DAE-Index and Convergence Analysis of Lumped Electric Circuits Refined by 3-D Magnetoquasistatic Conductor Models

Sebastian Schöps, Andreas Bartel, Herbert De Gersem, and Michael Günther

Abstract In this paper the field/circuit coupling is reconsidered for (non-linear) lumped electric circuits refined by 3-D magnetoquasistatic conductor models, where the circuit is described by modified nodal analysis and the field is discretized in terms of the finite integration technique. This leads to the coupling of systems of differential-algebraic equations, for which two numerical approaches are proposed, the weak coupling (co-simulation) and strong coupling (monolithic). The DAE-index of the subproblems and of the full problem are analyzed, then convergence properties of the co-simulation are studied. Finally computational results of a simple half rectifier circuit are exemplarily given to prove the applicability of the concepts.

1 Introduction

Basic elements in circuit analysis are described by (non-)linear relations, disregarding distributed field effects. Sometimes complex *companion models* are employed to meet reality. These give, however, only a partial insight into field effects. In contrast, *refined models* directly rely upon Maxwell's equations and are coupled here with electric network equations. We analyze this coupling with two distributed conductor types, which exhibit proximity and skin effects related to eddy currents.

The coupled problem is a system of differential-algebraic equations (DAEs) originating from Kirchhoff's laws and the discrete Maxwell equations. It can be directly addressed by solving one *monolithic* system using a field- or circuit-oriented approach. In the field approach, commonly the circuit is described using loop/branch techniques and is solved within the field simulator. This approach is quite successful and well understood [1], but it is neither efficient for coupling with very large

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circuits nor usable within modern circuit simulators that are based on modified nodal analysis (MNA). The circuit-oriented approach relies on MNA and although intensive research has been carried out [2], companion models are still widespread.

Obviously, the strongly coupled approaches do not have the advantages of problem-specific simulators. In this context *co-simulation* can becomes beneficial [3]. It allows to use different simulators for each subproblem, and thus provides a natural support for diversifying integration methods and time-stepping (*multirate*) with respect to the subproblems. Here, the coupling is given mathematically by a waveform relaxation scheme.

The paper is organized as follows: In Sections 2 and 3 the circuit and field settings are recalled; in Section 4 we analyze the index of the field-system; in Section 5 we introduce the weak and strong coupling and provide an index and convergence analysis; in Sections 6 and 7 we give an illustrative example and conclusions.

2 Lumped Electric Circuit

Electric circuits are described by basic element relations and Kirchhoff's laws. Using standard MNA, this yields a DAE system since the variables are redundant. In the charge-flux oriented formulation [4], the system reads

$$\begin{aligned} A_{\mathrm{C}}\tfrac{\mathrm{d}}{\mathrm{d}t}q + A_{\mathrm{R}}r(A_{\mathrm{R}}^{T}e,t) + A_{\mathrm{L}}i_{\mathrm{L}} + A_{\mathrm{V}}i_{\mathrm{V}} + A_{\mathrm{I}}i(t) + A_{\lambda}i_{\lambda}(A_{\lambda}^{T}e,t) &= 0,\\ \tfrac{\mathrm{d}}{\mathrm{d}t}\Phi - A_{\mathrm{L}}^{T}e = 0, \qquad A_{\mathrm{V}}^{T}e - v(t) &= 0,\\ q - q_{\mathrm{C}}(A_{\mathrm{C}}^{T}e,t) = 0, \quad \Phi - \Phi_{\mathrm{L}}(i_{\mathrm{L}},t) &= 0, \end{aligned} \tag{1}$$

with incidence matrices A, node potentials e, independent and controlled current and voltage sources i, i_λ and v, currents through voltage and flux controlled elements i_V and i_L, charges q and fluxes Φ, functions of charges, fluxes and resistances q_C, Φ_L and r (with positive definite derivatives), respectively.

Several index concepts were introduced to classify DAEs. Since these notations are equivalent for linear systems, we state here only the (differential) index, [5]: For the given system $F\left(t, \tfrac{\mathrm{d}}{\mathrm{d}t}x, x\right) = 0$, the index $\nu \in \mathbb{N}_0$ is the smallest number, such that the enlarged set of equations

$$F\left(t,\tfrac{\mathrm{d}}{\mathrm{d}t}x,x\right)=0,\quad \tfrac{\mathrm{d}}{\mathrm{d}t}F\left(t,\tfrac{\mathrm{d}}{\mathrm{d}t}x,x\right)=0,\quad \ldots,\quad \tfrac{\mathrm{d}^{\nu}}{\mathrm{d}t^{\nu}}F\left(t,\tfrac{\mathrm{d}}{\mathrm{d}t}x,x\right)=0$$

allows to deduce a system of ordinary differential equations (ODEs) by algebraic manipulations. In this way, ν denotes the inherent number of derivatives and measures the expected numerical difficulties.

In this respect, the numerical properties of (1) are well known, the DAE-index has been discussed by decomposing the unknown $(e, i_{\mathrm{V}}, i_{\mathrm{L}}, q, \Phi)$ into algebraic and differential parts using a projector Q_C onto the kernel of A_C^T, i.e.,

$$Q_C \ker A_C^T = \ker A_C^T \text{ and } A_C^T Q_C = 0$$

and its complement $P_C = I - Q_C$. We assume in the above terms:

C1 No loops of capacitors and voltage sources, i.e., $\ker Q_C^T A_V = \{0\}$.
C2 No cutsets of inductors and current sources[1], i.e., $\ker(A_C, A_R, A_V)^T = \{0\}$.
C3 Voltage controlled current sources parallel to capacitors, i.e., $Q_C^T A_\lambda i_\lambda = 0$.

This splits the unknown into a differential part $y := (P_C e, j_L)^T$ and an algebraic part $z := (Q_C e, j_V, q, \Phi)^T$, such that

$$\frac{\mathrm{d}}{\mathrm{d}t} y = f_1(y, z, i_\lambda), \qquad 0 = g_1(y, z), \tag{2}$$

is an index-1 description of (1) since the derivative $\frac{\partial}{\partial z} g_1$ can be shown to be non-singular assuming **C1-C3**. It is possible to prove [6]:

Theorem 1. *Let us consider a lumped electric circuit in form* (1) *that respects* ***C3****, then the flux/charged oriented MNA leads to an index-1 DAE iff* ***C1-C2*** *hold, it leads otherwise to an index-2 DAE.*

3 Electromagnetic Field

The electromagnetic field is described by Maxwell's equations. We assume a spatial discretization based on staggered grids (e.g. the finite integration technique) [7, 8]. In magnetoquasistatics with linear materials one can deduce the curl-curl equation

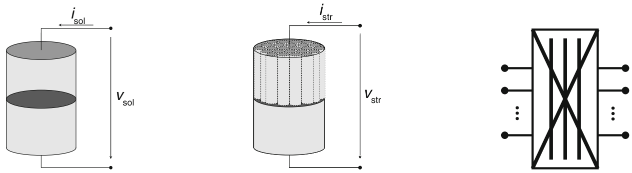
$$M_\sigma \frac{\mathrm{d}}{\mathrm{d}t} \widehat{a}(t) + K_\nu \widehat{a}(t) = \widehat{\widehat{j}}_{\mathrm{src}}(t) , \tag{3}$$

where $\widehat{a}$ denotes the discrete magnetic vector potential (MVP), M_σ the diagonal positive semi-definite conductivity matrix, $\widehat{\widehat{j}}_{\mathrm{src}}$ the source current density and $K_\nu := \tilde{C} M_\nu C$ is the curl-curl matrix composed of the curl-operators for the primary and dual grid C and $\tilde{C}$, respectively and the diagonal positive definite reluctivity matrix M_ν. Due to the non-trivial nullspace of M_σ this is a DAE, which is generally not uniquely solvable because of the additional nullspace of the curl-operators. Thus a gauge is needed to select one solution within the equivalent class $\widehat{\widehat{b}} = C\widehat{a}$, [10].

4 Field Models as Refined Network Elements

Conductor models for connecting field and circuit parts are well-known. Most common are solid and stranded conductors (Fig. 1). We use the given symbol for a (multiport) device that consists of (multiple) conductors of both types which are tightly coupled by the field. The field is described by the curl-curl equation and excited by $\widehat{\widehat{j}}_{\mathrm{src}}$ due to the connected circuit [9]. Typically voltage drops of solid conductors (v_{sol}) and the currents through stranded conductors (i_{str}) are considered to be given and thus the excitation reads

[1] neither independent nor voltage controlled current sources, i.e., solid/stranded conductors



(a) Solid **(b)** Stranded **(c)** Symbol

Fig. 1: Conductor models (**a**), (**b**) and device symbol (**c**) that embeds both into the circuit

$$\widehat{\widehat{j}}_{\mathrm{src}} = M_\sigma Q_{\mathrm{sol}} v_{\mathrm{sol}} + Q_{\mathrm{str}} i_{\mathrm{str}} \ . \tag{4}$$

Here $Q = [Q_{\mathrm{sol}}, Q_{\mathrm{str}}]$ denotes the coupling matrix. Each column corresponds to a conductor model and imposes currents/voltages onto edges of the grid. The unknown currents i_{sol} and voltages v_{str} are obtained by the additional equations

$$i_{sol} = G_{\mathrm{sol}} v_{\mathrm{sol}} - Q_{\mathrm{sol}}^T M_\sigma \tfrac{\mathrm{d}}{\mathrm{d}t} \widehat{a} \ , \qquad v_{\mathrm{str}} = R_{\mathrm{str}} i_{\mathrm{str}} + Q_{\mathrm{str}}^T \tfrac{\mathrm{d}}{\mathrm{d}t} \widehat{a} \ , \tag{5}$$

with the diagonal conductance matrices G_{sol} for solid and the diagonal resistance matrix $R_{\mathrm{str}} = G_{\mathrm{str}}^{-1}$ for stranded conductors. Let us assume the following:

F1 The matrix pencil is regular, i.e., $[M_\sigma, K_\nu] := \det(\lambda M_\sigma + K_\nu) \neq 0$ for a λ.
F2 The models are non-overlapping, i.e., $Q_{(i)}^T Q_{(j)} = 0$, for all $i \neq j$.
F3 The excitation is consistent, i.e., $\ker(CQ_{\mathrm{sol}}) = \{0\}$, $\ker(CM_{\sigma,\mathrm{aniso}}^+ Q_{\mathrm{str}}) = \{0\}$.

where $M_{\sigma,\mathrm{aniso}}^+$ is the pseudoinverse of the anisotropic conductivity matrix for stranded conductors. **F1** is equivalent to a gauging of (3) and **F2** prohibits the smearing of spatially separate models into each other, this allows to obtain (6) from (3-5),

$$M_{\sigma,\mathrm{fillin}} \tfrac{\mathrm{d}}{\mathrm{d}t} \widehat{a} + K_\nu \widehat{a} = M_\sigma Q_{\mathrm{sol}} v_{\mathrm{sol}} + Q_{\mathrm{str}} G_{\mathrm{str}} v_{\mathrm{str}} := \widehat{\widehat{j}}_{\mathrm{src}}^{\,*} \ , \tag{6a}$$

$$Q_{\mathrm{sol}}^T K_\nu \widehat{a} = i_{\mathrm{sol}} \ , \tag{6b}$$

$$G_{\mathrm{str}} Q_{\mathrm{str}}^T M_{\sigma,\mathrm{aniso}}^+ K_\nu \widehat{a} = i_{\mathrm{str}} \ , \tag{6c}$$

where $M_{\sigma,\mathrm{fillin}} := M_\sigma + Q_{\mathrm{str}} G_{\mathrm{str}} Q_{\mathrm{str}}^T$ is the (dense) conductivity matrix for both types.

Lemma 1. *Let the field problem consist of solid and stranded conductors which fulfill **F1**-**F2**, then the curl-curl equation* (6a) *is index-1 for given voltages and the algebraic part of the MVP is zero.*

Proof. By **F1**, the symmetric positive semi-definiteness of $M_{\sigma,\mathrm{fillin}}$ implies that (6a) is index-1 and the Kronecker Normal Form [5] for this system reads

$$\tfrac{\mathrm{d}}{\mathrm{d}t} \widehat{a}_1(t) + U_1 K_\nu V_1 \ \widehat{a}_1(t) = U_1 \ \widehat{\widehat{j}}_{\mathrm{src}}^{\,*} \ , \tag{7a}$$

$$\widehat{a}_2(t) = U_2 \ \widehat{\widehat{j}}_{\mathrm{src}}^{\,*} \ , \tag{7b}$$

and this splits the MVP $\widehat{a} = V_1 \widehat{a}_1 + V_2 \widehat{a}_2$ into differential and algebraic parts by using the regular matrices $U^T = \left(U_1^T \ , U_2^T\right)$ and $V = (V_1 \ , \ V_2)$. From

$$U_2 M_{\sigma,\text{fillin}} = U_2 \left(M_\sigma + Q_{\text{str}} G_{\text{str}} Q_{\text{str}}^T \right) = 0$$

follows that both $U_2 M_\sigma$ and $U_2 Q_{\text{str}} G_{\text{str}}$ vanish because the images of M_σ and Q_{str} are distinct, since **F2** is assumed. Hence we finally conclude that the algebraic part of the MVP is zero: $\widehat{a}_2 = U_2 \,\widehat{\widehat{j}}{}^*_{\text{src}} = 0$. □

Let us now study the full system (6) in the abstract form

$$\tfrac{\mathrm{d}}{\mathrm{d}t}\widehat{a} = f_{2a}(\widehat{a}, v_\lambda), \qquad 0 = f_{2b}(\widehat{a}, v_\lambda), \qquad 0 = g_2(\widehat{a}, i_\lambda), \tag{8}$$

where the voltages $v_\lambda = (v_{\text{sol}}, v_{\text{str}})^T$ and the currents $i_\lambda = (i_{\text{sol}}, i_{\text{str}})^T$ are combined in vectors. The algebraic evaluation f_{2b} is trivial in our case because of Lemma 1 and the algebraic function g_2 consists of (6b), (6c), which can be written in the form

$$0 = g_{\text{sol}}(\widehat{a}, i_{\text{sol}}), \qquad 0 = g_{\text{str}}(\widehat{a}, i_{\text{str}}).$$

System (8) establishes a relation between currents (i_{sol}, i_{str}) and voltages (v_{sol}, v_{str}) and we can choose which quantity is treated as unknown for each conductor type in the field system, since then the other quantity is defined by the coupled electric circuit. Therefore we will distinguish between the possible sets in the following.

Theorem 2. *Let the field problem consist of solid and stranded conductors which fulfill **F1**-**F3**. Iff all the voltages (v_{sol}, v_{str}) are given, then system* (6) *is index-1 and in all other cases it is index-2.*

Proof. In the case of given voltages the currents i_λ are obtained by evaluations of the algebraic equation g_2. Thus one differentiation with respect to time yields an ODE, hence we have index-1. In all other cases the arguments are analogue to the case of given i_{sol} and v_{str}. Now the function f_{2a} in (8) depends on the unknown v_{sol} and one time derivative yields the additional *hidden constraint*:

$$0 = \frac{\mathrm{d}}{\mathrm{d}t} g_{\text{sol}}(\widehat{a}, i_{\text{sol}}) = \frac{\partial}{\partial \widehat{a}} g_{\text{sol}} \cdot f_2(\widehat{a}, v_{\text{sol}}) + \frac{\mathrm{d}}{\mathrm{d}t} i_{\text{sol}} =: h_{\text{sol}}(\widehat{a}, v_{\text{sol}}, \frac{\mathrm{d}}{\mathrm{d}t} i_{\text{sol}}),$$

and since the conductivity matrices M_σ and $M_{\sigma,\text{aniso}}$ reflect **F2** ($M_{\sigma,\text{aniso}} Q_{\text{sol}} = 0$), another differentiation of this constraint gives

$$\frac{\partial}{\partial v_{\text{sol}}} h_{\text{sol}} = Q_{\text{sol}}^T K_\nu M_{\sigma,\text{fillin}}^+ M_\sigma Q_{\text{sol}} = Q_{\text{sol}}^T K_\nu Q_{\text{sol}} = Q_{\text{sol}}^T C^T M_\nu C Q_{\text{sol}} \,,$$

which is non-singular due to **F3**; thus it is index-2. □

If voltages are considered unknown, then (6) is an index-2 Hessenberg system (with index-1 evaluations), [11]. Since the index-2 variables enter only linearly and without time-dependence, the differential variables are not affected by the derivatives of perturbations and thus the numerical difficulties still correspond to index-1 [12].

5 Coupling

We assign v_λ to the differences of applied node potentials e for elements with topology A_λ and assign the i_λ to the currents through the conductors

$$v_\lambda = A_\lambda^T e\,, \qquad\qquad i_\lambda = (i_{\mathrm{sol}}, i_{\mathrm{str}})^T. \tag{9}$$

Now, the *monolithic* system is composed of (1), (6) and (9).

Theorem 3. *Let us consider an electric circuit in the form* (1) *with* ***C1-C2****, which is monolithically coupled via* (9) *to a field model* (6) *of solid and stranded conductors fulfilling* ***F1-F3****, then the full system is index-1.*

Proof. The algebraic components of the MVP are insignificant for solid and stranded conductors according to Lemma 1. Hence after embedding the field into the circuit system the separated unknowns of the full system read

$$y := (P_C e, j_L, \widehat{a}_1)^T, \qquad\qquad z := (Q_C e, j_V, q, \phi, i_\lambda)^T. \tag{10}$$

The critical partial derivative of the algebraic equation $\frac{\partial}{\partial z} g$ consisting of g_1 and g_2 is non-singular, since the first is regular due to **C1-C2** and the second is just an evaluation of a differential variable ($\widehat{a}_1$). Thus we have index-1. □

Assumption **C3** is not required in the monolithic coupling because the algebraic part of the MVP was shown to vanish for any excitement of solid and stranded conductors.

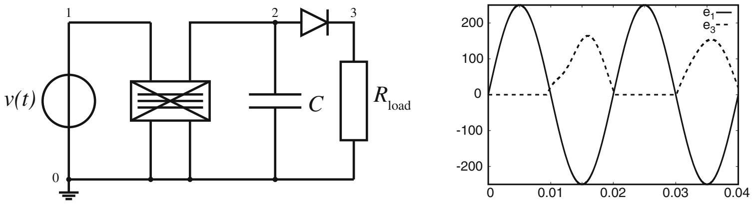
Alternatively, the subproblems could be treated separately by a *waveform relaxation scheme* (of Jacobi or Gau-Seidel type). When applying these schemes to DAEs one has to pay attention to algebraic constraints to avoid numerical instabilities, [13]. We suggest the Gau-Seidel scheme (11) that computes the functions $a^{(1)}$, $y^{(1)}$ and $z^{(1)}$ on a time frame $T = [t_0, t_0 + H]$ for given initial values at time t_0 and previous iterates $y^{(0)}$ and $z^{(0)}$.

$$\begin{aligned} \tfrac{\mathrm{d}}{\mathrm{d}t}\widehat{a}^{(1)} &= f_2(\widehat{a}^{(1)}, v^{(0)}), \quad v^{(0)} := A_\lambda^T(y^{(0)} + z^{(0)}), \quad & \tfrac{\mathrm{d}}{\mathrm{d}t} y^{(1)} &= f_1(y^{(1)}, z^{(1)}, i_\lambda^{(1)}), \\ 0 &= g_2(\widehat{a}^{(1)}, i_\lambda^{(1)}), & 0 &= g_1(y^{(1)}, z^{(1)}, i_\lambda^{(1)}). \end{aligned} \tag{11}$$

The convergence is guaranteed since there is no dependence in algebraic constraints (g_1, g_2) on previous algebraic iterates ($i_\lambda^{(0)}$, $z^{(0)}$), [14]. Hence we obtain:

Lemma 2. *Let us consider an electric circuit* (1) *fulfilling* ***C1-C2*** *and a field model* (6) *respecting* ***F1-F3*** *and employing the interface* (9)*. Then the waveform-relaxation* (11) *will converge.*

The additional assumption **C3** can eliminate the i_λ-dependence of the algebraic equation g_1 and allows us to exchanges the computational order of the subproblems (we may compute the circuit first) without losing the convergence guarantee.



(a) Half rectifier: $v_{\text{eff}} = 250\,\text{V}$, $f = 50\,\text{Hz}$, $R_{\text{load}} = 100\,\Omega$ and Shockley diode $I_s = 1\,\mu\text{A}$

(b) Voltage in nodes 1 and 3, obtained by `mono`, $H = 5\mu$s

Fig. 2: Refined half rectifier circuit and its input and computed output voltages

6 Numerical Experiments

The simulations were obtained with code that is implemented within the *COMSON DP* using field models constructed by *EM Studio* from *CST* (www.comson.org and www.cst.com). The code is capable of both, the monolithic (`mono`) and the co-simulation of non-linear circuits refined by conductor models. The co-simulation uses scheme (11) with no (`cosim1`) and two iterations (`cosim3`) of each time frame T. The integration was kept simple by applying backward Euler.

The example of Fig. 2 is a refined half rectifier with a transformer consisting of two stranded conductors and a solid core. `cosim1` performs slightly faster than `mono` using the step size H and it yields better results if the accuracy requirement is quite low. For decreasing step sizes `cosim1` does not linearly improve its accuracy as `mono` and `cosim3` do (Fig. 3), but `cosim3` suffers from an increased computational effort due to the additional iterations.

Adaptive time-integrators in the co-simulation apply the same step size to both subproblems, as long as they do not have multirate potential itself. This is in line with the fact that the field reflects the dynamics of the coupled circuit nodes.

7 Conclusions

The field problem is essentially an index-1 DAE, the monolithic coupled system is still index-1 and the convergence of the proposed co-simulation is guaranteed, as illustrated by the computation of a refined rectifier circuit. The co-simulation may use problem-specific software packages and exploits multirate potentials if available in the circuit, but its efficiency can be improved, for example by applying a time frame and iteration control, and the use of more complex equivalent circuits (e.g. additional inductivities) might require fewer field updates [15, 16].

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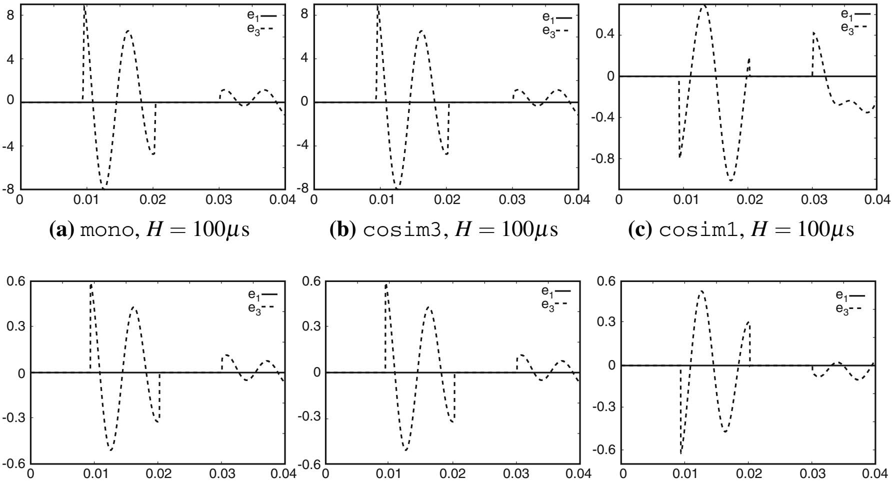

(a) mono, $H = 100\mu$s **(b)** cosim3, $H = 100\mu$s **(c)** cosim1, $H = 100\mu$s

(d) mono, $H = 10\mu$s **(e)** cosim3, $H = 10\mu$s **(f)** cosim1, $H = 10\mu$s

Fig. 3: Errors in the voltages compared to the results of mono, $H = 5 \cdot 10^{-6}$ from Fig. 2b

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