Impact of the Material Distribution Formalism on the Efficiency of Evolutionary Methods for Topology Optimization

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Summary. We consider an evolutionary method applied to a topology optimization problem. We compare two material distribution formalisms (static vs. Voronoi-based dynamic), and two sets of reproduction mechanisms (standard vs. topology-adapted). We test those four variants on both theoretical and practical test cases, to show that the Voronoi-based formalism combined with adapted reproduction mechanisms performs better and is less sensitive to its parameters.

1 Introduction

Optimization methods are used more and more frequently at increasingly early stages in the design process, with the goal of improving performance with respect to cost, weight or other criteria. One can distinguish three paradigms according to the type of design variable used: parametric, geometric and topology optimization.

Parametric optimization deals with a fixed geometry, chosen by the designer, and tries to find optimal choices of geometric parameters such as lengths, widths, etc. Geometric optimization considers instead design parameters which define various shapes in the object under study, using for example spline functions. The designer remains responsible for selecting the initial geometry and choosing which shapes (typically interfaces between materials) are optimized, and how they are parameterized.

In this work, we focus on topology optimization, where design parameters describe the distribution of some materials in a design space. This paradigm

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offers two significant advantages over the other two. First, it can be started from an empty design space, hence the designer does not have to provide a priori solutions or initial geometries. Second, potential solutions are not restricted in any way, and methods can find designs with a completely arbitrary topology. Topology optimization tools are generally composed of three functional blocks:

1. A material distribution formalism that converts a list of design parameters into a solution (i.e. a design);
2. An evaluation tool that computes the objective function(s) of solutions produced by the material distribution formalism;
3. An optimization algorithm that modifies the solution through its design parameters in order to improve the objective function(s).

These blocks are obviously not completely independent. The choice of one may influence more or less significantly the choice of others and how they are implemented [1, 2, 3, 4]. In this article, we focus on the following two aspects: what is the impact of the material distribution formalism on the performance of the optimization tool, and can the optimization algorithm be adapted to the specific problem of topology optimization.

The material distribution formalisms we consider are based on a subdivision of the design space into cells, each cell being filled homogeneously with a given material. The optimization algorithm used is NSGA-II [5], a genetic algorithm. The choice of an evolutionary meta-heuristic algorithm is motivated by our will to develop a generic optimization tool, completely independent of a particular physics or evaluation tool, that does not require the availability of derivative information and is able to handle discrete parameters (to decide the type of material in each cell) ; other non-evolutionary derivative-free algorithms [6], such as direct search methods, could also be appropriate but fall outside the scope of this work.

This article is structured as follows. Section 2 presents the two different material distribution formalisms we consider, one based on a static division of the design space and the other allowing dynamic divisions using the notion of Voronoi cells. Section 3 proposes one way to adapt, through its reproduction mechanisms, the genetic algorithm to the specific case of topology optimization. Section 4 describes the study cases used in Section 5 to assess the impact of the choice of a material distribution formalism and the adaptation of the optimization algorithm on the quality of the solution found. The results reveal notably that, for a given number of evaluations, a dynamic material distribution formalism leads to solutions with a better objective function, and that the proposed adaptation of the genetic algorithm improves robustness of the results with respect to variations in the number of design parameters used.

2 Material distribution formalisms

Material distribution formalisms can be either static or dynamic. In the first case, subdivision of the design space into cells is decided once and for all before

the optimization. Design parameters are then limited to materials constituting each cell, and their total number remains constant. In the second case, the subdivision may evolve during optimization through the number of cells, their shapes and their positions. Design parameters must therefore also include a geometric description of each cell, and their number can vary.

2.1 Static formalism

The static formalism we consider is based on a subdivision of the design space into a regular fixed rectangular grid with m rows and n columns (Fig. 1, left), which is the most frequently used configuration in the literature, see e.g. [8].

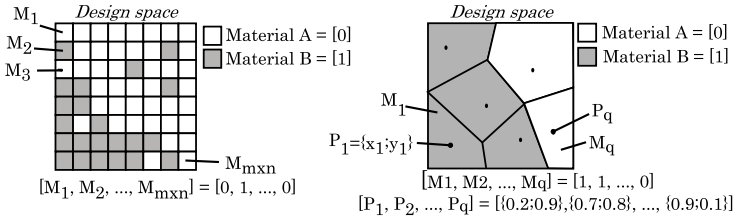

Fig. 1. Illustration of the static (left) and dynamic Voronoi (right) formalism.

Genetic algorithms manipulate the design parameters via a problem-dependent data structures called chromosomes. In this case, they are arrays where each element, called gene, is a discrete variable indicating the material of the cell. In this work, we only consider two materials, i.e. work with binary variables.

2.2 Dynamic formalism

The dynamic formalism we consider is based on the notion of Voronoi cells [9], whose use in the context of topology optimization was pioneered by Schoenauer (see e.g. [10, 7]). Each of the q cells is defined by its center, and includes all points of the design space that are nearest to this center (Fig. 1, right). In addition to the binary material chromosome of the static case, design parameters include the positions of each cell center, listed in a separate array of real x- and y-coordinates (i.e. $3q$ parameters in total).

3 Reproduction mechanisms

One of the main characteristics of meta-heuristic optimization algorithms is that they can be applied to various problems without requiring special adaptations. Indeed, genetic algorithms can be run as soon as the encoding of the design parameters characterizing a solution (called an individual) into one or more chromosomes is defined. These algorithms evolve a population of individuals by appropriate selection and reproduction mechanisms, with the aim of converging to an optimal solution (or to a set of non-dominated solutions if several objective functions are considered).

3.1 Standard mechanisms

The reproduction mechanisms involved in genetic algorithms are crossover and mutation. Crossover consists in exchanging some of the genes of two individuals, called parents, to produce two new individuals, called children. In its standard version, a pivot is randomly positioned inside the chromosome to determine the genes undergoing the exchange (Fig. 2, left). Mutation consists in modifying the (binary or real) value of a randomly chosen gene (Fig. 2, right).

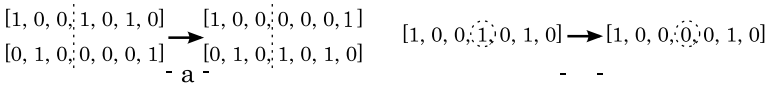

Fig. 2. Standard crossover (left) and mutation (right) reproduction mechanisms

These standard reproduction mechanisms may be applied to both static and dynamic material distribution formalisms (we must nevertheless ensure in the case of the dynamic formalism that crossovers are applied to the same parts of material and position chromosomes). Examples of these standard mechanisms are illustrated on Fig. 3.

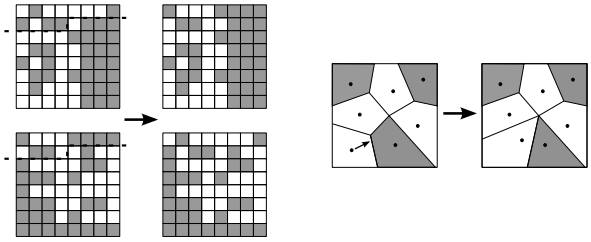

Fig. 3. Examples of standard reproduction mechanisms: crossover with static formalism (left) ; mutation with dynamic formalism (right)

3.2 Adapted mechanisms

The previous selection and reproduction mechanisms are completely generic and independent of the addressed problem. We now propose to use additional mechanisms better suited to the case of topology optimization and its geometric nature. More specifically, we suggest to apply the reproduction mechanisms graphically instead of working directly on chromosomes: a geometric region in the design space will be selected randomly and will then undergo a crossover or a mutation, after which the results will be translated back into the chromosome encoding.

In practice, the adapted crossovers we introduce in the static and dynamic cases are based on a random circle whose center and radius are randomly

chosen to fit within the design space. In the static cases, material genes within the circle are exchanged between the parents, while in the dynamic cases both position and material genes are exchanged (see an example on Fig. 4 left).

We also propose to introduce adapted mutations. In the static case, we set a whole randomly selected rectangle (instead of a single gene) to a single type of material (see Fig. 4 right). In the dynamic case, since standard mutations are already effective, we introduce a different type of adapted mutation that consists in randomly adding or deleting a Voronoi cell (note that the adapted crossover mechanism, in contrast with the standard mechanisms, already allows variations in the number of Voronoi cells, see again Fig. 4 left)[4].

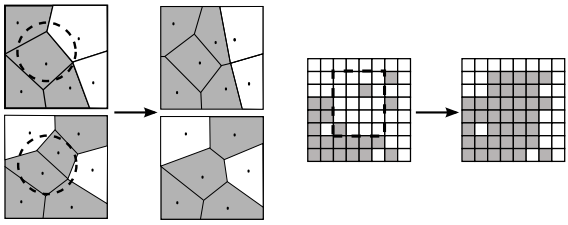

Fig. 4. Adapted mechanisms: dynamic crossover (left) static mutation (right)

4 Study cases

The dominating cost in a typical application of a genetic algorithm to an engineering design problem is the evaluation of the objective function, since computations required for population evolution are typically much cheaper. Therefore, in order to ensure a fair comparison between variants, we run each algorithm for a fixed number of generations, specifically 200 for the experiments reported in Section 5. We also use 1% mutation rates and 80% crossover rates, which have been empirically observed to give good results.

However, like others [10], we first consider a more theoretical test case where the objective function can be evaluated very cheaply. This allows us to run extensive experiments involving all proposed algorithm variants, and derive general observations about them. These conclusions are then validated on an actual engineering problem involving real physics but requiring much higher evaluation times.

4.1 Theoretical case

Our theoretical case study consists in searching for a hidden reference shape (Fig. 5, left). The corresponding objective function to minimize is given by the difference of concordance between the reference shape and that described using the material distribution formalisms. It is evaluated by projecting these two

[4] This however implies that standard crossovers are then no longer possible, because chromosomes can now have different lengths.

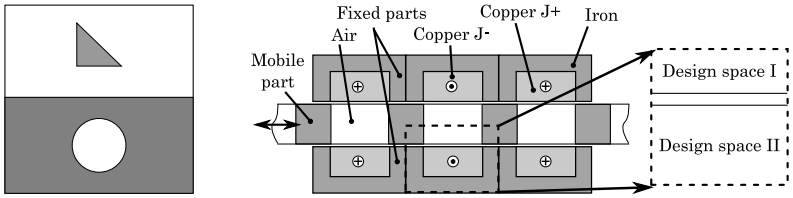

Fig. 5. Diagrams for theoretical (left, reference) and the practical (right) cases.

shapes onto a fine and identical $M \times N$ mesh (with $M \gg m$ and $N \gg n$). The objective function is therefore given by $\frac{\sum_{i=1}^{N}\sum_{j=1}^{N}(p_{ij} \oplus q_{ij})}{M \times N}$, where $\oplus$ denotes the exclusive or operation and p_{ij} and q_{ij} represent components on the fine mesh of the reference solution and of the solution to assess.

4.2 Practical case

Our practical study case concerns the design of a variable reluctance linear actuator (Fig. 5, right). The objective is to maximize the restoring force developed by the actuator between conjunction and opposition positions.

Given this objective and the symmetrical structure imposed on the actuator, the design space can be reduced to a small area (Fig. 5, right). This design space is partitioned into two subspaces, the first related to the mobile part and the other to the fixed part of the actuator.

The objective function to minimize is given by function $f = \psi_{opp} - \psi_{conj}$ [11], where ψ_{conj} and ψ_{opp} are the magnetic flux intercepted by the coils formed by the copper in the conjunction and opposition positions respectively. Evaluation of this function requires the use of a FEM software for calculating magnetic field distribution ; we used version 3.5 of COMSOL [12] (evaluation of a solution takes approximately 2 seconds on a 3 GHz computer).

5 Results and discussion

Whatever the formalism, one can expect that the (initial) number of cells significantly influences the behavior of the topology optimization tool. This is confirmed by results reported on Figs. 5 and 7 for all four combinations (static/dynamic and without/with adaptation). Note first that, in each situation, the smaller the number (initial) cells, the faster the convergence to a solution (a stable solution is even reached before the end of the 200 generations in the two smallest static cases 5×5 and 10×10). This is to be expected since a large number of cells, corresponding to a large number of design parameters, is naturally harder to optimize.

The effect of the proposed adaptation can be observed by comparing the left and right sides of Figs. 5 and 7. On the one hand, for the dynamic formalism, the adaptations are always beneficial, i.e. the final solution is always better. On the other hand, in the static case, results depend on the number

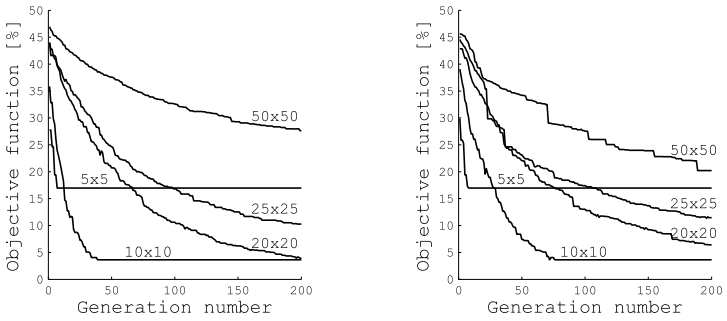

Fig. 6. Convergence of the objective function (theoretical case) for the classical formalism without (left) and with (right) adaptation for different grid sizes.

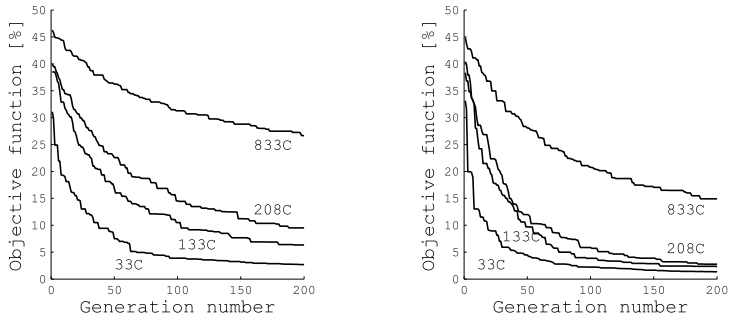

Fig. 7. Convergence of the objective function (theor. case) for the Voronoi formalism without (left) and with (right) adaptation for different initial numbers of cells.

of cells. For small grids, using the standard reproduction mechanisms leads to faster convergence, while the adapted mechanisms perform better for large grids. We explain this by noting that the adapted mutation mechanism, which works with groups of cells, can only speed up convergence when the number of cells is high, allowing more significant changes in the solution at each iteration. For lower number of cells, working with groups of cells has no effect or is even detrimental for the convergence.

Quality of the final solution obtained could be expected to increase when the number of cell increases, because this allows for more precise solutions. This is only partially confirmed by our results: while the static 10×10 result is better than its 5×5 counterpart, this trend does not continue with larger numbers of cells, nor with the dynamic formalism. The reason is that, when the number of cells is large, the 200-generation limit prevents the algorithm from reaching a stable solution. Running with an unlimited number of generations would show that larger numbers of cells lead to better final solutions, but this is of course unrealistic in practice.

Therefore, the initial number of cells becomes a key parameter in a topology optimization process. Too high, the slower convergence rate penalizes the

results because the solution does not have time to converge. Too low, the solution converges too quickly to a stable solution with lower quality and generations are wasted. Finding the optimum initial number of cells, one which ensures that the topological optimization tool converges to an optimal solution around the end of the fixed number of generations, is a crucial but difficult challenge, moreover likely to be heavily problem-dependent. Figures 8 and 9 illustrate this tradeoff for our theoretical case study (each box plot stands for 5 experiments).

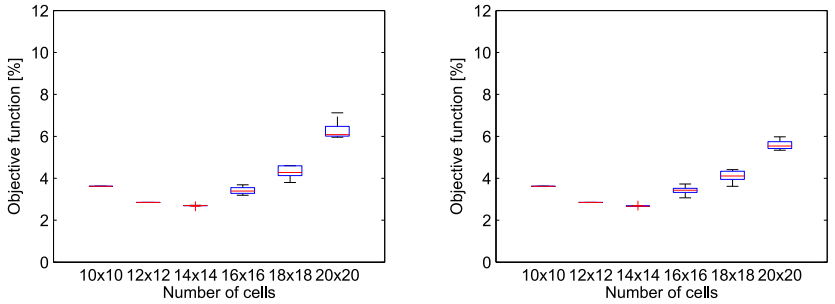

Fig. 8. Result (theoretical case) of the classical formalism without (left) and with (right) adaptation when the number of cells varies

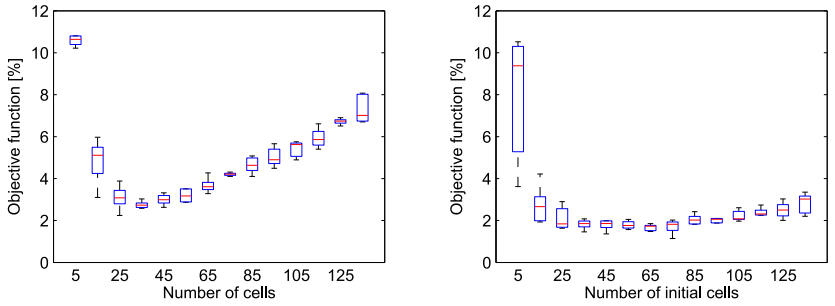

Fig. 9. Result (theoretical case) of the Voronoi formalism without (left) and with (right) adaptation when the number of cells varies

It appears that, when a static formalism is used, or when a dynamic formalism is used without adaptation, quality of the final solution returned by the genetic algorithm is very sensitive to the initial number of cells, the sweet spot for this particular problem being around a 14×14 grid or 25-35 Voronoi cells. However, the fourth combination, using a dynamic formalism with adaptations, is clearly much less sensitive to the initial conditions. Recall that this is the only variant where the number of cells can vary from individual to individual. We ascribe its better behaviour to this feature. Indeed, checking the number of cells present in the final solution confirms that this number naturally increases (resp. decreases) when it initially is too low (resp. too high). It is also worth noting that the absolute best objective function among all experiments (around 1.5%) is obtained by this fourth variant.

To conclude this section, we validate these observations on the practical case described at the end of the previous section, with a single run of each of the four versions of the optimization tool, again using a limit of 200 generations. We allocate roughly 200 parameters for both formalisms (a $2 \times 10 \times 10$ grid in the static case, and 67 initial cells in the dynamic case, which corresponds to $3 \times 67 = 201$ design parameters).

Results for the objective function reported in Table 1 are consistent with observations made on the theoretical case (objective values are normalized with respect to the baseline static case without adaptation). The advantage of the dynamic formalism over its static counterpart even seems to be larger than for the theoretical case, with solutions whose objective function is nearly an order of magnitude better than those obtained with the static formalism. Usefulness of the algorithm adaptation is also confirmed, at least in the case of the dynamic formalism.

Distribution formalism	Static	Static	Dynamic	Dynamic
Reproduction mechanisms	Standard	Adapted	Standard	Adapted
Objective function (normalized)	1.00	0.90	6.62	7.20

Table 1. Objective functions obtained after 200 generations in the practical case.

Finally, Fig. 10 displays solutions obtained in the two extreme cases: static formalism with standard reproduction mechanisms (left) and dynamic formalism coupled with adapted mechanisms (right). They suggest that the initial number of cells was too high in the static case, preventing the tool to converge over the course of the 200 generations (observe e.g. the mixture of materials in the lower part of the solution). The solution produced in the second case seems much closer to a stable design. However, the initial number of Voronoi cells was apparently not enough since it rose from 67 to 78 during the optimization. This confirms the observation that the optimization tool based on a combination of a dynamic formalism and an adapted optimization algorithm is much more robust with respect to variations in the initial number of cells.

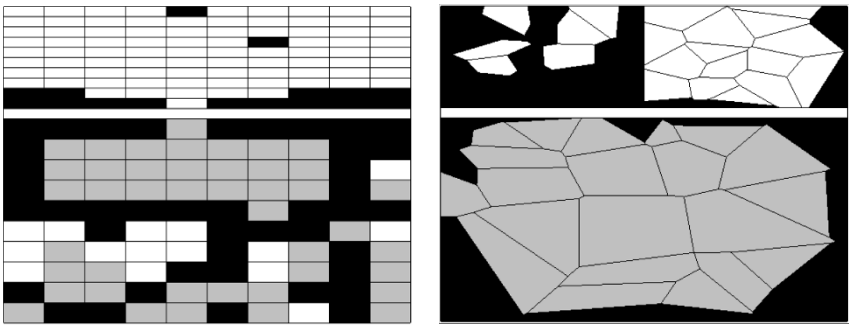

Fig. 10. Actuator design for the practical case obtained with a non-adapted static formalism (left) and adapted dynamic formalism (right).

To conclude, we relate our work with that of Schoenauer et al. (see e.g. [10, 7]), which demonstrates the potential of evolutionary algorithms when applied to the topology optimization of mechanical structures. We confirm their observation that the use of a dynamic formalism with adapted algorithms is beneficial for topology optimization, both on a theoretical case and on a practical application in electromagnetic design.

Our works differs however in several aspects: instead of waiting for convergence of the algorithm, which is unrealistic in many practical situations, we enforce a limit on the number of generations. We demonstrate that the initial number of cells provided to the algorithm is a key parameter influencing the quality of the final solution, but that it cannot be determined a priori. Nevertheless, we show that the quality of the solutions returned by our Voronoi-adapted variant is, through a regulation mechanism on the number of cells, less dependent on the initial number of cells while it converges towards better solutions.

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