Parametric Models of Transmission Lines Based on First Order Sensitivities

Alexandra Stefanescu, Daniel Ioan, and Gabriela Ciuprina

Abstract Further downscaling of the integrated circuits pushes the limits of lithographic technologies and certain variability effects previously considered negligible now should be taken into account. This paper proposes an efficient approach that addresses the problem of interconnect process variations. New models for line parameters parameterized with respect to the geometric transversal dimensions, subject to small or large variations are proposed. The parametric models are solely based on the computation of first order sensitivities. In the multiparametric case the use of multiplicative models can be a better choice than the use of traditional models based on first order Taylor Series truncation.

1 Introduction

Continuous improvements in today's fabrication processes determine smaller chip sizes and smaller device geometries. The impact of interconnect performances has become important as millions of closely spaced interconnections in one or more levels connect various components on the integrated circuit [1]. Process induced variations induce changes in the properties of metallic interconnect between devices, pushing the limits of lithographic technologies. Parasitic capacitances, resistances and inductances of the interconnections have become major factors in the evolution of very high speed IC technology. This paper focuses on the variability of the numerical extracted models for long interconnects modeled as transmission lines with respect to geometric parameters. The authors investigate promising alternatives beside the classic models of first-order truncations of Taylor expansions. The self - imposed restriction is to use in the extracted model exclusively the values of first-order sensitivities and not those of superior orders. The advantage of this approach

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is obvious. This represents one of the goals of the research carried out within the European project FP6/IST/Chameleon [2].

This paper is structured as follows: first the basic approach used is discussed, second, the approach is validated in the case of a microstrip line having one or multiple variable parameters. Next, results on technology variability are shown and conclusions are drawn at the end.

2 Parametric Models Based on First Order Sensitivities

First order sensitivities are essential for the analysis of the parameter variability [3, 4]. Parametric models are often obtained by truncating the Taylor series expansion for the quantity of interest. This requires the computation of the derivatives of the device characteristics with respect to the design parameters [5]. Let us assume that $y(p_1, p_2, \cdots, p_n) = y(\mathbf{p})$ is the device characteristic which depends on the design parameters $\mathbf{p} = [p_1, p_2, \cdots, p_n]$. The quantity y may be, for instance the real or the imaginary part of the device admittance at a given frequency. In our case this quantity is any of the p.u.l. parameters. The parameter variability is thus completely described by the real function, y, defined over the design space S, a subset of $\mathbb{R}^n$. The nominal design parameters correspond to the particular choice $\mathbf{p}_0 = [p_{01}\ p_{02}\ \cdots\ p_{0n}]$.

2.1 *Additive Model (A)*

If y is smooth enough then its truncated Taylor Series expansion is the best polynomial approximation in the vicinity of the expansion point $\mathbf{p}_0$. For one parameter ($n = 1$), the additive model is the first order truncation of the Taylor series:

$$\hat{y}(p) = y(p_0) + \frac{\partial y}{\partial p}(p_0)(p - p_0). \tag{1}$$

If we denote by $y(p_0) = y_0$ the nominal value of the output function, by $\frac{\partial y}{\partial p}(y_0)\frac{p_0}{y_0} = S_p^y$ the relative first order sensitivity and by $(p - p_0)/p_0 = \delta p$ the relative variation of the parameter p, then the variability model based on (1) defines an *affine* [6] or *additive* model (A):

$$\hat{y}(p) = y_0(1 + S_p^y \delta p). \tag{2}$$

According to the Taylor Series theory the neglected terms can be expressed function of the second order derivative in an intermediate point, ξ:

$$y(p) = y(p_0) + \frac{\partial y}{\partial p}(p_0)(p - p_0) + \frac{\partial^2 y}{\partial^2 p}(\xi)(p - p_0). \tag{3}$$

It follows that the relative variation of the output quantity $\delta y = (y(p) - y_0)/y_0$ can be expressed as

$$\delta y = S_p^y \delta p + \varepsilon, \tag{4}$$

where the approximation error ε depends on the second order derivative of the output quantity:

$$\varepsilon = \frac{p_0^2}{2y_0}\frac{\partial^2 y}{\partial^2 p}(\xi)(\delta y)^2. \tag{5}$$

Thus, to ensure a relative validity range of the first order approximation of the output quantity less a given threshold t_1, the absolute variation of the parameter must be less than

$$V_d = \sqrt{\frac{2y_0 t_1}{D_2}}, \tag{6}$$

where D_2 is an upper limit of the second order derivative of the output quantity y with respect to parameter p.

The validity range of the first approximation can be increased in some cases if the Taylor Series expansion is used for the "reversed" quantity $1/y(p)$. In this case, to obtain the same validity range of the first order approximation for the reversed output quantity, the variation of the parameter has to be less than

$$V_r = \sqrt{\frac{2t_1}{y_0 D_2'}}, \tag{7}$$

where D_2' is an upper limit of the second order derivative of the reversed output quantity.

For the multiparametric case, one gets:

$$y(\mathbf{p}) = y(\mathbf{p}_0) + \nabla y(\mathbf{p}_0)\cdot(\mathbf{p}-\mathbf{p}_0) = y_0 + \sum_{k=1}^{n}\frac{\partial y}{\partial p_k}(\mathbf{p}_0)(p_k - p_{0k}). \tag{8}$$

Similar with one parameter case, the relative sensitivities w.r.t. each parameter are denoted by $\frac{\partial y}{\partial p_k}(\mathbf{p}_0)\frac{p_{0k}}{y_0} = S_{p_k}^{y}$ and the relative variations of the parameters by $\delta p_k = (p_k - p_{0k})/p_{0k}$, the additive model (A) for n parameters being given by:

$$\hat{y}(\mathbf{p}) = y_0(1 + \sum_{k=1}^{n} S_{p_k}^{y}\delta p_k). \tag{9}$$

Thus, each new independent parameter taken into account adds a new term to the sum [7]. The additive model is simply a normalized standard version of a linearly truncated Taylor expansion. Instead of using this truncated expansion may be numerically favorable to expand some transformation $F(y)$ of y instead. Two particular choices for F have practical importance: identity and inversion as it will be indicated below. The originality of the algorithm for parametric model extraction proposed by authors is the automation of the choice of transformation F, based on the numerical estimation for the validity ranges (6), (7).

2.2 *Rational Model (R)*

The rational model is the additive model for the reverse quantity $1/y$. It is obtained from the first order truncation of the Taylor Series expansion for the function $1/y$. For $n = 1$, if we denote by $r(p) = \frac{1}{y(p)}$, it follows that:

$$\hat{r}(p) = r(p_0) + \frac{\partial r}{\partial p}(p_0)(p - p_0). \tag{10}$$

We define the relative first order sensitivity of the reverse circuit function: $\frac{\partial r}{\partial p}(p_0) \frac{p_0}{r(p_0)} = S_p^r = S_p^{1/y}$. Consequently, we obtain the rational model for $n = 1$:

$$y(p) = \frac{y_0}{1 + S_p^{1/y} \delta p}. \tag{11}$$

It can be easily shown that the reverse relative sensitivity is $S_p^{\frac{1}{y}} = -S_p^y$. For the multiple parameter case, the rational model is:

$$\hat{y}(\mathbf{p}) = \frac{y_0}{1 + \sum_{k=1}^n S_{p_k}^{1/y} \delta p_k}. \tag{12}$$

2.3 Multi-parametric Model (M)

Let us assume that in the multiparametric case the quantity of interest can be written as a product of functions with separated variables:

$$y(\mathbf{p}) = y_1(p_1) y_2(p_2) \cdots y_n(p_n). \tag{13}$$

Each component function, y_k depends only on a single parameter, p_k and for each one we can use either an additive or a rational model:

$$\hat{y}(\mathbf{p}) = \frac{y_0(1 + \sum_{k=1}^m S_{p_k}^y \delta p_k)}{1 + \sum_{k=m+1}^n S_{p_k}^{1/y} \delta p_k}. \tag{14}$$

The tensor product representation (13) seems to be a very particular case, however it fits perfectly the variation of RLC parameters w.r.t. geometric parameters extracted from uniform electric or magnetic field. The factorization and the choice of m are dictated by physics of the problem itself, however the modeling algorithm we propose is a numerical approach based on the expressions (6) and (7) for the validity ranges. For instance, in the case of two variable parameters, p_1, p_2 four versions of model M are possible:

- M_{AA}- additive models for both parameters

$$\hat{y}(\mathbf{p}) = y_0(1 + S_{p_1}^y)(1 + S_{p_2}^y); \tag{15}$$

- M_{RR} - rational models for both parameters

$$\hat{y}(\mathbf{p}) = y_0 \frac{1}{(1 + S_{p_1}^{1/y})(1 + S_{p_2}^{1/y})}; \tag{16}$$

- M_{AR} - additive model for the first parameter and rational model for the second one

$$\hat{y}(\mathbf{p}) = y_0 \frac{(1 + S_{p_1}^y)}{(1 + S_{p_2}^{1/y})}; \tag{17}$$

- M_{RA} - rational model for the first parameter and additive model for the second one

$$\hat{y}(\mathbf{p}) = y_0 \frac{(1 + S_{p_2}^{y})}{(1 + S_{p_1}^{1/y})}. \tag{18}$$

Together with the two "classical" A and R models, there are six possible parametric models for the two parameter case.

3 Case Study

In order to validate our approach and to evaluate different parametric models, several experiments have been performed on a test structure that consists of a microstrip (MS) transmission line having one Aluminum conductor embedded in a SIO_2 layer. The line has a rectangular cross section, parameterized by several parameters (Fig. 1). The return path is the grounded surface placed at $y = 0$. The nominal values used are: $x_{max} = 20\mu$m, $h_2 = 10\mu$m, $h_3 = 5\mu$m, $h_0 = 1\mu$m, $p_1 = 1\mu$m, $p_2 = 0.67\mu$m, $p_3 = 3\mu$m, $\sigma_{Si} = 10000$ MS/m, $\sigma_{Al} = 3.3$MS/m, $\varepsilon_{r-SiO_2} = 3.9$. In order to comply with designer's requirements, the model should include the field propagation along the line, taking into consideration the distributed parameters and the high frequency effects.

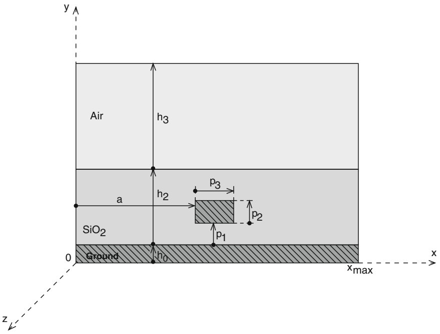

Fig. 1: Stripline parameterized structure

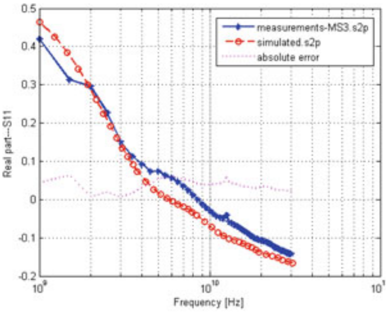

Fig. 2: Frequency characteristic $Re(S_{11})$: numerical model vs measurements

3.1 *Validation of the Nominal Model*

Before considering the parametric model, the results obtained for the nominal values of p.u.l. parameters were validated by deriving from them the scattering parameters (**S**) and compare the results with the measurements provided within the European project FP5/Codestar (www.imec.be/codestar). For the nominal case, by using dFIT + dELOB [7], at low frequencies, the following values are obtained:

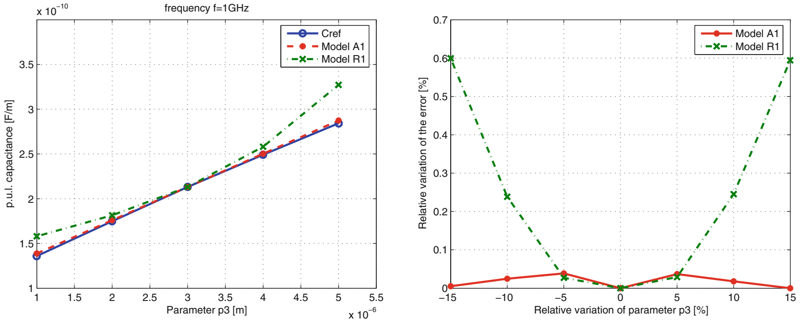

Fig. 3: *Left*: Reconstruction of the p.u.l. C from Taylor Series first order expansion; *Right*: Relative error w.r.t. the relative variation of parameter p_3

$$R = 18.11k\Omega/m, L = 322nH/m, C = 213pF/m \tag{19}$$

Actually p.u.l. resistance and inductance are frequency dependent, and they can be computed with the method described in [7]. The frequency response of the entire line having the length d was computed using Transmission Line equations [6]. The comparison between the simulations and the measurements is shown in Fig. 2 and validates the nominal model described before. The sensitivities of p.u.l. parameters are computed using the CHAMY software [2], by direct differentiation method applied to the state space equations [5]. They can also be computed by Adjoint Field Technique (AFT) [8,9].

3.2 Parametric Models

In this section, the accuracy of the A, R and M models for the line capacitance is investigated.

One Parameter Case

The first sets of tests considered only one parameter that varies, namely the width of the line, p_3. The nominal value chosen was $p_3 = 3\mu$m and samples in the interval $[1,5]\mu$m were considered. The reference result was obtained by doing "exact" simulations for the samples. These were compared with the approximate values obtained from models A and R (Fig. 3). As expected intuitively, the dependence w.r.t. p_3 is almost linear and the A model is better than the R model. Considering the relative variation of the parameters less than 15% (which is the typical limit for the technological variations nowadays) the relative variation of the output parameter is obtained (Fig. 3, right). The errors of both affine and rational first order models for p.u.l. parameters are given in Table 1. Model A based on the first order Taylor series approximation has a maximal error for technologic variations 1.78% for p.u.l. resistance when p_3 is variable, while model R has an approximation error of only 0.6% for the same range of the technological variations for p.u.l. capacitance when

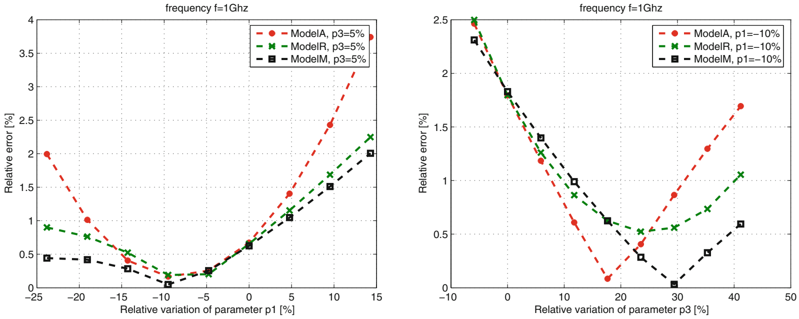

Fig. 4: *Left*: Relative error w.r.t. the relative variation of parameter p_1, for a variation of p_3 of 5%; *Right*: Relative error w.r.t. the relative variation of parameter p_3, for a variation of p_1 of 10%

p_3 is variable. Using (6) and (7) can be easily identified which is the best model in any case.

Table 1: Maximal errors [%] of p.u.l. parameters for technology variation of ± 15%

Parameter	Quantity	Affine (A1)	Rational (R1)
p_1	L	0.11	0.15
	C	0.65	0.25
p_3	R	1.78	0.22
	L	0.34	0.04
	C	0.035	0.6

Multiple Parametric Case

Let us consider now two parameters that vary simultaneously: p_1 and p_3. For reference, a set of samples in $[0.8, 1.2]\mu$m $\times$ $[2, 4]\mu$m were considered. The p.u.l. capacitance was approximated using the additive, rational and multiplicative models described above. In this case, model M is computed using an additive model for p_3 and a rational one for p_1, which is the best choice. Fig. 4a compares the relative variation of the errors w.r.t. a relative variation of parameter p_1 for a variation of p_3 of 5%. Model M provides lower errors (maximum error is 2%) than models A (3.7%) and R (2.2%). Fig. 4b illustrates that in the range from 20% to 40% model M is the best one if we look at the variation w.r.t. p_3 for a variation of p_1 of 10%. Thus, by using the appropriate multiplicative models in the modeling of the technological variability, the necessity of higher order approximations may be eliminated.

4 Conclusions

This paper analyzes variability models for TL structures considering the dependency of p.u.l. parameters w.r.t. geometric parameters, at a given frequency. A detailed study of the line sensitivity was made by using numeric techniques. For one parameter case, the proposed methods avoid the evaluation of higher order sensitivities, maintaining the accuracy by introducing rational models. The multi-parametric case has been analyzed, in addition, a multiplicative parametric model (M) has been proposed. This is based on the assumption that the quantity of interest can be expressed with separated variables, for which A and/or R models are used. Model M is sometimes better than A and R models obtained from Taylor Series expansion. Its specific terms (products of first order sensitivities) can thus approximate higher order, cross-terms of Taylor Series. In order to automatically select the best first order model for a multiparametric problem, the validity ranges of direct and reversed quantities have to be evaluated. Once we establish the best model (A or R) for each parameter, the M model will be easily computed by multiplication of individual submodels. Our numerical experiments with the proposed algorithm in all particular structures we investigated prove that the technological variability (e.g. ±20% variation of geometric parameters, which is typical for the technology node of 65 nm) can be modeled with acceptable accuracy (relative errors under 5%) using only first order parametric models for line parameters.

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