Introduction of Symbolic Simplified Expressions in Circuit Optimization

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Summary. The computation time required for the analysis and optimization of a complete circuit can be too long. The use of symbolic analysis together with model order reduction techniques can reduce it and make the optimization a practicable way in the circuit design. To evaluate the possibilities offered by the technique, firstly a linear test case has been considered. The problem of an inductor simulation has been analyzed by introducing simplified analytical expressions and different optimization algorithms in the fitting/optimization process. Then the optimization technique has been applied to a real circuit, a voltage reference, trying to improve the stability of the reference over the temperature.

1 Introduction

Optimization techniques, symbolic analysis and simplification techniques have been applied to two examples of electronics circuits: firstly to an equivalent lumped circuit of a micro inductor, and then to a band voltage reference. Both of them are important circuital blocks really employed in the nowadays electronics. A band voltage reference (BVR) is a voltage reference that gives an output proportional to the band gap energy of a transistor. The BVR circuit balances the negative temperature coefficient of a pn junction with the positive temperature coefficient of the "thermal voltage": Vt = kT\q. Integrated inductors improve both reliability and efficiency of silicon-integrated RF cells; they can offer circuit solutions with superior performance and contribute to a higher level of integration The inductance of an integrated inductor can be computed exactly by solving Maxwell's equations [1] but to facilitate the design of such components, significant work has gone into modelling spiral inductors using lumped circuit models that takes into account the parasitic resistors and capacitors. Generally, the adopted method to extract model

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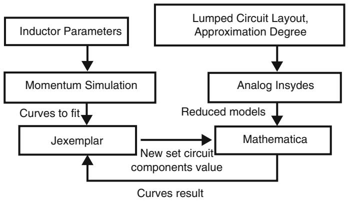

Fig. 1. Inductor Simulation Flow diagram

parameters from an event, is based on the Least Squares Method, that is, minimizing the l^2 norm of the difference between the output and the measured (or required) value, and consists in an optimization problem. In our research work six different optimization methods, both deterministic and stochastic have been tested, in combination with symbolic analysis and simplification techniques, for the fitting of inductor Y parameters. Symbolic analysis is a formal technique to write and solve equations describing circuits behaviour without introducing the numerical value of variables which are symbolically represented. Dimensions of symbolic expression of a circuit actually increase rapidly with the complexity of the network, and in order to make use of the symbolic technique not limited to small circuits, it is necessary to simplify generated expressions by keeping the *dominant terms* only.

2 Inductor Simulation Flow

A flow to permit designers to apply precise inductor model in their integrated circuits simulations was developed by the STMicroelectronics CAD group [8]. Following Fig. 1 it is possible to see how the simulation flow has been modified by introducing the use of Jexemplar, Mathematica and Analog Insydes software in order to obtain and employ an approximated Y parameters symbolic expression. This expression is used by the fitting process in order to find the appropriate set of circuit component values and to reproduce the *target curves*. Now the employed optimization software is Jexemplar; it reads the file with the numerical expression of Y parameters and compares it with the target file, then it generates a new set of component values or ends the process if the fixed approximation is rejoined. Mathematica has the duty to calculate and save the numerical value of Y parameters using the approximated symbolic Y parameters expression obtained by Analog Insydes. Analog Insydes, making use of the Mathematica calculus routines and, receiving as input the description of the inductor lumped circuit, provides the approximated Y parameters expressions. The approximation software offers the possibility to approximate equation systems both before and after the symbolic solution of the systems [10], and both these approximation techniques have been applied.

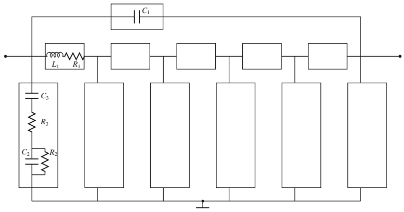

Fig. 2. Inductor layout

Table 1. Lower and upper bounds of the Inductor problem variables

VAR.	LOWER BOUND	UPPER BOUND	VAR.	LOWER BOUND	UPPER BOUND
Rb1	1.00×10^{-8}	1.00×10^{-4}	Cox1	1.00×10^{-17}	1.00×10^{-13}
Rb2	1.00×10^{-4}	1	Cox2	1.00×10^{-17}	1.00×10^{-13}
Rox1	1.00×10^{-6}	1.00×10^{-2}	Rl	1.00×10^{-4}	1
Rox2	1.00×10^{-4}	1	LL	1.00×10^{-13}	1.00×10^{-9}
Cb1	1.00×10^{-26}	1.00×10^{-22}	Cl	1.00×10^{-21}	1.00×10^{-17}
Cb2	1.00×10^{-25}	1.00×10^{-21}			

The structure of the lumped circuit is fixed, the inductor is represented as an equivalent distributed inductor model with a variable cell number to better describe the inductor behaviour at high frequencies [1,9]. The complete inductor model has the layout illustrated in Fig. 2.

The computer platform used was based on Xeon 3.2 Ghz Intel processor 32 bit, RedHat Enterprise Linux release 3 update 4 operating system. We have tested six of the state-of-the-art optimization algorithms for real world applications; in particular, we use *Controlled Random Search* (CRS) [4], *Controlled Random Search Enhanced* (CRS-E) [5], the *immune algorithm* (OPTIA) [6] *Powell's algorithm* [2], *Direct method* (DIRECT) [3] and *Differential Evolution*, DE [7].

3 Simulations

3.1 Inductor

The optimization process has been executed with both approximated and not approximated Y parameter expressions. Our black box function takes as input 11 variables which bounds are presented in Table 1.

Three different simplified expressions of the Y parameters obtained with a maximum relative error of 10^{-1}, 10^{-4} and 10^{-6} have been tested. For each algorithm, we fixed the number of objective function evaluation to 10^4 and a tolerance factor value of $\epsilon = 10^{-12}$; if the algorithm find a configuration with a Y value that differs less than ϵ from the target, it will be stopped. In addiction, for the DIRECT algorithm, we stop the algorithm if the volume of the hyper rectangle is less than 10^{-12}. Moreover, for the OPTIA algorithm, we use a population of ten candidate solutions, $d = 10$, a duplication parameter value equal to two, *dup* = 2, and the elitist selection operator. For DE algorithm, after a series of test to individuate the best option, we set the real factor which controls the amplification of the differential variation to 0.2 and the crossover probability to 0.5. Finally, for DIRECT and POWELL algorithms, we set a initial point where each variable is centred into the given lower and upper bound, instead for CRS, CRS-E and OPTIA we start totally from random points.

By inspecting the results we can note that the best found solution for the circuit using no approximation is clustered at $Y = 4.86 \times 10^{-3}$; excluding DIRECT, every algorithm reaches this solution and the difference between each algorithm is negligible.

Analyzing the results using the approximated function at different level of tolerance, Table 2, we can observe that the algorithms are robust and the results scale accordingly to the fixed tolerance; in Fig. 3, we show the convergence plot. Each algorithm found a solution that generally differs from the one obtained using a non approximated model of 0.8×10^{-3} and this suggests that the model is accurate enough to perform a complete optimization of the sizing of the circuit using only the symbolic model.

From an optimization point of view, we can infer that the produced approximated expression can be optimized with both deterministic and stochastic algorithms obtaining comparable results; instead, by inspecting the convergence plots, we can note that the stochastic approach guarantees a better speed of convergence than the deterministic ones.

3.2 Band Voltage Reference

The objective of the optimization with the BVG circuit was to stabilize, working on the project parameters, the reference voltage to 650 mv over a temperature interval between −40 and +125°C, and with two different values of the voltage supply 0.9 and 1.2 V. The optimization variables are the physical dimensions of the circuit components; designers suggested a first list of 32 parameters, then a sensitivity analysis made with the functionalities of ELDO by MENTOR GRAPHICS highlighted the three most important variables to set the output voltage. The variables have been considered continuous. The result of the sensitivity analysis is a consequence of the circuit structure: the voltage

Table 2. Performance of the optimization algorithms using the symbolic model with different tolerance settings

Algorithm	10^{-6}	10^{-4}	10^{-1}
CRS	$\mathbf{4.871 \times 10^{-3}}$	$\mathbf{5.111 \times 10^{-3}}$	5.621×10^{-3}
CRS-E	$\mathbf{4.871 \times 10^{-3}}$	5.113×10^{-3}	5.621×10^{-3}
POWELL	4.871×10^{-3}	5.137×10^{-3}	5.624×10^{-3}
DIRECT	9.828×10^{-3}	8.477×10^{-3}	9.144×10^{-3}
OPTIA	4.872×10^{-3}	5.134×10^{-3}	$\mathbf{5.616 \times 10^{-3}}$
DE	4.875×10^{-3}	5.308×10^{-3}	$\mathbf{6.016 \times 10^{-3}}$

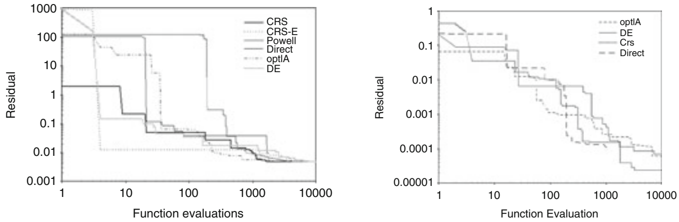

Fig. 3. *Left*: Inductor convergence plot with the error set to 10^{-6}. CRS algorithm obtains the best convergence curve. *Right*: BVR convergence plot

Table 3. Inductor : performance of the optimization algorithms without approximations

ALGORITHM	BEST RESIDUAL	ALGORITHM	BEST RESIDUAL
DE	$\mathbf{2.402 \times 10^{-5}}$	CRS	6.9564×10^{-5}
OPTIA	6.0517×10^{-5}	DIRECT	1.1019×10^{-4}

reference is generated by injecting a controlled current on the resistors. In this configuration the role of the resistors is crucial.

By inspecting the results, Table 3, it is possible to see that the performances of all the algorithms, with the exception of DIRECT, are good; DE showed to be able to get a results slightly better. In Fig. 3.1 is showed the convergence plot for the algorithms tested. In Fig. 4 is plotted the curve voltage vs temperature for the circuit both optimized that not optimized. It is possible to see how the The optimization has reduced the peak to peak value of the curve but has also moved maximum versus lower temperature, instead of keeping it at 27°C.

4 Conclusion

The possibility to introduce simplification techniques in the optimization flow of an inductor simulation has been evaluated; as a test case was used the spiral inductor. The symbolic circuit simulator ANALOG INSYDES was linked with an optimization framework, which take into account deterministic and stochastic optimization algorithms. The results show that it has been possible to find solutions using the simplified parameter expressions comparable to the solutions obtained without approximated expressions. It has been proved that the time is dramatically decreased. In order to extend the same strategy to more complicate circuits, we started by verifying the utility of the application of optimization techniques to a band voltage reference circuit. The result obtained has showed the possibility to improve the designers job; future works are led to complete the flow by applying the symbolic analysis also to similar circuits.

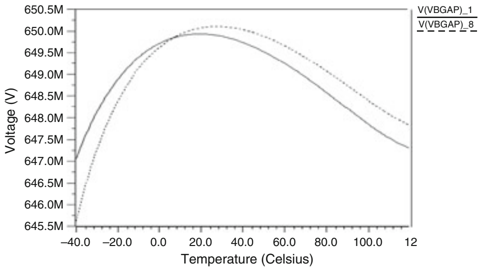

Fig. 4. Voltage vs temperature, comparison between optimized solution (*continuous line*)and starting one *dotted line*)

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