IIE-4, AOE-3, IAA-3, OAE-3, IAA-4, OOA-1, OIA-1, OIA-3, AOE-1, AIA-2, EOA-3, AIA-4, AOA-4, EOA-4, OAA-4, OAE-4, EOA-1, EOA-2
Uncertain; undecided; ratio=0.5	6	AIA-1, AIO-1, AIA-3, AIO-3, AOA-3, AOO-3
Likely; rather true; 0.5<ratio<1.0	100	EOO-1, EOO-2, OIO-1, OOO-1, OIO-3, AIO-2, EOO-3, AIO-4, AOI-1, AOO-4, EOO-4, OAI-4, OAO-4, IAO-3, IAO-4, OAI-3, AOI-3, III-1, III-2, III-3, III-4, OOO-3, OOI-2, OOO-2, IOI-1, IOO-1, OII-2, OIO-2, IOI-2, IOO-2, OII-4, OIO-4, IAI-1, OAO-1, OEO-1, AII-2, OEO-3, IAI-2, AII-4, AAI-2, AAO-2, EEI-2, EEO-2, EEI-3, EEO-3, EEI-4, EEO-4, EEI-1, EEO-1, IIO-1, IIO-2, IIO-3, IIO-4, IEO-1, IEO-2, IEO-3, IEO-4, OII-1, OOI-1, IOI-3, IOO-3, OII-3, IOI-4, IOO-4, OOI-4, OOO-4, EOI-1, EOI-2, OEI-4, OEI-2, OEO-2, OEO-4, AEI-1, AEO-1, AAO-3, AEI-3, AEO-3, EAI-3, EAI-4, OOI-3, AOO-1, IAO-1, OAI-1, OEI-1, IAO-2, EOI-3, OEI-3, AOI-4, EOI-4, AOI-2, OAI-2, OAO-2, IEI-1, EII-1, EII-2, IEI-2, EII-3, IEI-3, EII-4, IEI-4
Certainly; true; ratio=1.0	25	AAA-1, AAI-1, AII-1, EAE-1, EAO-1, EIO-1, AEE-2, AEO-2, AOO-2, EAE-2, EAO-2, EIO-2, AAI-3, AII-3, EAO-3, EIO-3, IAI-3, OAO-3, AAI-4, AAO-4, AEE-4, AEO-4, EAO-4, EIO-4, IAI-4