An Update of Hopkins' Analysis of the Optical Disc Player Using Singular-System Theory

Roy Pike

Clerk Maxwell Professor of Theoretical Physics, King's College, London, UK
roy.pike@kcl.ac.uk

Summary. In this paper we describe a new approach to the analysis of scanning optical imaging systems which uses singular function expansions, rather than Fourier optics, to update the well-known low-aperture treatment of Hopkins of 1979 (J. Opt. Soc. Am. 69:4–24). This new approach can also be used to update the widely used theory of optical transfer functions for general imaging systems at arbitrary numerical apertures.

1 Introduction

We consider an optical system with an illumination lens and an objective lens combination as depicted schematically in Fig. 1.

The same lens serves for both illumination and imaging functions in a reflective system. This is a special case of the general partially coherent optics described, for example, in early work of Hopkins [3]. We will use $\mathbf{x}$, $\mathbf{k}$ and $\mathbf{y}$ as the two-dimensional disc-plane, pupil-plane and image-plane coordinates, respectively. The action of each lens is described by a linear integral equation, relating object $f(\mathbf{x})$ to image $g(\mathbf{y})$,

$$g(\mathbf{y}) = \int d\mathbf{x} W(\mathbf{y} - \mathbf{x}) f(\mathbf{x}), \tag{1}$$

where W is its point-spread function (PSF). Following Hopkins, even in the reflective case, we allow the incoming and outgoing illumination to be described using different PSFs, W_{in}, and W_{out}, respectively, which can include a non-uniform beam profile and aberration corrections, particularly on the high-aperture side in an optical-disc system. It is normally adequate to use a paraxial approximation on the detector side for W_{out}. The supports of both object and image in $\mathbf{x}$ and $\mathbf{y}$ respectively are, in theory, infinite but in practice will be defined by the rapid fall-off of the illumination or by a finite detector aperture.

A.D. Fitt et al. (eds.), *Progress in Industrial Mathematics at ECMI 2008*,
Mathematics in Industry 15, DOI 10.1007/978-3-642-12110-4_45,

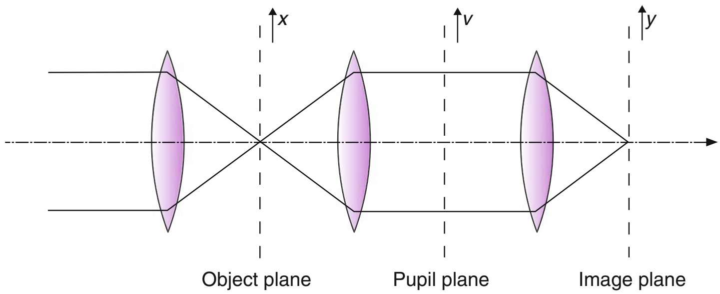

Fig. 1. Object, pupil and image planes

In contrast to the theory of Hopkins, which only applies to low-aperture (paraxial) scalar theory, our theoretical treatment will not depend formally on the numerical apertures, which will occur simply as numerical parameters in the calculation. Nevertheless, the cylindrical symmetry of paraxial systems can be used to improve the numerical efficiency of our calculations when this approximation may be made. The optics of scanning systems of high numerical aperture are discussed in terms of singular function expansions in [4].

The description of the imaging process uses (1) twice, first with the illumination of the disc surface as the image plane of W_{in}, and then using the reflected light from the disc using W_{out} to form the image seen by the detector. Thus, using $\mathbf{s}$ for the scanning variable, the objective lens sees a (complex) field, $f(\mathbf{x})$, as its object, equal to $W_{in}(\mathbf{x})R(\mathbf{x}-\mathbf{s})$, where $R(\mathbf{x})$ is the disc reflectance (or transmittance). The field in the image plane of the objective (ignoring magnification) is thus defined by the linear integral operator, A, where

$$g(\mathbf{y},\mathbf{s}) = (AR)(\mathbf{y},\mathbf{s}) = \int d\mathbf{x} W_{out}(\mathbf{y}-\mathbf{x})W_{in}(\mathbf{x})R(\mathbf{x}-\mathbf{s})). \tag{2}$$

2 Hopkins' Analysis

Using Fourier optics, which is applicable in the paraxial approximation, Hopkins writes the pupil-plane field amplitude as

$$\begin{aligned} E(\mathbf{k},\mathbf{s}) &= \mathcal{F}[R(\mathbf{x}-\mathbf{s}).W_{in}(\mathbf{x})] = \hat{R}(\mathbf{k},\mathbf{s}) \otimes \hat{W}_{in}(\mathbf{k}) \\ &= \int dk' \hat{R}(\mathbf{k}')\hat{W}_{in}(\mathbf{k}-\mathbf{k}')e^{i\mathbf{k}'.\mathbf{s}}, \end{aligned} \tag{3}$$

where $\mathcal{F}$ denotes the Fourier transform, $\otimes$ denotes convolution, the overhat denotes Fourier-transformed functions and we have used the Fourier shift theorem. The pupil-plane intensity is given by

$$\begin{aligned} I(\mathbf{k},\mathbf{s}) &= |E(\mathbf{k},\mathbf{s})|^2 \\ &= \int\int \mathbf{dk}'\mathbf{dk}'' \hat{R}(\mathbf{k}')\hat{R}(\mathbf{k}'')\hat{W}_{in}(\mathbf{k}-\mathbf{k}')\hat{W}_{in}(\mathbf{k}-\mathbf{k}'')e^{i(\mathbf{k}'-\mathbf{k}'').\mathbf{s}} \end{aligned} \tag{4}$$

and the integrated intensity over the pupil-plane is

$$I(\mathbf{s}) = \int_{pupil} \mathbf{dk} I(\mathbf{k}, \mathbf{s}) = \int\int \mathbf{dk}'\mathbf{dk}'' \hat{R}(\mathbf{k}')\hat{R}(\mathbf{k}'')e^{i(\mathbf{k}'-\mathbf{k}'').\mathbf{s}} D(\mathbf{k}', \mathbf{k}''), \tag{5}$$

where

$$D(\mathbf{k}', \mathbf{k}'') = \int_{pupil} \mathbf{dk} \hat{W}_{in}(\mathbf{k} - \mathbf{k}')\hat{W}_{in}(\mathbf{k} - \mathbf{k}''). \tag{6}$$

We put $\mathbf{k}' - \mathbf{k}'' = \mu$ so that

$$I(\mathbf{s}) = \int d\mu e^{i\mu\mathbf{s}} \int_{pupil} \mathbf{dk}' \hat{R}(\mathbf{k}')\hat{R}(\mathbf{k}' + \mu)D(\mathbf{k}', \mathbf{k}' + \mu) = \int d\mu I(\mu)e^{i\mu\mathbf{s}}, \tag{7}$$

where

$$I(\mu) = \int_{pupil} \mathbf{dk} \hat{R}(\mathbf{k})\hat{R}(\mathbf{k} + \mu)D(\mathbf{k}, \mathbf{k} + \mu). \tag{8}$$

We normalise by

$$\int_{pupil} \mathbf{dk} D(0, 0) = \int_{pupil} \mathbf{dk} |\hat{W}_{in}(\mathbf{k})|^2. \tag{9}$$

To calculate the output signal from an extended image-plane square-law detector we use Parseval's theorem

$$\int_{image} d\mathbf{y} I(\mathbf{y}) = \int_{pupil} d\mathbf{k} I(\mathbf{k}), \tag{10}$$

so that it is given directly by (7). Calculations are performed by constructing reflection functions for various periodic arrangements in two euclidean local dimensions (along and across track) of specified pits on the disc surface, with a sufficiently fine discretisation for numerical integration in those two dimensions in the $\mathbf{x}$ and $\mathbf{k}$ planes.

3 Singular Function Analysis

To economise on notation from here on we will use the Dirac bra-ket notation for wave-amplitude functions in the $\mathbf{x}$, $\mathbf{k}$ and $\mathbf{y}$ planes and automatic summation on repeated indices. The image-plane amplitude is given by the operator form of the imaging equation (2)

$$|g(\mathbf{s}) >= A|R(\mathbf{s}) >, \tag{11}$$

where the operator A maps the **x** plane into the **y** plane (considered as L^2 function spaces). The singular value decomposition of A is

$$A = \alpha_i |v_i >< u_i|, \tag{12}$$

where the singular values α_i are real and the singular functions $|u_i>$ and $|v_i>$ are orthonormal basis functions in the **x**- and **y**-planes, respectively. Using this decomposition and the following singular function expansions of R and g:

$$\begin{aligned} |R(\mathbf{s})> &= < u_i|R(\mathbf{s}) > |u_i >= R_i(\mathbf{s})|u_i > \\ |g(\mathbf{s})> &= < v_i|g(\mathbf{s}) > |v_i >= g_i(\mathbf{s})|v_i >, \end{aligned} \tag{13}$$

we find that

$$|g(\mathbf{s}) >= \alpha_i R_i(\mathbf{s})|v_i >, \tag{14}$$

The integrated intensity over the image plane is then

$$\begin{aligned} I(\mathbf{s}) &= < g|g >= |\alpha_i R_i(\mathbf{s})|^2 \\ &= \alpha_i^2 R_i(\mathbf{s}) R_i^*(\mathbf{s}), \end{aligned} \tag{15}$$

When scanning we need to recover only the axial values of $R(\mathbf{s})$.

In the paraxial approximation the calculation of the two-dimensional singular system may be performed in one dimension by using the axial symmetry of the optical system [1]; otherwise a full two-dimensional calculation is needed. The singular functions and singular values are precomputed and the calculation then simply needs the scalar product of $R(s)$ with the desired very small number of $u_i s$ over the disc plane. Column 1 of Table 1 of Bertero et al. [1] shows that in the paraxial approximation the values of α_i^2 fall off as shown in Fig. 2; it can be seen that the contribution of the second function in the expansion is less than 4% and the third less that 1% of that of the first.

In contrast to the Hopkins method, in which the R are all coupled together by the functions D of (6), the object components are completely decoupled from each other in our calculation due to the orthonormality of the singular functions. This allows the simplification which may be seen between (15) and (7). Of course, the small number of terms required in the new method is also helpful.

Acknowledgements

I would like to acknowledge gratefully collaboration over a number of years with Drs. Jan Grochmalicki and Deeph Chana at King's College, London and with the members of the optical group at Philips Laboratories, Eindhoven, particularly Drs. Sjoerd Stallinga and Paul Urbach on theoretical questions. This work has been supported by EPSRC Grant No GR/L13964 and by the European Union FET programme 2000-26479 "Super Laser Array Memory".

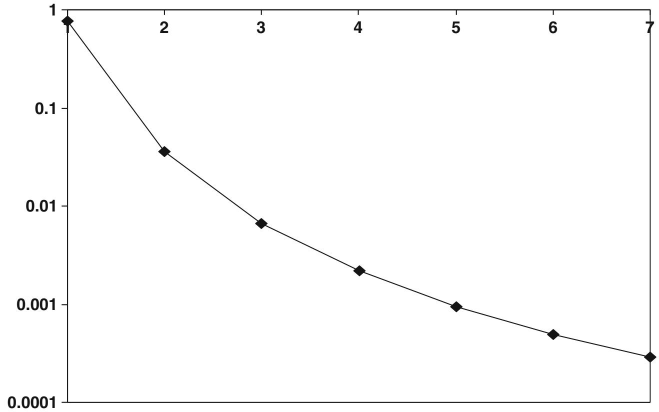

Fig. 2. The squared spectrum of singular values in the paraxial approximation

References

1. Bertero, M., Boccacci, P., Davies, R.E., Pike, E.R.: Inverse Probl. **7**, 655–74 (1991)
2. Hopkins, H.: J. Opt. Soc. Am. **69**, 4–24 (1979)
3. Hopkins, H.: Proc. R. Soc. Lond. **A208**, 263–277 (1951)
4. Grochmalicki, J., Pike, E.R.: App. Opt. **39**, 6341–6349 (2000)