Evolutionary Approaches to Joint Nash – Pareto Equilibria

D. Dumitrescu, Rodica Ioana Lung, and Tudor Dan Mihoc

Abstract. A new type of equilibrium incorporating different rationality types for finite non cooperative games with perfect information is introduced. The concept of strategic game is generalized in order to admit players with different rationalities. Generative relations are used to characterize several types of equilibria with respect to players rationality. An evolutionary technique for detecting it is considered. Numerical experiments show the potential of the method.

1 Introduction

Equilibrium concepts are the most common solutions proposed in game theory. In a particular game it is usually considered that players interact according to a unique equilibrium concept, i.e. only players guided by the same kind of equilibrium are allowed to interact. This restriction induces unrealistic predictions. For example, the concept of Nash equilibrium sometimes can lead to deceptive results [5].

In real life players (agents) can be more or less cooperative, more or less competitive and more or less rational. In order to cope with more complex situations a concept of generalized game is presented. Players are allowed to have different behaviors/rationality types considering an adequate meta-strategy concept.

According to [3] game equilibria can be characterized using appropriate generative relations. Thus Nash equilibrium is characterized by the ascendancy relation [8] and Pareto equilibrium by the Pareto domination. Combining the two relations may lead to different types of joint Nash–Pareto equilibria.

An evolutionary technique for detecting the joint Nash–Pareto equilibrium for the generalized game is used.

D. Dumitrescu · Rodica Ioana Lung · Tudor Dan Mihoc
Babes Bolyai University
e-mail: ddumitr@cs.ubbcluj.ro, rodica.lung@econ.ubbcluj.ro, mihoct@cs.ubbcluj.ro

J.R. González et al. (Eds.): NICSO 2010, SCI 284, pp. 233–243, 2010.
springerlink.com

2 Generalized Games

In order to cope with different rationality types the concept of generalized game is defined [3].

Definition 2.1. A finite strategic *generalized game* is defined as a system by $G = (N, M, U)$ where:

- $N = \{1, ..., n\}$, represents the set of players, n is the number of players;
- for each player $i \in N$, S_i represents the set of actions available to him, $S_i = \{s_{i_1}, s_{i_2}, ..., s_{i_{m_i}}\}$; $S = S_1 \times S_2 \times ... \times S_N$ is the set of all possible situations of the game;
- for each player $i \in N$, M_i represents the set of available meta-strategies, a meta-strategy is a system $(s_i|r_i)$ where $s_i \in S_i$ and r_i is the i^{th} player rationality type;
- $M = M_1 \times M_2 \times ... \times M_N$ is the set of all possible situations of the generalized game and $(s_1|r_1, s_2|r_2, ..., s_n|r_n) \in M$ is a meta-strategy profile.
- for each player $i \in N$, $u_i : S \to \mathbf{R}$ represents the payoff function.

$$U = \{u_1, ..., u_n\}.$$

Remark 2.1. In a generalized game the set of all possible meta-strategies represents the meta-strategy search space.

3 Generative Relations for Generalized Games

Three generative relations are considered in this section. Two of them correspond to Pareto and Nash equilibria. The third induces a new type of joint Nash–Pareto equilibrium.

3.1 n_P–Strict Pareto Domination

We introduce the n_P–strict Pareto domination in order to be able to combine the concepts of Nash and Pareto domination.

In a finite strategic generalized game consider the set of players Pareto biased

$$I_P = \{j \in \{1, ..., n\} | r_j = \text{Pareto}\}$$

and $n_P = card\ I_P$, where *card* A denotes the number of elements in the set A.

Let us consider two meta strategy profiles x and y from M.

Definition 3.1. The meta strategy profile x n_P–strict Pareto dominates the meta strategy profile y if the payoff of each Pareto biased player from I_P using meta strategy x is strictly greater than the payoff associated to the meta strategy y, i.e.

$$u_i(x) > u_i(y),\ \forall i \in I_P.$$

Remark 3.1. The set of non dominated meta strategies with respect to the n_P–strict Pareto domination relation when $n_P = n$ is a subset of the Pareto front.

3.2 Nash – Ascendancy

Similar to Pareto equilibrium a particular relation between strategy profiles can be used in order to describe Nash rationality. This relation is called Nash-ascendancy (NA).

A strategy is called Nash equilibrium [7] if each player has no incentive to unilaterally deviate i.e. it can not improve the payoff by modifying its strategy while the others do not modify theirs.

We denote by (s_{i_j}, s^*_{-i}) the strategy profile obtained from s^* by replacing the strategy of player i with s_{i_j} i.e.

$$(s_{i_j}, s^*_{-i}) = (s^*_1, s^*_2, ..., s^*_{i-1}, s_{i_j}, s^*_{i+1}, ..., s^*_1).$$

Definition 3.2. The strategy profile x Nash-ascends the strategy profile y, and we write $x <_{NA} y$ if there are less players i that can increase their payoffs by switching their strategy from x_i to y_i then vice versa.

In [8] is introduced an operator

$$k : S \times S \to \mathbf{N},$$

$$k(y,x) = card\{i \in \{1,...,n\} | u_i(x_i, y_{-i}) \geq u_i(y), x_i \neq y_i\}.$$

$k(y,x)$ denotes the number of players which benefit by switching from y to x.

Proposition 3.1. *The strategy x Nash-ascends y (x is NA-preferred to y), and we write $x <_{NA} y$, if the inequality*

$$k(x,y) < k(y,x),$$

holds.

According to [8] the set of all strategies from S non-dominated by respect of Nash ascendancy relation equals the set of Nash equilibria.

This result proves that the Nash ascendancy is the generative relation for the Nash equilibrium.

3.3 Joint Nash–Pareto Domination

Let us consider two meta-strategies

$$x = (x_1|r_1, x_2|r_2, ..., x_n|r_n) \text{ and } y = (y_1|r_1, y_2|r_2, ..., y_n|r_n).$$

Let us denote by I_N the set of Nash biased players (N-players) and by I_P the set of Pareto biased players (P-players). Therefore we have

$$I_N = \{i \in \{1,...,n\} | r_i = \text{Nash}\}.$$

We consider the operators k_P and k_N defined as:

$$k_P(x,y) = card\{j \in I_P | u_j(x) > u_j(y), x \neq y\}$$

and respectively

$$k_N(x,y) = card\{i \in I_N | u_i(y_i, x_{-i}) \geq u_i(x), x_i \neq y_i\}.$$

Remark 3.2. $k_P(x,y)$ measures the *relative efficiency* of the meta strategies x and y with respect to Pareto rationality and $k_N(x,y)$ measures the *relative efficiency* of the meta strategies x and y with respect to Nash rationality.

Definition 3.3. The meta strategy x N–P dominates the meta strategy y if and only if the following statements hold

1. $k_P(x,y) = n_P$
2. $k_N(x,y) < k_N(y,x)$

In what follows we consider that efficiency relation induces a new type of equilibrium called *joint Nash-Pareto equilibrium.*

Remark 3.3. Joint Nash-Pareto equilibrium defined in this section is a concept completely different from the existing concept of Pareto-Nash equilibria [10].

4 Detecting Joint N–P Equilibria in Generalized Games

Consider a three player non-cooperative game. Let r_i be the rationality type of player i.

If $r_1 = r_2 = r_3 =$ Nash then all players are Nash biased and the corresponding solution concept is the Nash equilibrium.

If $r_1 = r_2 = r_3 =$ Pareto then all players are Pareto biased and the corresponding equilibria are described by the set of strictly non dominated strategies (Pareto front).

We also intend to explore the joint cases where one of the players is Nash biased and others are Pareto and the one where one is Pareto and the others are Nash biased.

In order to detect the joint Nash–Pareto equilibria of the generalized game an evolutionary approach is used.

Let us consider an initial population $P(0)$ of p meta strategies for the generalized three player game. Each member of the population has the form

$$x = (s_1|r_1, s_2|r_2, s_3|r_3).$$

Pairs of meta-strategies are randomly chosen from the current population $P(t)$. For each pair a binary tournament is considered. The meta strategies are compared by means of the domination relation. An arbitrary tie breaking is used if the two meta strategies have the same efficiency. The winers of two binary tournaments are recombined using the simulated binary crossover (SBX) operator [11] resulting two offspring. Offspring population is mutated using real polynomial mutation [2], resulting an intermediate population P'. Population $P(t)$ and P' are merged.

The resulting set of meta strategies is sorted with respect to the efficiency relation using a fast non dominant sorting approach [2]. For each meta strategy M' the number expressing how many meta strategies in the merged population are less efficient then M' is computed. On this basis the first p meta strategies are selected from the merged population. Selected meta strategies represent the new population $P(t+1)$.

Let us remark that in the proposed technique selection for recombination and survival is driven by the efficiency relation. Therefore the population of meta-strategies is expected to converge toward the joint Nash–Pareto front. According to the proposed approach the members of this front represent the joint N–P equilibria of the generalized game.

5 Numerical Experiments

In order to illustrate the proposed concepts the oligopoly Cournot model is considered (see for instance [6]).

Let q_1, q_2 and q_3 denote the quantities of a product. This unique product is produced by three companies. The market price denoted by $P(Q)$ is given by

$$P(Q) = \begin{cases} a - Q, & \text{for } Q < a, \\ 0, & \text{for } Q \geq a. \end{cases}$$

where

$$Q = q_1 + q_2 + q_3,$$

is the aggregate quantity on the market and $a > 0$ is a constant characterizing the market.

The cost for the company i of producing q_i units is $C_i(q_i)$

$$C_i\ (q_i) = c_i q_i,$$

where $c_i < a$. Suppose that the companies choose their quantities simultaneously. The payoff for the company i is its profit, which can be expressed as:

$$\begin{aligned} \pi_i(q_1, q_2, q_3) &= q_i P(Q) - C_i(q_i) \\ &= q_i \left[a - (q_1 + q_2 + q_3) - c_i\right], i = 1, 2, 3. \end{aligned}$$

A game strategy is a triple

$$s = (q_1, q_2, q_3).$$

Several experiments have been performed for this game by using RED technique [3].

5.1 Symmetric Games

The symmetric Cournot model with parameters $a = 24$ and $c_1 = c_2 = c_3 = 9$ is considered.The payoff corresponding to Nash equilibrium is (14.00, 14.00, 14.00).

Table 1 Average payoff and standard deviation (St, Dev.) of the final populations in 30 runs with 100 meta-strategies after 30 generations for the symmetric Cournot model where all three players are Nash biased

N-N-N	Average payoff			St. dev.			Maximum payoff			Minimum payoff		
player	p1	p2	p3	p1	p2	p3	p1	p2	p3	p1	p2	p3
Average	14,05	14,06	14,05	0,03	0,04	0,04	14,85	15,57	15,00	12,25	12,49	12,45
St. Dev.	0,02	0,02	0,02	0,08	0,09	0,08	1,39	2,80	1,83	3,25	3,00	3,05

According to the data from the Table 1 in less than 30 generations the algorithm converges to the Nash equilibrium point.

Table 2 Average payoff and standard deviation (St. Dev.) of the final populations in 30 runs with 100 meta-strategies after 30 generations for the symmetric Cournot model where two player are Nash biased and one is Pareto

N-N-P	Average payoff			St. dev.			Maximum payoff			Minimum payoff		
player	p1	p2	p3	p1	p2	p3	p1	p2	p3	p1	p2	p3
Average	10,99	11,01	29,80	52,81	53,02	182,28	25,92	25,71	56,24	0,00	0,00	0,49
St. Dev.	0,36	0,33	0,78	1,75	2,33	17,62	0,92	0,88	0,00	0,00	0,00	1,67

The resulting front in the Nash-Nash-Pareto case spreads from the standard Nash equilibrium corresponding to the two player–Cournot game (25.00, 25.00) to the Nash equilibrium corresponding to the three player–Cournot game, and from there to the edges of Pareto front. The equilibrium set is depicted in Figure 1 and Figure 2 from two angles, for a better view. The numerical results are presented in Table 2.

As we can see in the Figure 3 in the Nash-Pareto-Pareto case the result is similar to the Pareto front, an result that is determined by the strength of the Pareto component in the generative relation for the joint Nash–Pareto equilibrium. As we can see in Table 3 the minimum values for all three players are 0.00 and the maximum are 56.24, the same like the ones for the Pareto front.

5.2 Asymmetric Games

First, let us consider the two player asymmetric Cournot game with parameters $a = 24$, $c_1 = 9$ and $c_2 = 12$.

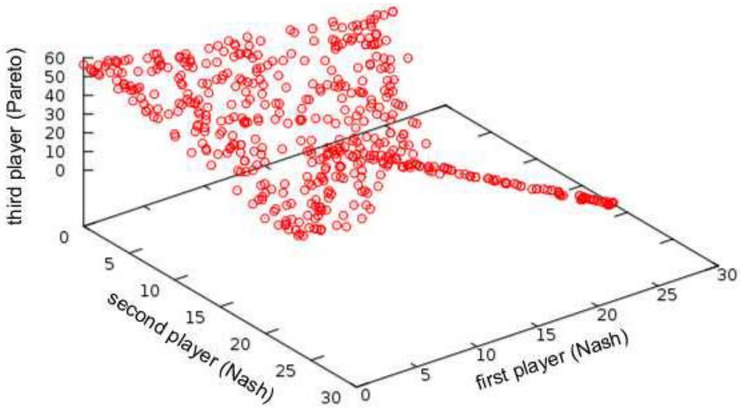

Fig. 1 The payoffs for the Nash-Nash-Pareto front detected in less than 30 iterations for the symmetric Cournot game

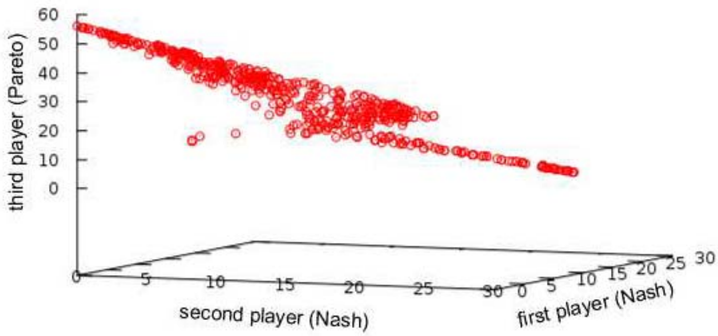

Fig. 2 The payoffs for the Nash-Nash-Pareto front detected in less than 30 iterations for the symmetric Cournot game

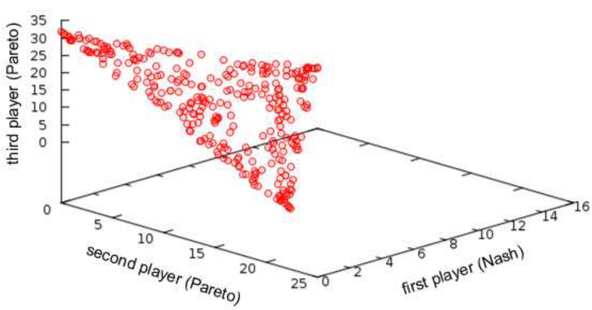

Fig. 3 The payoffs for the Nash-Pareto-Pareto front detected in less than 30 iterations for the symmetric Cournot game

Table 3 Average payoff and standard deviation (St. Dev.) of the final populations in 30 runs with 100 meta-strategies after 30 generations for the symmetric Cournot model where one player is Nash biased and the other two Pareto

N-P-P	Average payoff			St. dev.			Maximum payoff			Minimum payoff		
player	p1	p2	p3	p1	p2	p3	p1	p2	p3	p1	p2	p3
Average	17,74	18,52	18,44	242,42	247,83	247,97	56,23	56,24	56,24	0,00	0,00	0,00
St. Dev.	0,40	0,36	0,42	7,36	6,98	6,82	0,04	0,00	0,00	0,00	0,00	0,00

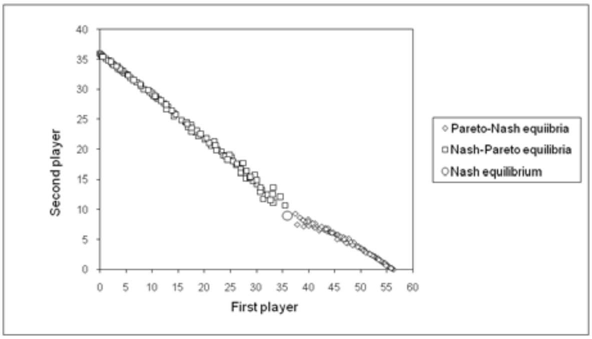

Fig. 4 The payoffs for the Nash-Pareto, Pareto-Nash and Nash-Nash equilibria approximations detected after 30 iterations for Cournot's model with $a = 24$, $c_1 = 9$, and $c_2 = 12$

Table 4 Numerical results for the two asymmetric games with a Nash-Nash-Pareto rationality in final population in 30 runs, for 100 meta strategies and after 30 iterations

N-N-P	First game			Second game		
	$c_1 = 9$	$c_2 = 12$	$c_3 = 9$	$c_1 = 9$	$c_2 = 12$	$c_3 = 5$
Player	p1	p2	p3	p1	p2	p3
Average payoff	17,19	2,72	46,86	16,12	3,93	29,19
St. Dev.	10,59	2,49	25,90	11,29	2,67	15,45
Minimum payoff	0,00	0,00	0,18	0,00	0,00	0,11
Maximum payoff	36,72	9,24	90,25	36,61	9,43	56,25

In Figure 4 are depicted the payoff functions for two players. The results are in concordance with those obtained in [3]. The difference between c_1 and c_2 determines an asymmetry for the represented detected equilibria.

The asymmetric three player Cournot games with parameters $a = 24$, $c_1 = 9$, $c_2 = 12$, $c_3 = 5$ and respectively $a = 24$, $c_1 = 9$, $c_2 = 12$, $c_3 = 9$ are considered. The asymmetries allow us to better understand the players behavior in the joint Nash–Pareto equilibrium.

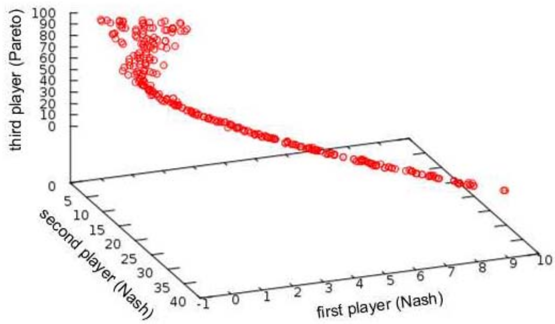

Fig. 5 The payoffs for the Nash-Nash-Pareto front detected in less than 30 iterations for the asymmetric Cournot game with parameters $a = 24$, $c_1 = 9$, $c_2 = 12$, $c_3 = 5$

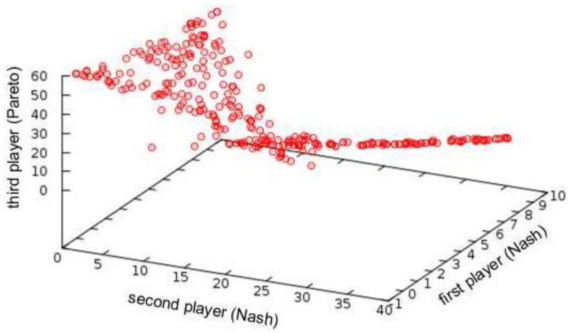

Fig. 6 The payoffs for the Nash-Nash-Pareto front detected in less than 30 iterations for the symmetric Cournot game with parameters $a = 24$, $c_1 = 9$, $c_2 = 12$, $c_3 = 9$

Comparing Figure 5 and Figure 6 one can observe the influence of the Pareto component in to the final front. If parameter c_3 is close to parameters c_1 and c_2 the Pareto influence determines the set so spread out in a plane similar to the pure Pareto front. The distribution between the Nash equilibrium for two players (as the third gains nothing) and three players Nash equilibrium remains also for the asymmetric game.

Analyzing the results one can observe that for these particular cases of joint Nash–Pareto rationalities, symmetric or not, there is no minimum guaranteed payoff for any rationality.

As regarding the maximum payoffs an interesting feature appears if we construct a new game based on the maximum payoffs. The players strategies will be their rationality (Nash or Pareto) and their payoffs the maximum gains in the joint Nash-Pareto equilibria. Solving this game in GAMBIT the pure strategies Nash equilibrium is the Pareto-Pareto-Pareto rationality.

6 Conclusions and Future Work

A concept of generalized game is used in order to capture the behavior of players with several types of rationalities. A new generative relation between meta strategies induces a new solution concept called joint Nash–Pareto equilibrium. Proposed method allows the combination of different types of equilibria in a game.

An evolutionary technique for detecting an approximation of the generalized equilibria is used. The idea are exemplified for Cournot games with three players and two types of rationality.

Results indicate the potential of the proposed technique. Different analyses prove that for the presented games there is no minimal payoff but the possibility of a maximum gain is obtained for the Pareto rationality. These experimental results offer an inside view of the problems arising when two different type of equilibria are considered in the same game.

Future work will address generalized games having other rationality types then Nash and Pareto and other methods of combining them.

Acknowledgements. This research is supported partially by the CNCSIS Grant ID508 *"New Computational paradigms for dynamic complex problems"* funded by the MEC and from the SECTORAL OPERATIONAL PROGRAMME HUMAN RESOURCES DEVELOPMENT, Contract POSDRU 6/1.5/S/3 "Doctoral studies: through science towards society", Babeş - Bolyai University, Cluj - Napoca, România.

References

[1] Bade, S., Haeringer, G., Renou, L.: More strategies, more Nash equilibria, Working Paper 2004-15, School of Economics University of Adelaide University (2004)

[2] Deb, K., Agrawal, S., Pratab, A., Meyarivan, T.: A Fast Elitist Non-Dominated Sorting Genetic Algorithm for Multi-Objective Optimization: NSGA-II. In: Schoenauer, M., Deb, K., Rudolph, G., Yao, X., Lutton, E., Merelo, J.J., Schwefel, H.-P. (eds.) PPSN 2000. LNCS, vol. 1917, pp. 849–858. Springer, Heidelberg (2000)

[3] Dumitrescu, D., Lung, R.I., Mihoc, T.D.: Evolutionary Equilibria Detection in Non-cooperative Games. In: Giacobini, M., Brabazon, A., Cagnoni, S., Di Caro, G.A., Ekárt, A., Esparcia-Alcázar, A.I., Farooq, M., Fink, A., Machado, P. (eds.) EvoCOMNET. LNCS, vol. 5484, pp. 253–262. Springer, Heidelberg (2009)

[4] McKelvey, R.D., McLennan, A.: Computation of equilibria in finite games. In: Amman, H.M., Kendrick, D.A., Rust, J. (eds.) Handbook of Computational Economics. Elsevier, Amsterdam (1996)

[5] McKelvey, R.D., Palfrey, T.: An experimental study of the centipede game. Econometrica 60(4), 803–836 (1992)

[6] Lung, R.I., Muresan, A.S., Filip, D.A.: Solving multi-objective optimization problems by means of natural computing with application in finance. In: Aplimat 2006, Bratislava, February 2006, pp. 445–452 (2006)

[7] Nash, J.: Non-cooperative games. Annals of Mathematics 54, 286–295 (1951)

[8] Lung, R.I., Dumitrescu, D.: Computing Nash Equilibria by Means of Evolutionary Computation. Int. J. of Computers, Communications & Control, 364–368 (2008)

[9] Osborne, M.J., Rubinstein, A.: A Course in Game Theory. MIT Press, Cambridge (1994)
[10] Maskin, E.: The theory of implementation in Nash equilibrium:A survey. In: Hurwicz, L., Schmeidler, D., Sonnenschein, H. (eds.) Social Goals and Social Organization, pp. 173–204. Cambridge University Press, Cambridge (1985)
[11] Deb, K., Beyer, H.: Self-adaptive genetic algorithms with simulated binary crossover. Complex Systems 9, 431–454 (1995)