Modelling, Analysis and Stability of Milling Processes Including Workpiece Effects

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Summary. A common model for milling dynamics, i.e. a harmonic oscillator, was extended by a continuous, linear thermo-elastic model describing the dynamic behaviour of the workpiece. A widely studied, empirical cutting force model is used to describe the coupling of both systems. Finally, a numerical solution strategy for the coupled system is outlined and complemented by numerical simulations that show the work piece effect on the stability of the cutting process.

1 Introduction

A milling machine is a machine tool for the shaping of metal or other solids. Its basic components are a rotating cutter, a spindle, a z-slider that is attached to a moving portal and a table on which the workpiece is mounted. The modelling of milling dynamics, the determination of stable cutting conditions and the design of more efficient milling machines are important research fields in production technology. Effective methods to predict stable processes have been developed in recent years [2, 6]. An essential part of these methods is an abstract dynamical model that reproduces the local characteristics of the actual milling system in terms of the dynamics at the tip of the cutter. In combination with a process model to describe the cutting forces it leads to a delay-differential equation (DDE), whose stability characteristics have been widely studied in the last decades [6, 8].

The focus of this paper is a detailed study of the machine work piece interactions. Hence, in addition to the DDE model for the cutter the workpiece is accounted for by a visco-elastic material model. The coupling is realised through the cutting force. This approach allows for a refined stability analysis and will eventually lead to a refined prognosis of stable cutting conditions.

The paper is organised as follows: In Sect. 2 we derive the model equations. An algorithm for the numerical approximation of the new milling model is outlined in Sect. 3. Numerical results for different scenarios corresponding to stable and unstable cutting conditions are included. The last section is devoted to some concluding remarks.

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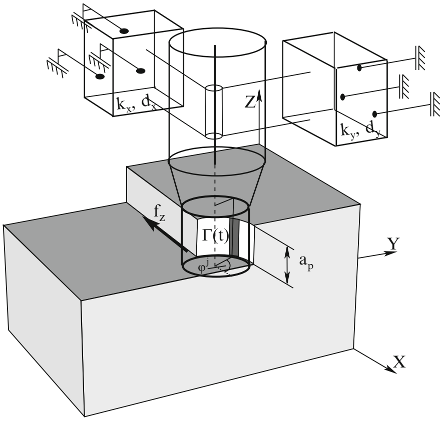

Fig. 1. Schematic representation of the milling process

2 Modelling

2.1 Model Equations

To avoid technicalities we represent the machine dynamics by an harmonic oscillator equation for the centre of mass $q = (q_x, q_y, q_z)^T$, of the cutter with mass m_c, written in the inertial reference frame (x, y, z), cf. Fig. 1. Note that this model reproduces at least one relevant mode of the milling machine. Furthermore, the cutter oscillates only in the x, y-plane. The additional z-component has been introduced to ease the coupling with the workpiece model. The coordinates in the oscillator reference frame are related to those of the workpiece reference frame by a linear, time dependent transformation $(x, y, z) = (X, Y, Z) - b(t)$, where $b(t) = (X_0 - f_z \frac{t}{\tau}, Y_0, Z_0)$ denotes the translation vector given in the workpiece frame. Hence, the equation of motion for the cutter model reads:

$$m_c \ddot{q} + D\dot{q} + Kq = [F_x, F_y, 0]^T, \tag{1}$$

where D denotes the damping and K the stiffness matrix. The right hand side of (1) takes into account the cutting force, a sum of the forces acting on each tooth in cut.

We assume that the largest part of the workpiece behaves like a visco-elastic solid. Only in the vicinity of the cutting edge, visco-elasto-plastic effects have to be taken into account. To some extend, these effects are already included in the empirical cutting force model. Therefore, we focus here on the visco-elastic behaviour of the workpiece, which we model with the standard equations of linear visco-elasticity with Kelvin–Voigt damping (see, e.g., [5]) assuming small strains.

2.2 Coupling

Boundary Conditions

The basis for the coupling of workpiece and machine model is the cutting force. Usually, the latter is computed in terms of the so-called *uncut chip thickness* (see, e.g., [1]), which describes the thickness of the material to be removed by the tooth which is in cut . Here we use the following algebraic relation between uncut chip thickness and the cutting forces due to Weck [9]:

$$\hat{F} = \left(\hat{F}_R, \hat{F}_T, \hat{F}_Z\right)^T = a_P \hat{K}(v_C) \max(h, 0), \tag{2}$$

where $\hat{K}(v_C)$ denotes the vector of cutting constants which can be a function of the cutting speed v_C [6]. Note that the precise form of $\hat{K}(v_C)$ has to be found experimentally.

An expression for the uncut chip thickness h can be derived by considering a two dimensional, independently vibrating work-piece-cutter system, for details we refer to [3]:

$$h = -f_z \cos\varphi^j + (q(t) - q(t-\tau)) \cdot e_r^j - (u(t, R_P) - u(t-\tau, R_Q)) \cdot e_r^j. \tag{3}$$

Here, R_P and R_Q denote the two work piece material points being currently machined. We notice that the uncut chip thickness consists of three different parts. The first one represents just the cutter displacement due to the given feed f_z. Projected on the radial direction, it yields the stationary uncut chip thickness. The second part represents the machine oscillations and produces the modulation of the chip thickness that has been identified to be the main reason for chatter. The third contribution to the uncut chip thickness is related to the workpiece deformation.

On the cutting edge the forces act in three directions: perpendicular to the cutting velocity, in opposite direction to the cutting velocity and parallel to the rotation axis of the cutter. Note that the z-component of $\hat{K}(v_C)$ vanishes for orthogonal cutting. We transform (2) into the workpiece reference frame and sum up for all teeth to obtain:

$$F = (F_x, F_y, F_z)^T = -\sum_{j=1}^{N_z} g(\varphi^j(t)) O(\varphi^j(t)) \hat{F}. \tag{4}$$

Here, $g = 1$ if the corresponding tooth ‘j’ is in cut and $g = 0$, otherwise. The orthogonal matrix $O(\varphi^j)$ transforms the forces $\hat{F}$ into the workpiece reference frame. N_z denotes the number of cutter teeth.

Now we define the boundary condition for the momentum balance equation as:

$$u = 0 \qquad \text{on} \qquad \Gamma_D,$$

$$\sigma \cdot n = \begin{cases} \frac{F}{|\Gamma(t)|} & \text{on } \Gamma(t) \times (0, t_e). \\ 0 & \text{otherwise.} \end{cases} \tag{5}$$

$|\,\Gamma(t)\,|$ denotes the measure of the area, where the cutting force acts on the workpiece (cf. Fig. 1).

3 Numerical Simulations

3.1 The Algorithm

We present a straightforward numerical algorithm to compute an approximate solution of the coupled system in time domain. Recall that in a milling process with nonzero feed material is removed from the workpiece. Since we use a visco-elastic work piece representation that cannot take the chip removal into account directly, we introduce the following approximation of this process. At first we divide the time interval $[0, t_e]$ into subintervals with the length τ, tacitly assuming t_e to be a multiple of τ. Based on these subintervals, we construct a sequence of space-time-cylinders $Q_l = \Omega_l \times (k\tau, (k+1)\tau]$ with an update of the work piece shape according to the theoretical tooth path given by the static chip thickness (i.e. the first term in (3)).

This approach allows us to use the methods of steps to solve the coupled PDE/DDE system: For given initial data on the interval $[-\tau, 0]$ we solve the system in $[0, \tau]$. Then iteratively, we use the solution in $((l-1)\tau, l\tau]$ as initial data for the following tooth period and perform the analysis for the interval $(l\tau, (l+1)\tau]$. With the help of this technique we proved the existence of a unique weak solution of the coupled system in the entire time interval $[0, t_e]$, see e.g. [3].

With the above considerations we may now introduce a time integration scheme for an arbitrary tooth period $(l\tau, (l+1)\tau]$. To this end, we discretise the pde part of the coupled system with linear finite elements in space. Thus, we obtain a system of ordinary differential equations with delay. In our time stepping strategy we make use of an incremental decoupling such that the momentum balance can be solved in each time step with a Newmark scheme [7]. We integrate the remaining DDE with a standard ode-solver, i.e. Runge–Kutta-54 [4]. All retarded and coupling terms are provided by means of interpolation. The data transfer from one tooth period to another is also carried out by interpolation.

Note that the presented method is restricted to cutting conditions where only one tooth is in cut. However, form a practical point of view this poses no severe limitation.

3.2 Simulation Results

The system of equations has unstable and stable solutions depending on the parameters rotation speed n, number of teeth N_z, axial depth of cut a_p and radial depth of cut. For the simulations we fix the cutting parameters $f_z = 0.2\,\text{mm}$, $N_z = 4$, $D = 15\,\text{mm}$ and $n = 7{,}500\,\text{rpm}$. For the first example the

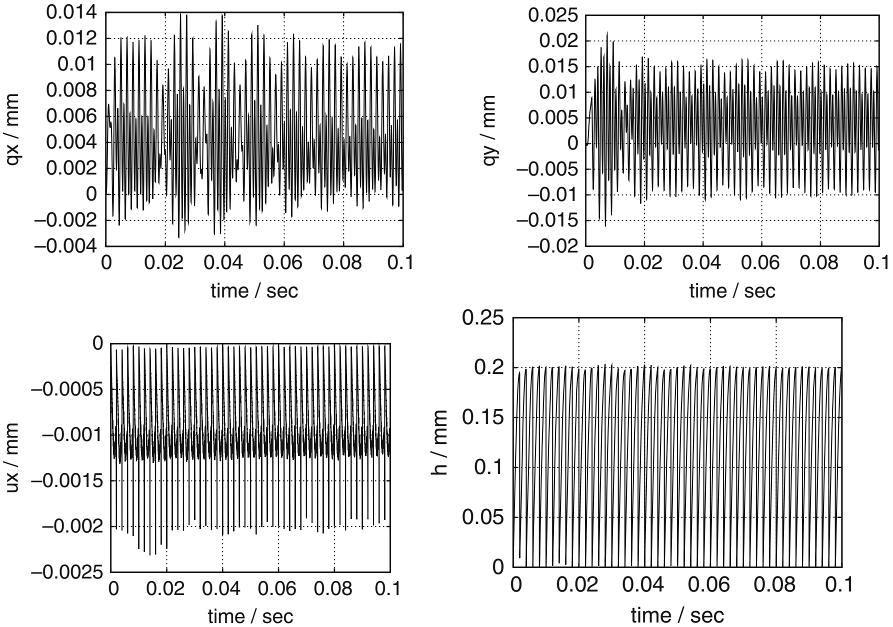

Fig. 2. x,y-component of the cutter vibrations (q_x, q_y), mean workpiece deformations (u_x) at the cutting edge and the uncut chip thickness (h)

axial depth of cut is $a_p = 1\,\text{mm}$, the entry and exit angles are $\varphi_s = \pi/2$, $\varphi_E = \pi$ and we have chosen a rather rigid workpiece geometry, corresponding to the work piece depicted in Fig. 1. We set $t_e = 50\tau$, i.e., for the choice of n and N_z, $t_e = 0.1\,\text{s}$. In Fig. 2, we see the results of the first simulation run. Since the uncut chip thickness converges to a stationary state, we identify this milling process as stable. The induced deformations do not interfere with the stable cutting conditions. The workpiece deformations are one order of magnitude smaller than the cutter vibrations.

The second example shows the work piece effect on the stability of milling process. The cutter was assumed to be rigid, which means that only the work piece may cause unstable cutting conditions. We performed the simulations for a rather rigid work piece geometry, as shown in Fig. 1 and a beam-like work piece which has a low bending stiffness. The cutting parameters are $a_p = 15\,\text{mm}$, $\varphi_s = \pi/2$ and $\varphi_E = 3\pi/4$ and we simulated 20 tooth periods, i.e. for the choice of n and N_z, $t_e = 0.04\,\text{s}$. While the uncut chip thickness converges for the stable workpiece geometry to the stationary evolution, we observe for the beam like, unstable workpiece geometry a divergence of the uncut chip thickness indicating the occurrence of unstable cutting conditions, i.e. chatter.

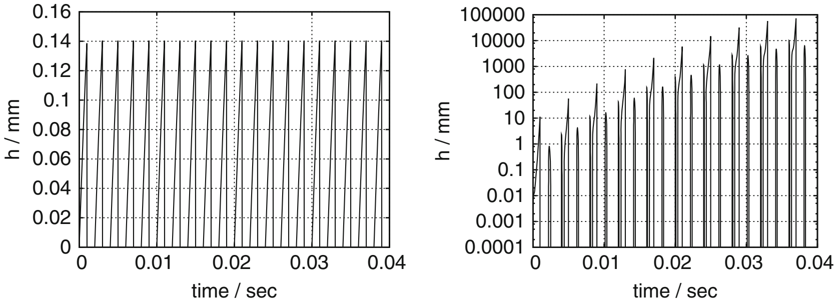

Fig. 3. Uncut chip thickness for a stable and an unstable work piece

4 Conclusions

The goal of this paper was to enhance existing models of the milling process to allow for the consideration of the workpiece influence. The simulations in Sect. 3 show that the model is capable of reproducing instability effects due to a lack of workpiece stiffness.

The results are promising and open up various directions for future research, such as the development of an efficient numerical tool for the systematic derivation of stability diagrams for the coupled system.

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