Matrices Representation of Multi Soft-Sets and Its Application

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Abstract. In previous paper, we introduced a concept of multi-soft sets and used it for finding reducts. However, the comparison of the proposed reduct has not been presented yet, especially with rough-set based reduct. In this paper, we present matrices representation of multi-soft sets. We define AND and OR operations on a collection of such matrices and apply it for finding reducts and core of attributes in a multi-valued information system. Finally, we prove that our proposed technique for reduct is equivalent to Pawlak's rough reduct.

Keywords: Multi-valued information system; Multi-soft sets, Matrices representation, Reducts and core of attributes.

1 Introduction

Soft set theory [1], proposed by Molodtsov in 1999, is a new general method for dealing with uncertain data. In recent years, research on soft set theory has been active, and great progress has been achieved, including the works of theoretical soft set, soft set theory in abstract algebra, parameterization reduction, decision making and forecasting. Let $S=(U,A,V,f)$ be an information system as in [2]. The "standard" soft set deals with a binary-valued information system $S=(U,A,V_{\{0,1\}},f)$. For a multi valued information system, we introduced a concept of multi soft-sets [3]. The idea is based on a decomposition of a multi-valued information system $S=(U,A,V,f)$, into $|A|$ number of binary-valued information systems $S=(U,A,V_{\{0,1\}},f)$, where $|A|$ denotes the cardinality of A. Consequently, the $|A|$ binary-valued information systems define *multi-soft sets*, denoted by $(F,A)=\{(F,a_i):1\le i\le |A|\}$. In [4], we used the concept of multi-soft sets and AND operation for finding reducts in a multi-valued information system. However, the comparison of the proposed reduct has not been presented yet, especially with rough-set based reduct [2,5]. In this paper, we present the notion of matrices representation of multi-soft sets. We present a definition of AND and OR operations on a collection

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of such matrices. Further, we apply the notion of AND operation for finding reducts and core attributes in a multi-valued information system. We prove that the proposed technique of reduct is equivalent with Pawlak's rough reduct.

The rest of this paper is organized as follows. Section 2 describes the notion of information system. Section 3 describes fundamental concept of soft set theory. Section 4 describes multi soft sets construction in a multi-valued information system. Section 5 describes matrices representation of multi-soft sets, AND and OR operations, as well a simple example. Section 6 describes applications of matrices representation of multi-soft sets for finding reducts and core of attributes. Further, we prove that our proposed technique for reduction is equivalent to Pawlak's rough reduction. Finally, we conclude and describe future activities of our works in section 7.

2 Information System

Data are often presented as a table, columns of which are labeled by *attributes*, rows by *objects* of interest and entries of the table are *attribute values*. By an *information system*, we mean a 4-tuple (quadruple) $S=(U,A,V,f)$, where $U=\{u_1,u_2,u_3,\cdots,u_{|U|}\}$ is a non-empty finite set of objects, $A=\{a_1,a_2,a_3,\cdots,a_{|A|}\}$ is a non-empty finite set of attributes, $V=\bigcup_{a\in A}V_a$, V_a is the domain (value set) of attribute a, $f:U\times A\to V$ is an information function such that $f(u,a)\in V_a$, for every $(u,a)\in U\times A$, called information (knowledge) function. An information system can be intuitively expressed in terms of an information table (refers to Table 1).

Table 1. An information system

U	a_1	$\cdots$	a_k	$\cdots$	$a_{\lvert A\rvert}$
u_1	$f(u_1,a_1)$	$\cdots$	$f(u_1,a_k)$	$\cdots$	$f(u_1,a_{\lvert A\rvert})$
u_2	$f(u_2,a_1)$	$\cdots$	$f(u_2,a_k)$	$\cdots$	$f(u_2,a_{\lvert A\rvert})$
$\vdots$	$\vdots$	$\ddots$	$\vdots$	$\ddots$	$\vdots$
$u_{\lvert U\rvert}$	$f(u_{\lvert U\rvert},a_1)$	$\cdots$	$f(u_{\lvert U\rvert},a_k)$	$\cdots$	$f(u_{\lvert U\rvert},a_{\lvert A\rvert})$

A relational database may be considered as an information system in which rows are labeled by the objects (entities), columns are labeled by attributes and the entry in row u and column a has the value $f(u,a)$. We note that a each map $f(u,a):U\times A\to V$ is a tupple $t_i=(f(u_i,a_1),f(u_i,a_2),f(u_i,a_3),\cdots,f(u_i,a_{|A|}))$, for $1\le i\le |U|$. In many applications, there is an outcome of classification that is known. Note that the tuple t is not necessarily associated with entity uniquely (refer to objects 3 and 4 in Table 3). In an information table, two distinct entities could have the same tuple representation (duplicated/redundant tuple), which is *not permissible* in relational databases. Thus, the concepts in information systems are a generalization of the same concepts in relational databases.

This a posteriori knowledge is expressed by one (or more) distinguished attribute called decision attribute; the process is known as supervised learning. An information system of this kind is called a decision system. A *decision system* is an information system of the form $D = (U, A \cup \{d\}, V, f)$, where $d \notin A$ is the decision attribute. The elements of A are called condition attributes. An example of a decision system is given in Table 2.

3 Soft Set Theory

The theory of soft set [1] proposed, by Molodtsov 1999 is a new method for handling uncertain data. The soft set is a mapping from parameter to the crisp subset of universe. From such case, we may see the structure of a soft set can classify the objects into two classes (yes/1 or no/0).

Definition 1. (See [1].) *A pair (F, A) is called a soft set over U, where F is a mapping given by*

$$F : A \to P(U).$$

In other words, a soft set over *U* is a parameterized family of subsets of the universe *U*. For $\varepsilon \in A$, $F(\varepsilon)$ may be considered as the set of ε-elements of the soft set (F, A) or as the set of ε-approximate elements of the soft set. Clearly, a soft set is not a (crisp) set.

Based on the definition of an information system and a soft set, in this section we show that a soft set is a special type of information systems, i.e., a binary-valued information system.

Proposition 2. *If (F, A) is a soft set over the universe U, then (F, A) is a binary-valued information system $S = (U, A, V_{\{0,1\}}, f)$.*

Proof. Let (F, A) be a soft set over the universe *U*, we define a mapping

$$F = \{f_1, f_2, \cdots, f_n\},$$

where

$$f_i : U \to V_i \text{ and } f_i(x) = \begin{cases} 1, & x \in F(a_i) \\ 0, & x \notin F(a_i) \end{cases}, \text{ for } 1 \le i \le |A|.$$

Hence, if $V = \bigcup_{a_i \in A} V_{a_i}$, where $V_{e_i} = \{0,1\}$, then a soft set (F, A) can be considered as a binary-valued information system $S = (U, A, V_{\{0,1\}}, f)$. □

From Proposition 2, it is easily to understand that a binary-valued information system can be represented as a soft set. Thus, we can make a one-to-one correspondence between (F, E) over *U* and $S = (U, A, V_{\{0,1\}}, f)$.

Definition 3. (See [6].) *The class of all value sets of a soft set (F, E) is called value-class of the soft set and is denoted by $C_{(F,E)}$.*

4 Multi-Soft Sets Construction in Information Systems

The idea of multi-soft sets is based on a decomposition of a multi-valued information system $S=(U,A,V,f)$, into $|A|$ number of binary-valued information systems $S=(U,A,V_{\{0,1\}},f)$, where $|A|$ denotes the cardinality of A. Consequently, the $|A|$ binary-valued information systems define *multi-soft sets* $(F,A)=\{(F,a_i):1\le i\le |A|\}$.

4.1 Decomposition of Multi-valued Information Systems

The decomposition of $S=(U,A,V,f)$ is based on decomposition of $A=\{a_1,a_2,\cdots,a_{|A|}\}$ into the disjoint-singleton attribute $\{a_1\},\{a_2\},\cdots,\{a_{|A|}\}$. Here, we only consider for complete information systems. Let $S=(U,A,V,f)$ be an information system such that for every $a\in A$, $V_a=f(U,A)$ is a finite non-empty set and for every $u\in U$, $|f(u,a)|=1$. For every a_i under i th-attribute consideration, $a_i\in A$ and $v\in V_a$, we define the map $a_v^i:U\to\{0,1\}$ such that $a_v^i(u)=1$ if $f(u,a)=v$, otherwise $a_v^i(u)=0$. The next result, we define a binary-valued information system as a quadruple $S^i=(U,a_i,V_{\{0,1\}},f)$. The information systems $S^i=(U,a_i,V_{\{0,1\}},f)$, $1\le i\le |A|$ is referred to as a decomposition of a multi-valued information system $S=(U,A,V,f)$ into $|A|$ binary-valued information systems, as depicted in Figure 1. Every information system $S^i=(U,a_i,V_{ai},f)$, $1\le i\le |A|$ is a deterministic information system since for every $a\in A$ and for every $u\in U$, $|f(u,a)|=1$ such that the structure of a multi-valued information system and $|A|$ number of binary-valued information systems give the same value of attribute related to objects.

4.2 Multi-soft Sets in Information Systems

Based on the notion of a decomposition of a multi-valued information system in the previous sub-section, in this sub-section we present the notion of multi-soft set representing multi-valued information systems. Let $S=(U,A,V,f)$ be a multi-valued information system and $S^i=(U,a_i,V_{ai},f)$, $1\le i\le |A|$ be the $|A|$ binary-valued information systems. From Proposition 2, we have

$$S=(U,A,V,f)=\begin{cases} S^1=(U,a_1,V_{\{0,1\}},f) & \Leftrightarrow (F,a_1) \\ S^2=(U,a_2,V_{\{0,1\}},f) & \Leftrightarrow (F,a_2) \\ \quad\vdots & \quad\vdots \quad \vdots \\ S^{|A|}=(U,a_{|A|},V_{\{0,1\}},f) & \Leftrightarrow (F,a_{|A|}) \end{cases}$$

$$=\left((F,a_1),(F,a_2),\cdots,(F,a_{|A|})\right)$$

We define $(F,A)=\left((F,a_1),(F,a_2),\cdots,(F,a_{|A|})\right)$ as a *multi-soft sets* over universe U representing a multi-valued information system $S=(U,A,V,f)$.

U	a_1	a_2	$\cdots$	a_k	$\cdots$	$a_{\|A\|}$
u_1	$f(u_1,a_1)$	$f(u_1,a_2)$	$\cdots$	$f(u_1,a_k)$	$\cdots$	$f(u_1,a_{\|A\|})$
u_2	$f(u_2,a_1)$	$f(u_2,a_2)$	$\cdots$	$f(u_2,a_k)$	$\cdots$	$f(u_2,a_{\|A\|})$
u_3	$f(u_3,a_1)$	$f(u_3,a_2)$	$\cdots$	$f(u_3,a_k)$	$\cdots$	$f(u_3,a_{\|A\|})$
$\vdots$	$\vdots$	$\vdots$	$\ddots$	$\vdots$	$\ddots$	$\vdots$
$u_{\|U\|}$	$f(u_{\|U\|},a_1)$	$f(u_{\|U\|},a_2)$	$\cdots$	$f(u_{\|U\|},a_k)$	$\cdots$	$f(u_{\|U\|},a_{\|A\|})$

Binary-valued information system-1

	a_1					
U	Va_{11}	Va_{12}	$\cdots$	Va_{1k}	$\cdots$	Va_{1n}
u_1	0	1	$\cdots$	0	$\cdots$	0
u_2	0	0	$\cdots$	1	$\cdots$	0
u_3	0	1	$\cdots$	0	$\cdots$	0
$\vdots$	$\vdots$	$\vdots$	$\ddots$	$\vdots$	$\ddots$	$\vdots$
$u_{\|U\|}$	1	0	$\cdots$	0	$\cdots$	0

$\vdots$

Binary-valued information system-$|A|$

	$a_{\|A\|}$					
U	$Va_{\|A\|1}$	$Va_{\|A\|2}$	$\cdots$	$Va_{\|A\|k}$	$\cdots$	$Va_{\|A\|n}$
u_1	0	0	$\cdots$	1	$\cdots$	0
u_2	0	0	$\cdots$	0	$\cdots$	1
u_3	0	1	$\cdots$	0	$\cdots$	0
$\vdots$	$\vdots$	$\vdots$	$\ddots$	$\vdots$	$\ddots$	$\vdots$
$u_{\|U\|}$	1	0	$\cdots$	0	$\cdots$	0

Fig. 1. A decomposition of information systems

Example 4. *Hiring* [7]: an example of a multi-valued information system is presented in Table 2. In Table 2, six students are characterized by four conditional attributes; Diploma, Experience, French, Reference and one decision attribute; Decision.

Let $A=\{\text{Diploma, Experience, French, Reference, Decision}\}$. Therefore, the multi-soft set representing Table 2 is given in Figure 2. Note that the class value of every soft set is a partition of U.

Table 2. *Hiring*: an information system from [7]

Student	Diploma	Experience	French	Reference	Decision
1	MBA	Medium	Yes	Excellent	Accept
2	MBA	Low	Yes	Neutral	Reject
3	MCE	Low	Yes	Good	Reject
4	MSc	High	Yes	Neutral	Accept
5	MSc	Medium	Yes	Neutral	Reject
6	MSc	High	Yes	Excellent	Accept
7	MBA	High	No	Good	Accept
8	MCE	Low	No	Excellent	Reject

$$(F,A)=\begin{pmatrix}\{\text{MBA}=\{1,2,7\},\text{MCE}=\{3,8\},\text{MSc}=\{4,5,6\}\},\\ \{\text{Medium}=\{1,5\},\text{Low}=\{2,3,8\},\text{High}=\{4,6,7\}\},\\ \{\text{Yes}=\{1,2,3,4,5,6\},\text{No}=\{7,8\}\},\\ \{\text{Excellent}=\{1,6,8\},\text{Neutral}=\{2,4,5\},\text{Good}=\{3,7\}\},\\ \{\text{Accept}=\{1,4,6,7\},\text{Reject}=\{2,3,5,8\}\}\end{pmatrix}$$

Fig. 2. Multi soft-sets representing Table 2

5 Matrices Representation of Multi-soft Sets

5.1 Matrix Representation

The concept of matrix representing multi-soft sets is described here. In this sub-section, the notation (F,A) represents a multi-soft set over universe U of a multi-valued information system $S=(U,A,V,f)$.

$$(F,A)\Leftrightarrow\begin{cases}(F,a_1) \Leftrightarrow S^1=(U,a_1,V_{\{0,1\}},f) \Leftrightarrow M_{a_1}\\ (F,a_2) \Leftrightarrow S^2=(U,a_2,V_{\{0,1\}},f) \Leftrightarrow M_{a_2}\\ \quad\vdots \qquad\qquad \vdots \qquad\qquad \vdots\\ (F,a_{|A|})\Leftrightarrow S^{|A|}=(U,a_{|A|},V_{\{0,1\}},f)\Leftrightarrow M_{|A|}\end{cases}.$$

Definition 5. *Matrix* $M_{a_i}, 1\le i\le |A|$ *is called matrix representation of soft set* (F,a_1) *over universe U. The dimension of matrices is defined by* $\dim(M_{a_i})=|U|\times|Va_i|$. *All entries of* $M_{a_i}=[a_{ij}]$ *is belong to a set* $\{0,1\}$, *where*

$$a_{ij}=\begin{cases}0, & \text{if } |f(u,\alpha)|=0\\ 1, & \text{if } |f(u,\alpha)|=1\end{cases},\ 1\le i\le|U|,\ 1\le j\le|Va_i|,\ u\in U \text{ and } \alpha\in Va_i.$$

The collection of all matrices representing (F,A) *is denoted by* $\mathcal{M}_A$, *i.e.,*

$$\mathcal{M}_A=\{M_{a_i}:1\le i\le|A|\}.$$

Definition 6. *Let* $M_{a_i} \in \mathcal{M}_A$ *be a matrix representation of a soft set* (F, a_i) *over U. The value-class of* M_{a_i}, *i.e., class of all value sets of* M_{a_i}, *denoted* $C_{M_{a_i}}$ *is defined by*

$$C_{M_{a_i}} = \left\{ \{u : |f(u,\alpha_1)| = 1\}, \cdots, \{u : |f(u,\alpha_{|Va_i|})| = 1\} \right\},$$

where $1 \le i \le |Va_i|$, $u \in U$ and $\alpha \in Va_i$.
Clearly $C_{M_{a_i}} \subseteq P(U)$.

Example 7. The collection of matrices representing (F, A), is given as

$$\mathcal{M}_A = \left\{ M_{a_1}, M_{a_2}, M_{a_3}, M_{a_4}, M_{a_5} \right\},$$

where

$$M_{\text{Diploma}} = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 1 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}, \ M_{\text{Experience}} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix}, \ M_{\text{French}} = \begin{bmatrix} 1 & 0 \\ 1 & 0 \\ 1 & 0 \\ 1 & 0 \\ 1 & 0 \\ 1 & 0 \\ 0 & 1 \\ 0 & 1 \end{bmatrix},$$

$$M_{\text{Reference}} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{bmatrix} \text{ and } M_{\text{Decision}} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 0 & 1 \\ 1 & 0 \\ 0 & 1 \\ 1 & 0 \\ 1 & 0 \\ 0 & 1 \end{bmatrix}.$$

where

$$C_{M_{\text{Diploma}}} = \{\{1,2,7\},\{3,8\},\{4,5,6\}\},\ C_{M_{\text{Experience}}} = \{\{1,5\},\{2,3,8\},\{4,6,7\}\},$$
$$C_{M_{\text{French}}} = \{\{1,2,3,4,5,6\},\{7,8\}\},\ C_{M_{\text{Reference}}} = \{\{1,6,8\},\{2,4,5\},\{3,7\}\},$$
$$\text{and } C_{M_{\text{Decision}}} = \{\{1,4,6,7\},\{2,3,5,8\}\},$$

respectively.

5.2 AND and OR Operations in a Collection of Matrices

The AND and OR operation of the matrices are described in this sub-section.

Definition 8. *Let* $M_{a_i} = [a_{kl}]$, $1 \le k \le |U|$, $1 \le l \le |V_{a_i}|$ *and* $M_{a_j} = [a_{mn}]$, $1 \le m \le |U|$, $1 \le n \le |V_{a_i}|$ *be two matrices in* $\mathcal{M}_A$. *The* "AND" *operation* $M_{a_i}\text{AND}M_{a_j}$ *of matrices* M_{a_i} *and* M_{a_j} *is defined as follows*

$$M_{a_i}\text{AND}M_{a_j} = M_{a_{ij}} = [a_{pq}] \text{ with } \dim\left(M_{a_{ij}}\right) = |U| \times \left(|Va_i| \times |Va_j|\right),$$

where

$$a_{p1} = \min\{a_{k1}, a_{m1}\},\ a_{p2} = \min\{a_{k1}, a_{m2}\}, \ldots, a_{p\left(|Va_i| \times |Va_j|\right)} = \min\left\{a_{k|Va_i|}, a_{m|Va_j|}\right\}.$$

Example 9. *Let* $M_{\text{Diploma}}, M_{\text{Experience}} \in \mathcal{M}_E$, from Definition 8, we have

$$M_{\text{Diploma}}\text{AND}M_{\text{Experience}} = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 1 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix} \text{AND} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \end{bmatrix},$$

where

$$C_{M_{\text{Diploma}}\text{AND}M_{\text{Experience}}} = \{\{1\}, \{2\}, \{3,8\}, \{4,6\}, \{5\}, \{7\}\}.$$

Definition 10. *Let* $M_{a_i} = [a_{kl}]$ *and* $M_{a_j} = [b_{mn}]$ *be two matrices in* $\mathcal{M}_A$. *The* "OR" *operation* $M_{a_i}\text{OR}M_{a_j}$ *of matrices* M_{a_i} *and* M_{a_j} *is defined as follows*

$$M_{a_i}\text{OR}M_{a_j} = M_{a_{ij}} = [c_{pq}] \text{ with } \dim\left(M_{a_{ij}}\right) = |U| \times \left(|Va_i| \times |Va_j|\right),$$

where

$$a_{p1} = \max\{a_{k1}, a_{m1}\},\ a_{p2} = \max\{a_{k1}, a_{m2}\}, \ldots, a_{p\left(|Va_i| \times |Va_j|\right)} = \max\left\{a_{k|Va_i|}, a_{m|Va_j|}\right\}.$$

Example 11. *Let* $M_{\text{Diploma}}, M_{\text{Experience}} \in \mathcal{M}_E$, from Definition 10, we have

$$M_{\text{Diploma}}\text{OR}M_{\text{Experience}} = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 1 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix} \text{OR} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 & 1 & 0 & 0 & 1 & 0 & 0 \\ 1 & 1 & 1 & 0 & 1 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 & 1 & 1 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 & 1 & 1 & 1 \\ 1 & 0 & 0 & 1 & 0 & 0 & 1 & 1 & 1 \\ 0 & 0 & 1 & 0 & 0 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 0 & 0 & 1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 & 1 & 1 & 0 & 1 & 0 \end{bmatrix},$$

where

$$C_{M_{\text{Diploma}}\text{OR}M_{\text{Experience}}} = \{\{1\},\{2\},\{3,8\},\{4,6\},\{5\},\{7\}\}.$$

We can see that, since $\dim\left(M_{a_{ij}}\right)=|U|\times\left|Va_i \cup Va_j\right|$, thus $\mathcal{M}_A$ is not closed under AND and OR operations, thus $(\mathcal{M}_A, \text{AND})$ and $(\mathcal{M}_A, \text{OR})$ are not groupoids.

Proposition 12. *Let M_{a_i}, M_{a_j} and M_{a_k} be matrices in $\mathcal{M}_A$. The following properties are holds.*

a. $M_{a_i}\text{AND}M_{a_i} = M_{a_i}$, *idempotent*

b. $M_{a_i}\text{OR}M_{a_i} = M_{a_i}$, *idempotent*

c. $M_{a_i}\text{AND}\left(M_{a_j}\text{AND}M_{a_k}\right)=\left(M_{a_i}\text{AND}M_{a_j}\right)\text{AND}M_{a_k}$ *associative*

d. $M_{a_i}\text{OR}\left(M_{a_j}\text{OR}M_{a_k}\right)=\left(M_{a_i}\text{OR}M_{a_j}\right)\text{OR}M_{a_k}$ *associative*

AND and OR operations are not commutative.

6 Application

In this section, we present an application of matrices representation of multi-soft sets for dimensionality reduction and finding core attributes.

6.1 Reduct and Core

In [4], we presented the application of multi-soft sets for finding reducts. The proposed approach is based on AND operation in multi-soft sets [3]. In this sub-section, we explore the applicability of matrices representation for finding reducts and core attributes. We show that the reducts obtained are equivalent with that in [4]. The notion of reduct in [4] is redefined and given as follow.

Definition 13. *Let $(F,A)=\left((F,a_i):1\le i\le|A|\right)$ be multi-soft set over U representing a multi-valued information system $S=(U,A,V,f)$. A set of attributes $B\subseteq A$ is called a reduct for A if*

$$C_{F\left(b_1\times\cdots\times b_{|B|}\right)} = C_{F\left(a_1\times\cdots\times a_{|A|}\right)} \text{ and } C_{F\left(b_1\times\cdots\times b_{|B^*|}\right)} \ne C_{F\left(a_1\times\cdots\times a_{|A|}\right)},\ B^*\subset B,$$

where

$$\left(F,a_i\times a_j\right)=\left(F,a_i\right)\text{AND}\left(F,a_j\right).$$

A core of A is defined as

$$\text{CORE}(A)=\bigcap \text{RED}(A),$$

where $\text{RED}(A)$ *is the set of all reducts of A.*

Proposition 14. *Let $\mathcal{M}_A$ be a multi-soft set over U representing multi-soft sets (F,A). A set of attributes $B \subseteq A$ is a reduct for A if only if $C_{ANDM_b : b \in B} = C_{ANDM_a : a \in B}$.*

Proof. It is clear, since every (F,a_i) can be represented as M_{a_i}, $1 \le i \le |A|$ and vice versa. □

Example 15. For simple example of soft reduction in multi-valued information system, we consider a small dataset as used in [4].

Table 3. An information system

U	a_1	a_2	a_3	a_4
1	low	bad	loss	small
2	low	good	loss	large
3	high	good	loss	medium
4	high	good	loss	medium
5	low	good	profit	large

Let $A = \{a_1, a_2, a_3, a_4\}$, the multi-soft set representing Table 3 is given as

$$(F,A) = \begin{pmatrix} \{\{\text{low} = 1,2,5\},\{\text{high} = 3,4\}\},\{\{\text{bad} = 1\},\{\text{good} = 2,3,4,5\}\}, \\ \{\{\text{loss} = 1,2,3,4\},\{\text{profit} = 5\}\},\{\{\text{small} = 1\},\{\text{large} = 2,5\},\{\text{medium} = 3,4\}\} \end{pmatrix},$$

and the matrices representing (F,A) is $\mathcal{M}_A = \{M_{a_1}, M_{a_2}, M_{a_3}, M_{a_4}\}$, where

$$M_{a_1} = \begin{bmatrix} 1 & 0 \\ 1 & 0 \\ 0 & 1 \\ 0 & 1 \\ 1 & 0 \end{bmatrix}, \; M_{a_2} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 0 & 1 \\ 0 & 1 \\ 0 & 1 \end{bmatrix}, \; M_{a_3} = \begin{bmatrix} 1 & 0 \\ 1 & 0 \\ 1 & 0 \\ 1 & 0 \\ 0 & 1 \end{bmatrix} \text{ and } M_{a_4} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix}.$$

Since $A = \{a_1, a_2, a_3, a_4\}$, then

$$M_{ANDa_i : 1 \le i \le 4} = \begin{bmatrix} 1&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0 \\ 0&0&0&0&0&0&0&1&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0 \\ 0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&1&0&0&0 \\ 0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&1&0&0&0 \\ 0&0&0&0&0&0&0&0&0&0&1&0&0&0&0&0&0&0&0&0&0&0&0&0 \end{bmatrix}$$

where

$$C_{M_{ANDa_i : 1 \le i \le 4}} = \{\{1\},\{2\},\{3,4\},\{5\}\}.$$

Let, given $B=\{a_1,a_2,a_3\}$ and $C=\{a_3,a_4\}$, then we have

$$M_{a_1}\text{AND}M_{a_2}\text{AND}M_{a_3}=\begin{bmatrix}1&0&0&0&0&0&0&0\\0&0&1&0&0&0&0&0\\0&0&0&0&0&0&1&0\\0&0&0&0&0&0&1&0\\0&0&0&1&0&0&0&0\end{bmatrix}$$

and

$$M_{a_3}\text{AND}M_{a_4}=\begin{bmatrix}1&0&0&0&0&0\\0&1&0&0&0&0\\0&0&1&0&0&0\\0&0&1&0&0&0\\0&0&0&0&1&0\end{bmatrix}$$

where

$$C_{M_{a_1}\text{AND}M_{a_2}\text{AND}M_{a_3}}=\{\{1\},\{2\},\{3,4\},\{5\}\}\text{ and }C_{M_{a_1}\text{AND}M_{a_2}\text{AND}M_{a_3}}=\{\{1\},\{2\},\{3,4\},\{5\}\},$$

respectively. Thus, we have $\{a_1,a_2,a_3\}$ and $\{a_3,a_4\}$ are reducts of *A*. Furthermore, core is

$$\{a_1,a_2,a_3\}\cap\{a_3,a_4\}=\{a_3\}.$$

6.2 Equivalence with Pawlak's Rough Reduct

In rough set theory [2,5], the aim of dimensionality reduction is to keep only those attributes that preserve the indiscernibility relation and, consequently, set approximation. The remaining attributes are redundant since their removal does not worsen the classification. There are usually several such subsets of attributes and those which are minimal are called reducts. While computing equivalence classes is straightforward, finding all reducts is NP-hard. This reduction is different in soft set theory. The existing techniques of soft parameterization are still based on a binary information system [8,9]. Soft parameterization reduction is obtained based on the optimal and sub-optimal choice related to each object. Thus, the idea of reduction under rough set theory generally cannot be applied directly in reduction under soft set theory. In this sub-section, we show that our proposed technique on dimensionality reduction in an multi-valued information system is equivalent with that in [2,5]. Let, firstly, we recall the notion of rough reduction as follow.

Let $S=(U,A,V,f)$ be an information system and let *B* be any subset of *A*. Two elements $x,y\in U$ are said to be *B-indiscernible* (indiscernible by the set of attribute *B*) if and only if $f(x,a)=f(y,a)$, for every $a\in B$. Obviously, every subset of *A* induces unique indiscernibility relation. Notice that, an indiscernibility relation is an *equivalence relation*. The *partition* of *U* induced by $IND(B)$ in $S=(U,A,V,f)$ is denoted by U/B.

Definition 16. *Let $S=(U,A,V,f)$ be an information system and let B be any subset of A. A subset $B\subseteq A$ is called a reduct of A if B satisfies the following conditions*

a. $U/B=U/A$
b. $U/(B-\{b\})\neq U/A, \quad \forall b\in B$

A core of A is defined as

$$\mathrm{CORE}(A)=\bigcap \mathrm{RED}(A),$$

where $\mathrm{RED}(A)$ is the set of all reducts of A.

Proposition 17. *Soft reduction in Definition 13 is equivalent with rough reduction in Definition 16.*

Proof. $(\Rightarrow)$ It is clear that $(F,A)=(U,A,V,f)$, thus $C_{F(a_i)}=U/a_i,\ 1\leq i\leq |A|$. From the hypothesis, since *B* is a reduct of *A*, then

$$\begin{gathered}
C_{F\left(b_1\times\cdots\times b_{|B|}\right)}=C_{F\left(a_1\times\cdots\times a_{|A|}\right)}\\
(F,b_1)\mathrm{AND}\cdots\mathrm{AND}\left(F,b_{|B|}\right)=(F,a_1)\mathrm{AND}\cdots\mathrm{AND}\left(F,a_{|A|}\right)\\
C_{F(b_1)}\cap\cdots\cap C_{F\left(b_{|B|}\right)}=C_{F(a_1)}\cap\cdots\cap C_{F\left(a_{|A|}\right)}\\
\bigcap_{1\leq i\leq |B|}C_{F(b_i)}=\bigcap_{1\leq i\leq |A|}C_{F(a_i)}\\
\bigcap_{1\leq i\leq |B|}U/b_i=\bigcap_{1\leq i\leq |A|}U/a_i\\
U/B=U/A
\end{gathered}$$

Say, $B^*=B-\{b\}$, where $b\in B$, then

$$\begin{gathered}
C_{F\left(b_1\times\cdots\times b_{|B^*|}\right)}\neq C_{F\left(a_1\times\cdots\times a_{|A|}\right)}\\
U/(B-\{b\})\neq U/A
\end{gathered}$$

$(\Leftarrow)$ Obvious. □

Example 18. From Table 2, let

$$B=\{\text{Diploma, Experience, French, Reference}\}.$$

Using rough set approach, we have

$$U/B=\{\{1\},\{2\},\{3\},\{4\},\{5\},\{6\},\{7\},\{8\}\}.$$

Therefore, reduct of *B* is $B^*=\{\text{Experience, Reference}\}$.

For matrices approach, from Example 7, we have

$$M_{\text{Diploma}}\text{AND}M_{\text{Experience}}\text{AND}M_{\text{French}}M_{\text{Reference}} =$$

$$= \begin{bmatrix} 1 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 1 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix} \text{AND} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix} \text{AND} \begin{bmatrix} 1 & 0 \\ 1 & 0 \\ 1 & 0 \\ 1 & 0 \\ 1 & 0 \\ 1 & 0 \\ 0 & 1 \\ 0 & 1 \end{bmatrix} \text{AND} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{bmatrix}$$

The calculation of the above AND operation is given as follows

$$= \left(\begin{bmatrix} 1 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 1 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix} \text{AND} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix} \right) \text{AND} \left(\begin{bmatrix} 1 & 0 \\ 1 & 0 \\ 1 & 0 \\ 1 & 0 \\ 1 & 0 \\ 1 & 0 \\ 0 & 1 \\ 0 & 1 \end{bmatrix} \text{AND} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{bmatrix} \right)$$

$$= \left(\begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \end{bmatrix} \right) \text{AND} \left(\begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 & 0 \end{bmatrix} \right)$$

$$= \begin{bmatrix}
1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0
\end{bmatrix}$$

Thus we have

$$C_{M_{\text{Diploma}}\text{AND}M_{\text{Experience}}\text{AND}M_{\text{French}}M_{\text{Reference}}} = \{\{1\},\{2\},\{3\},\{4\},\{5\},\{6\},\{7\},\{8\}\},$$

Therefore, reduct of B is $B^* = \{\text{Experience}, \text{Reference}\}$. It is shown that, using rough and matrices approaches, reduct obtained is equivalent.

7 Conclusion

The "standard" soft set deals with a binary-valued information system. For a multi-valued information system, the notion of multi-soft sets has been proposed. The idea of multi-soft sets is based on a decomposition of a multi-valued information system into binary-valued information systems. In this paper, we have presented the notion of matrices representation of such multi-sot sets. The AND and OR operations in a collection of matrices sets also presented. The AND and OR operations satisfy idempotent and associative properties, neither closed nor commutative. Further, we have proven that AND operation can be applied for finding reducts. Finally, we have proven that our proposed technique provide the same results with that rough reduct. For future research, we further elaborate our approach for finding functional, identity and attribute dependencies in information systems. Since, the calculation of matrices is complex, we will use MATLAB to solve this problem. We believe that our proposed approach can be used for feature selection and data cleansing in multi-valued information system under soft set theory.

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