A Dynamical Game Model for Sustainable Development

D. Dumitrescu and Andrei Sîrghi

Abstract. The paper addresses the possibility to combine nature inspired optimization techniques, Dynamical Systems and Game Theory in order to solve a complex real-world problem. A computational model for *Sustainable Development* (SD) problem, called *Dynamical Game for Sustainable Development* (DGSD) is proposed. This model combines ideas from *Dynamical Systems* and *Game Theory* in a new paradigm for adaptive behavior of systems. The actors of SD: *Economy*, *Environment* and *Society*, are viewed as evolvable systems. The main aim is to ensure a balanced coevolution of SD actors. A chain of control points are used to guide the evolution toward system equilibrium (*sustainability*). Each control point is represented as a three player game. In order to guide system to *sustainability*, the local equilibrium at each control point is used to determine further development strategies for SD actors. The local equilibrium is conceived as a game equilibrium. Several kinds of equilibria are possible. For detecting these equilibria an evolutionary approach may be used.

Introduction

The concept and methodology of sustainable development [3] appeared over the past few decades as result to a set of interdependent issues like: *climate change*, *pollution control*, *preservation of biodiversity*, etc. Crises, degradation and risks affecting human health, social and economic stability have fostered public suspicions on the evolution of technology and economic growth. These suspicions gave rise to this new concept, and further, a new branch of science. SD concept lead us to principles of organizing and controlling the development and complex interactions

D. Dumitrescu · Andrei Sîrghi
Center for the Complexity Studies, Babeş-Bolyai University,
Department of Computer Science, Babeş-Bolyai University,
Cluj-Napoca, Romania
e-mail: ddumitr@cs.ubbcluj.ro, andreisirghi@yahoo.com

J.R. González et al. (Eds.): NICSO 2010, SCI 284, pp. 151–162, 2010.
springerlink.com

between society, production activities and natural resources in such a way that results in a constructive coexistence between these three big areas.

The paper proposes a new mathematical model for SD problem. This model, called *Dynamical Game for Sustainable Development* (DGSD), combines ideas from Game Theory and Dynamical Systems in a new paradigm for self-organizing coevolutive systems. We use *Dynamical Systems* to describe the evolution of each area included in SD. *Game Theory* is used to model the decision process of our model in each control point. Each control point represents a three player game, corresponding to SD actors. DGSD model is intended to show how much, the decisions taken in one area influence the other areas of the region.

The conflict of interests between players and limited amount of resources in the region, enforce players to be very "careful" in the process of decision making. If a player does not consider this principle of "carefulness", and as result, exploits resources from the other areas, his further decisions are constrained by big limits in resource usage imposed by the other players. Therefore, the area represented by this player collapses.

The principle of *carefulness* represents an important factor of self-organization which ensures a balanced systems coevolution for achieving SD goals.

The DGSD model has an adaptive behavior. This behavior is ensured by the game, which finds compromise solutions between the actors in any situation of region development described by a control point. The goal of these compromise solutions is to converge region development to SD. Each compromise represents the game equilibrium in a specific point of region development. To obtain game equilibrium, we consider an evolutionary approach based on generative relations [5].

1 Sustainable Development Problem

At the base of SD concept stays the principle that objectives of *Society*, *Economy* and *Environment* should be complementary and interdependent in the development process of a region.

The problem related to SD which we propose to solve in this paper can be defined as:

Create a mathematical model having the next characteristics:

1. Represents the real development process of a region from the three aspects corresponding to the major areas: *Economy*, *Environment* and *Society*;
2. Suitable for the robust prediction of the future state and behavior of the real process;
3. Valuable in the real decision making process;
4. Every area has a particular set of objectives and decision functions;
5. Proposes strategies and control elements that leads region to Sustainable Development.

Our aim is to develop such a model by considering a specific region and supposing to have complete information about economy, environment and society.

2 Related Work

SD problem received high attention from its first apparition, but until now does not exist a powerful mathematical model that can be used to represent this problem. The result of researches of almost all communities that analyze this problem is a set of indicators which can be used to measure the quality of sustainable development process for specific regions.

There are two widely accepted methods to measure the sustainable development of a country:

1. Sustainable Development Gauging Matrix (SDGM) [11]. The measure technique of SDGM consists in the aggregation of three dimension indices: economic (I_{ec}), ecological (I_e) and social (I_s) in the index of sustainable development (I_{sd}). Further, each of these indices is calculated by using other six global indices widely used in Statistics communities.

2. IPAT equation [1]. Expresses the relationship between technological innovation and environmental impact. *IPAT* states that human impact (I) on the environment equals the product of population (P), affluence (A - consumption per capita) and technology (T - environmental impact per unit of consumption).

3 Dynamical Systems of Areas Evolution

To describe the evolution of SD areas we use a system of dynamical models. Each area included in SD has an own model of evolution, which contains one or more dynamical functions, and interacts with the models of the other areas. Every particular model has two types of parameters:

- internal parameters - that are indices of the area represented by the model and which describe the evolution of the area, and
- external parameters - that are important in decision making process and indicates the dependences between the current area and the other areas. These parameters represent a key element in the process of SD self-organization because they are used by individual areas to influence the other areas.

The DGSD model is extensive, and it can be used with different evolution models of the areas, depending on different circumstances. In this paper we work with an abstract model which reflects the basic relations between SD areas and their structures. This model can be simple replaced by a more descriptive one in specific cases.

The abstract model is built as a system of individual functions of evolution for sustainable development areas: *Economy*, *Environment* and *Society*. The correlations between particular functions are very important for our approach. They represent the base criteria to analyze and control the sustainability of region development. Further we describe the functions of areas evolution and their correlations.

Economy plans the optimal amount of products outcome by choosing corresponding quantities of capital ($K(t)$), nonrenewable natural resources ($h(t)$), renewable natural resources ($r(t)$) and social capital or labor ($l(t)$). The function of economic development (denoted *EC*) may be represented as a dynamical system given by a particular *production function* [9] *ec*:

$$EC(t+1) = ec(K(t), h(t), r(t), l(t)). \tag{1}$$

where t stands for time period ($t = [t_0, T]$).

The production function *ec* in the *Cobb-Douglas* [7] form is:

$$\begin{cases} ec(K,h,r,l) = AK(t)^{\alpha}h(t)^{\beta}r(t)^{\gamma}l(t)^{\delta} \\ \alpha+\beta+\gamma+\delta = 1 | \alpha,\beta,\gamma,\delta \in (0,1] \end{cases}, \tag{2}$$

where A represents *total factor productivity*, and the exponents α,β, γ, and δ represent the elasticities of production related to capital, nonrenewable resources, renewable resources and labor respectively.

Accumulated capital stock evolution depends on rate of capital deprecation (σ), and economic products consumption ($c(t)$):

$$K(t+1) = (1-\sigma)K(t) + ec(K(t), h(t), r(t), l(t)) - c(t). \tag{3}$$

The goal of the *Economy* is to maximize production in condition of sustainability which implies an activity constrained by actual and future benefits of all areas. The optimization problem of this area may be represented as:

$$\begin{cases} ec(K,h,r,l) \to max \\ \text{subject to sustainability (SD) constraints.} \end{cases} \tag{4}$$

Environment tries to achieve a sustainable trajectory in the development of nonrenewable (R_n) and renewable (R_r) resources stocks by restricting as much as possible the natural resources consumption. *Environment* development may be represented as a dynamical system which represents the evolution of natural resources stocks:

$$\begin{cases} R_n(t+1) = R_n(t) - h(t) \\ R_r(t+1) = R_r(t) - r(t) + g(R_r(t) - r(t)) \end{cases}, \tag{5}$$

where g represents *Environment*'s regenerative capacity and can have multiple forms [2].

The goal of *Environment* is to maximize stock of renewable resources and to preserve actual stock of nonrenewable resources by imposing limits in resources consumption:

$$\begin{cases} R_n \sim const \\ h(t) \to h_n(t) \\ R_r \to max \\ r(t) \to r_n(t) \end{cases}, \tag{6}$$

where $h_n(t)$ is the limit of nonrenewable resources consumption imposed by the *Environment* to *Economy*, and $r_n(t)$ is the limit of renewable resources consumption.

The *Environment* and the *Economy* influence the living conditions in the region, which can be suitable or not for people life. Analyzing these conditions, the *Society* has to choose, to stay in this system or not.

Society's goal is to achieve normal values for indicators of social development such as: birthrate, mortality rate, migration rate and unemployment rate. *Society* development may be represented as the dynamical system:

$$S(t+1) = S(t) + s(R_n(t), R_r(t), \omega_m(t), \omega_b(t), l(t), c(t)), \tag{7}$$

where $S(t)$ represents society size in period t, and s represents *society growth*. Society growth depends on natural resources availability, mortality rate ($\omega_m(t)$), birthrate ($\omega_b(t)$), used labor and economic products consumption. A simple form of society growth function can be represented as:

$$s = \omega_b(t)S(t) - \omega_m(t)S(t) - m(u_r(t), R_n(t), R_r(t), c(t))S(t), \tag{8}$$

where $m(u_r(t), R_n(t), R_r(t), c(t))$ represents migration rate, and is influenced by unemployment rate (u_r), per capita consumption ($c(t)/S(t)$), and per capita natural resources availability($R_n(t)/S(t)$ and $R_r(t)/S(t)$).

The goal of the *Society* is to maximize the living conditions in the region. This goal may be expressed as the system:

$$\begin{cases} \omega_m \to \omega_{nm} \\ \omega_b \to \omega_{nb} \\ m \to nm \\ u_r \to u_{nr} \end{cases}, \tag{9}$$

where ω_{nm} is the normal rate of mortality, ω_{nb} is the normal birthrate, nm is the normal migration rate and u_{nr} is the normal unemployment rate.

The areas goals are mutually contradictory. Thus a sustainable strategy must coordinate and manage the development process of these areas in a balanced manner, which must result in the long-term viability of the system. In following Section we analyze how these goals can be balanced using Game Theory.

4 Sustainable Development Game

At the beginning of each development iteration t, the SD actors should propose a development strategy for their specific areas. Within DGSD model, Game Theory is used to represent the decision process of SD actors in each control point of region evolution. The SD game involves three players, or agents: *Economy* (*EC*), *Environment* (*EV*) and *Society* (*S*). Using this game, each agent chooses best available development strategy by combining the information about the state of their area with their "belief" about the behavior of the other agents.

4.1 Extended Form of the SD Game

SD decision process may be represented as an extended form game by using a tree which levels corresponds to the player information sets (see Fig. 1).

Player EC has two pure strategies:

1. to choose a quantity of natural resources that follow environmental standards. Let us denote this pure strategy *E*;
2. to exploit environment resources - strategy denoted *NE*.

EC is represented by the root of the game tree, as depicted in Fig. 1.

The second information set in the game tree is denoted by EV and represents the *Environment*. EV has two nodes:

1. n_1, which represents the behavior, or potential actions of *Environment* when *Economy* plays *E*, and
2. n_2, that represent the behavior of EV when *Economy* plays *NE*.

EV assigns the probability x to the node n_1 and the probability $(1-x)$ to the node n_2. This means that EV "*believes*" that EC will choose a good environmental policy with probability x.

Environment has two pure strategies:

1. to be suitable for human life and for economy (to restrict as much as possible natural resource consumption). Let us call this strategy *ST*;
2. to be not suitable - strategy *NST*.

The third player in the game is the *Society*. This player has two information sets:

1. l_1, which corresponds to the choice *E* of economic agent, and
2. l_2, which corresponds to the choice *NE*.

Each set has two nodes: l_{11} and l_{12} for l_1, and l_{21} and l_{22} for l_2. l_{11} and l_{21} follow the decision *ST* of the *Environment* agent. The nodes l_{12} and l_{22} follow the pure strategy *NST* of the *Environment*.

Society has two pure strategies:

1. to stay in this region, to live and to work here - strategy denoted by *L*;
2. to escape from the region - strategy denoted by *NL*.

In SD game, *Economy* plans the optimal production quantity for the next period of region development, by choosing either an economic strategy that follows environmental standards (*E*), or one strategy that destroys the environment (*NE*). Then *Environment* must move. *Environment* move can be suitable for people life and for economy (strategy *ST*) or not (strategy *NST*). But *Environment* is not informed about EC choice, it has just a belief about behavior of EC. Selecting an appropriate strategy, the *Environment* imposes the admissible values of resource usage for EC. Eventually, the *Society* must choose to live in this system (strategy *L*), or not (strategy *NL*), without any information about the other players moves, excluding its belief.

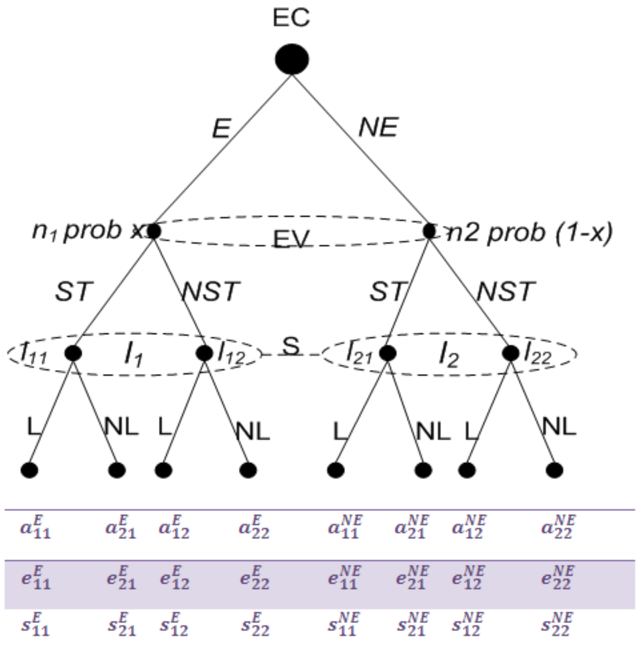

Fig. 1 SD Game Tree: tree representation of the sustainable development game

By choosing a strategy, each agent controls the development of their area, and influences the development of the other areas through external parameters of dynamical models presented in Section 3. Economy plans the development strategy by choosing corresponding quantities of natural resources, labor and capital, thus influencing Environment (through natural resources consumption) and Society (through used labor). Environment controls the quantities of natural resources that can be used by Economy, and can impose a penalty for natural resources overconsumption to Economy. Therefore, Environment influences the Economy through the limits in natural resources usage, and the Society through the availability of natural resources per capita. Finally, Society can influence the Economy and Environment through migration.

4.2 Payoffs in the SD Game

Vector (a,e,s) from SD Game Tree (see Fig. 1) represents player payoffs, where real numbers a^i_{hk}, $i \in \{E, NE\}$, $h,k \in \{1,2\}$ represents the payoff of EC corresponding to different strategy profiles, analogously, e^i_{hk}, s^i_{hk} represents the payoffs corresponding to the *Environment* (EV) and *Society* (S) respectively. *Environment* assigns probability x to play the game given by strategy E of *Economy* and $(1-x)$ to play the game given by strategy *NE*. Similarly *Society* assigns its belief parameters $(q, 1-q)$,

$0 \le q \le 1$ for information set l_1 and $(q', 1-q')$, $0 \le q' \le 1$ for information set l_2. The probabilities q and q', represents the belief of *Society* that region will be suitable for people life, when *Economy* plays *E*, or *NE* respectively. The belief of economic agent is represented by probabilities $P_x(ST)$ and $P_x(NST)$. $P_x(ST)$ is the belief of *Economy* that the region will be suitable for economic activity. $P_x(NST)$ represents *Economy*'s belief that region will not be suitable for economic activity in next period, and is computed as: $P_x(NST) = 1 - P_x(ST)$.

Expected values for different strategies of each agent may now be computed. The expected value that *Environment* assign to the strategy *ST* is:

$$E_{EV}(ST) = x[qe_{11}^{E} + (1-q)e_{21}^{E}] + (1-x)[q'e_{11}^{NE} + (1-q')e_{21}^{NE}]. \tag{10}$$

Expected value of the *Environment* for the strategy *NST* is:

$$E_{EV}(NST) = x[qe_{12}^{E} + (1-q)e_{22}^{E}] + (1-x)[q'e_{12}^{NE} + (1-q')e_{22}^{NE}]. \tag{11}$$

The actual and future benefit of the *Economy* depends on the states of the *Environment* and *Society*. Analyzing these states and short term and long term goals, the economic agent chooses the most profitable strategy. The expected value that EC assigns to the strategy *E* is:

$$E_{EC}(E) = P_x(ST)[qa_{11}^{E} + (1-q)a_{21}^{E}] + P_x(NST)[qa_{12}^{E} + (1-q)a_{22}^{E}]. \tag{12}$$

Expected value for the strategy *NE* is:

$$\begin{aligned} E_{EC}(NE) = & P_x(ST)[q'a_{11}^{NE} + (1-q')a_{21}^{NE}] \\ & + P_x(NST)[q'a_{12}^{NE} + (1-q')a_{22}^{NE}] - p(h(t), r(t)), \end{aligned} \tag{13}$$

where $p(h(t), r(t))$ represents the penalty paid by *Economy* if the consumption of natural resources is larger than admissible quantity of resource consumption imposed by the *Environment*.

The expected value assigned by *Society* to the strategy *L* is:

$$E_S(L) = q(s_{11}^{E} + s_{12}^{E}) + q'(s_{11}^{NE} + s_{12}^{NE}). \tag{14}$$

Expected value that *Society* assigns to the strategy *NL* is:

$$E_S(NL) = (1-q)(s_{21}^{E} + s_{22}^{E}) + (1-q')(s_{21}^{NE} + s_{22}^{NE}). \tag{15}$$

In order to simplify the representation of SD decision process, we considered each player has to choose a pure strategy. In real life each player usually prefer to play a *mixed strategy game* [10]. Using mixed strategies, players control the intensity of their SD policy which is situated between two limits: sustainable development policy, and unsustainable development policy. An important remark about DGSD model is that the concept of *game strategy* is abstract. In presented game, each player has two strategies, but the intensity of their strategies can be very different, and each strategy can be instantiated from a large set of values.

5 Sustainable Development Game Equilibrium

The development strategies of SD areas are given by the game which is played in each control point. In SD game, we assume that each player chooses the best available strategy. A strategy containing the best choice of each player represents the *Game Equilibrium* (or *Nash Equilibrium*). Game equilibrium depends, in general, on the potential choices of the other players. Each player has to form a hypothesis about the behavior of the concurrent players. For each game, Nash Equilibrium always exists for mixed strategies. Therefore SD game always has an equilibrium, which means, the players always find a compromise solution according to sustainable development criteria.

For each development iteration, every player in SD game chooses a combination between the two available pure strategies. In other words, each player plays the first pure strategy with a probability *p* and the other strategy with the probability (*1-p*).

Hence *Economy* plays *E* with the probability p_{EC}, and *NE* with the probability $(1-p_{EC})$. *Environment* plays *ST* with the probability p_{EV}, and *NST* with the probability$(1-p_{EV})$. Finally, *Society* chooses *L* with probability p_S, and *NL* with the probability $(1-p_S)$.

To compute game equilibrium, an evolutionary technique is considered [4] [5] [6]. The main advantage of this technique is the possibility to model the game having different types of rationality. Depending on these rationalities, multiple types of equilibrium exist: Nash, Pareto, Mixed. Therefore the most representative type of equilibrium may be chosen. To keep the model simple, we consider only the Nash Equilibrium.

Applying this evolutionary technique the Nash equilibrium of the SD game is detected. Detected equilibrium is represented by the set $\{p_{EC}, p_{EV}, p_S\}$. At each iteration, or control point, this set actually redirects the development of the region to a direction formed by the combination of the new goals of players. This combination represents a compromise solution between the players goals. The equilibrium corresponding to the control points actually enforces the players to follow sustainable criteria of development.

6 System Equilibrium

An important concept to study DGSD model is *balanced system evolution* based on SD criteria. In essence, DGSD system evolution is balanced if weak perturbations cause just small variations in the trajectories with respect to desired SD trajectory . The most commonly SD trajectory is that of equilibrium.

Basically, equilibrium of a dynamical system corresponds to a situation where the evolution is stopped or, as in our case, has a stable behavior in the sense that the system states become steady. In this situation we can say that the SD system acquired a sustainable development behavior. The direction of development for DGSD system is guided by its decision process for each iteration. The only way for the system to achieve equilibrium is to take optimal and viable decisions at each development iteration.

Mathematically, this steady state can be described through a system which includes the goals of areas included in SD. This system is described by equations: 4, 6 and 9.

7 Numerical Experiments

We analyze the behavior of our model in two situations:

Situation 1 is described by: *Environment* possesses sufficient stocks of renewable and nonrenewable resources for over 50 iterations, *Economy* has a capital stock for about 10 iterations, *Society* is described by normal values for almost all indicators.

Situation 2 is characterized by: the stock of nonrenewable resources is sufficient for about 30 iterations, the initial stock of renewable resources is sufficient for 2 iterations, and capital stock will be consumed in 2 iterations.

Each situation represents a start state which describes a region in terms of DGSD model. Applying DGSD model on each situation we obtain the evolution of region which is guided by DGSD model toward sustainable development. The evolution of the region for this two situations is presented in Fig. 2 and Fig. 3.

Four indices are used to represent the general sustainability of the region, and the sustainabilities of individual areas: Economy, Environment and Society. Each sustainability index of individual areas shows the state of the area for a development period, relative to the best possible state and the worst possible state. General sustainability of the region represents the arithmetic average of the indices for individual areas. Each sustainability index takes values from the interval [0, 1].

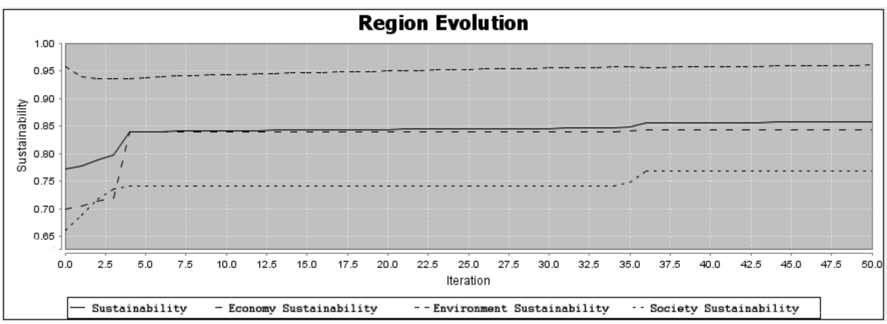

Fig. 2 The evolution of the region in Situation 1. *Economy* starts with an increase in evolution which causes society development and general sustainability to grow. But *Economy* increase collapses *Environment* evolution. Hence, the amount of natural resources which can be consumed by *Economy* is restricted. Further the region development tends to have a stable behavior with small variations

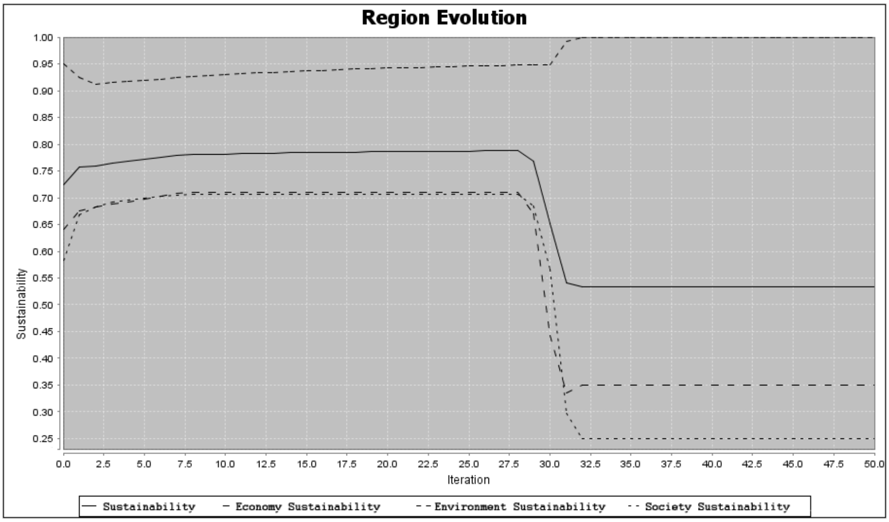

Fig. 3 The evolution of the region in Situation 2. In this case, the initial variations in areas evolution are weaker, but region achieves stability in a longer interval. After iteration 28 the development indicators of *Economy* and *Society* decrease significantly, because of nonrenewable resources deficiency. In this case, model cannot propose viable solutions to continue region evolution in a sustainable manner

The evolution diagrams show how *Environment*, *Society* and *Economy* contribute to the general sustainability of region. Here is easy to observe control principles which were described in SD game and dynamical systems:

- when *Economy* tend to have an explosive evolution, the *Environment* restricts the natural resources consumption;
- an increase in economic or environment sustainability results in a smaller increase in society sustainability;
- all areas involved in SD model participate with the same rate in general sustainability of analyzed region.

8 Conclusion and Further Work

A new model for Sustainable Development problem is proposed. This model, called Dynamical Game for Sustainable Development (DGSD), combines nature inspired optimization techniques, ideas from Dynamical Systems and Game Theory. Dynamical Systems were used to describe the evolution of each area included in SD. Using Game Theory we modeled the control points of DGSD model, as a game between these areas. Game equilibrium is computed using an evolutionary technique based on generative relations. Game equilibrium controls the evolution of each area in conformance with region sustainability principles.

Game Theory captures very well strategic situations of each player involved in the game and the conflicts existing between the areas. We proved that using game equilibrium, each area involved in region development is enforced to implement just "sustainable" decisions, otherwise they are constrained by the other areas.

However, game equilibrium alone, does not guarantee to drive the region to maximum sustainability in all the cases. It finds just the local optimum, and as it is, does not use the idea or concept of long term optimum. Nash equilibrium imposes players to be selfish, but in our case the players must put the interest of the whole region development over their individual interest. In other words, the player must consider an *altruistic behavior* related to the development of the region where they act.

To avoid the drawback of local optimum we intend to combine our model with approaches that guarantees the convergence of the model to the global optimum, which in our case is maximum sustainability. Further, we intend to integrate DGSD model with different decision making approaches. We may consider an alternative approach for decision process which is based on Public Good Games(PGG) [8]. PGGs describe social behavior through public goods, and models players diversity depending on their percentage of collaboration.

We hope that DGSD model is a good start point to study Sustainable Development. Even if with present form, this model does not provide best strategies to drive any region to SD development, there are a lot of possibilities to extend and to improve it, preserving its fundamental ideas.

References

[1] Chertow, M.R.: The IPAT Equation and Its Variants. Journal of Industrial Technology 4.4, 13–29 (2001)
[2] De Lara, M., Doyen, L.: Sustainable Management of Natural Resources. Mathematical Models and Methods. Springer, Berlin (2008)
[3] Dalal-Clayton, B., Bass, S.: Sustainable Development Strategies. Earthscan Publications Ltd., London (2002)
[4] Dumitrescu, D., Lung, R.I., Mihoc, T.D.: Evolutionary Equilibria Detection in Non-cooperative Games. In: Giacobini, M., Brabazon, A., Cagnoni, S., Di Caro, G.A., Ekárt, A., Esparcia-Alcázar, A.I., Farooq, M., Fink, A., Machado, P. (eds.) EvoCOMNET. LNCS, vol. 5484, pp. 253–262. Springer, Heidelberg (2009)
[5] Dumitrescu, D., Lung, R.I., Mihoc, T.D.: Generative Relations for Evolutionary Equilibria Detection. In: GECCO 2009, pp. 1507–1512 (2009)
[6] Dumitrescu, D., Lung, R.I.: ESCA: A new Evolutionary-Swarm Cooperative Algorithm. In: NICSO 2007, pp. 105–114 (2007)
[7] Gujarati, D.: Basic Econometrics. McGraw-Hill, Columbus (2003)
[8] Janssen, M.A., Ahn, T.: Adaptation vs. Anticipation in Public-Good Games (2003), http://www.allacademic.com/meta/p64827_index.html
[9] Mishra, S.K.: A Brief History of Production Functions. Social Science Research Network (2007)
[10] Osborne, M.J., Rubinstein, A.: A Course in Game Theory. MIT Press, Cambridge (1994)
[11] Zgurovsky, M.Z.: Sustainable Development Global Simulation: Opportunities and Treats to the Planet, Kyiv (2007)