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# Application of Nonlinear Dynamics in Civil Aerospace

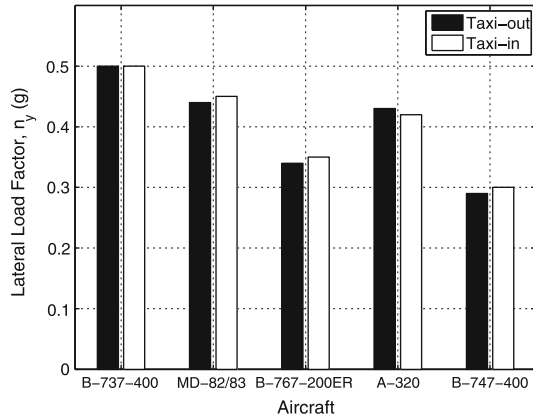
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**Summary.** Nonlinear analysis techniques, especially methods from bifurcation theory, have emerged as valuable tools over the last 20 years, particularly due to the advent of the modern computer. Originally developed as part of dynamical systems and chaos theory, they gradually are finding their way into applications areas from all walks of life. As far as the aerospace industry is concerned, methods from nonlinear dynamics were used initially for the prediction of aircraft flight dynamics at high angle of attack flight regimes, where traditional methods have failed. They are now being used within Airbus to analyse aspects of the dynamics of aircraft on the ground. Specific aerospace applications, where nonlinear dynamics techniques are expected to make an impact, include the design of flexible structures and mechanisms, and the dynamics of a braking wheel. Challenges related to the industrialisation of such methods are also discussed.

## 1 Introduction

Landing gear engineers observe nonlinear phenomena such as hysteresis, backlash and stiction on a daily basis, without necessarily appreciating the full meaning behind these observations. A wheel that locks up during braking is a good example. Many conflicting requirements need to be considered during the design, where the weight and pavement loading needs to be minimised, and the shock absorption maximised. The lateral stability on the ground is determined by the position of the gears, along with the tyre and oleo (shock damper) characteristics. Experience has shown that the use of different tyres can mean the difference between a stable and an unstable aircraft. Landing gears contain highly nonlinear components, including tyres, brakes and oleos, and therefore traditional analysis is usually done at some very specific design conditions. There is a perceived need to characterise the behaviour of the system over a wide variety of parameters, and this is the industrial domain where methods from nonlinear dynamics can and should be brought to bear. We discuss here some of the open avenues for this approach within the specific context of ground dynamics of passenger aircraft.

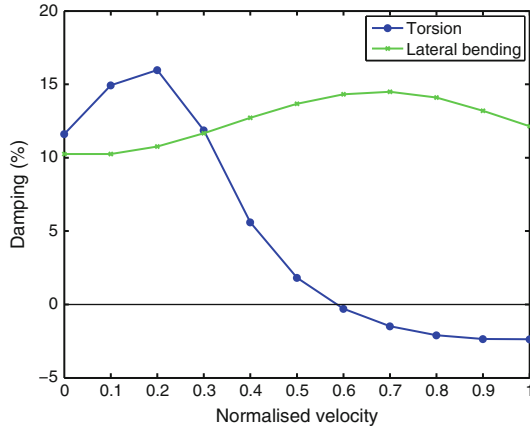


**Fig. 1.** Comparison of equal probability lateral load factors during ground turning for five aircraft after [1]

## 2 Aircraft Ground Manoeuvres

Ground operations tend to be performed at constant thrust settings, because the thrust is adjusted only occasionally by the pilot with the aim of altering the velocity. One issue is to find points (in terms of operational input) where the aircraft becomes uncontrollable during a turn. A loss of stability is dependant on several parameters, such as the steering angle, entry velocity of the turn, tyre properties, and the runway condition. Mathematically, stability loss corresponds to a limit point (or fold) bifurcation or a Hopf bifurcation, which makes it possible to classify the dynamics of a turning aircraft on the ground with the use of continuation methods. In this way, physical causes for the loss of stability have been identified [2–4]. Specifically, limit points and Hopf bifurcations bound regions in parameter space where the tyres are saturated, so that they cannot provide enough side force to maintain a specific manoeuvre.

An ongoing study by the FAA has been aiming to identify what type of lateral loading conditions can be experienced by in-service aircraft. The goal is to validate the conservative design factors that are currently required during the design phase. Current regulations require an 0.5  $g$ -level at the centre of gravity, even though it is known from experience that such high  $g$ -levels are not possible in larger aircraft. The results from the study indicates that the actual  $g$ -levels experienced by airline operators are approximately 0.3  $g$  for wide-body aircraft, such as the Boeing 747, and 0.43  $g$  for narrow-body aircraft, such as the Airbus A320; Fig. 1 shows a summary of the expected peak  $g$ -levels as extracted from the report [1]. It would be of great benefit if the influence of the main parameters could be studied during the preliminary design phases of a project, where some analysis is indeed already done by means of detailed nonlinear simulations. Our experience with the bifurcation study of ground



**Fig. 2.** An example of linear shimmy analysis

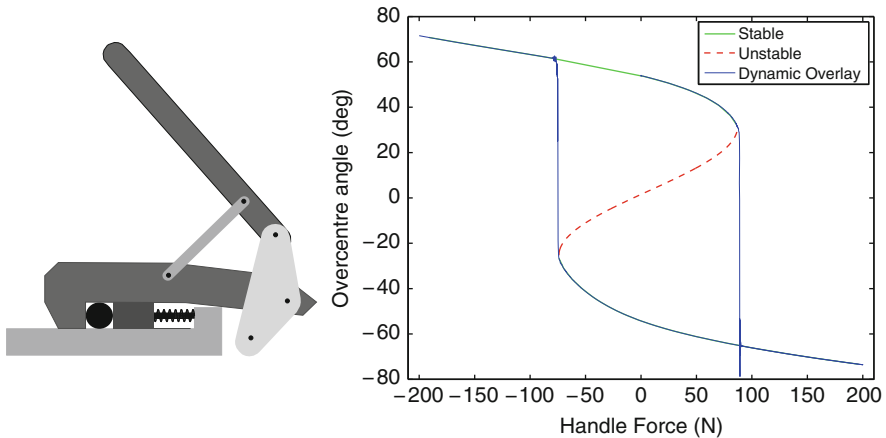
manoeuvres indicates that continuation methods may be used as a new tool to provide a reasonable estimate of the maximum  $g$ -levels, as well as where such operating conditions will occur.

### 3 Landing Gear Shimmy

Shimmy oscillations of a landing gear are undesirable due to the safety and maintenance aspects involved with the occurrence of this phenomena. Linear shimmy analysis is typically done at specific operating points for design purposes, while detailed nonlinear simulations are usually performed only after an incident occurred. Torsional and/or lateral motion can be observed during shimmy oscillations, and the contribution of each mode may be dependant on the initial conditions of the system. Linear shimmy methods calculate the damping in the system while the velocity is varied to identify the onset of shimmy as a point where either the torsional mode or the lateral mode has zero damping. Figure 2 shows an example of such an analysis.

Pilots often report the onset and disappearance of shimmy oscillations between certain velocities, indicating a trajectory across a boundary of Hopf-bifurcations. There are still many differing opinions with regards to the main parameters that influence shimmy, and they result in differing maintenance actions that are recommended when shimmy occurs. Hydraulic shimmy dampers are installed on some aircraft to prevent oscillations in the steering system, but this adds weight.

The development of a nonlinear model of a nose landing gear, and its subsequent bifurcation analysis, has demonstrated the coupled nature of the torsional and the lateral modes via nonlinear tyre forces in the presence of geometric nonlinearities [5, 6]. Future research will focus on the construction



**Fig. 3.** A mechanism with a hysteresis loop of its force-reaction diagram

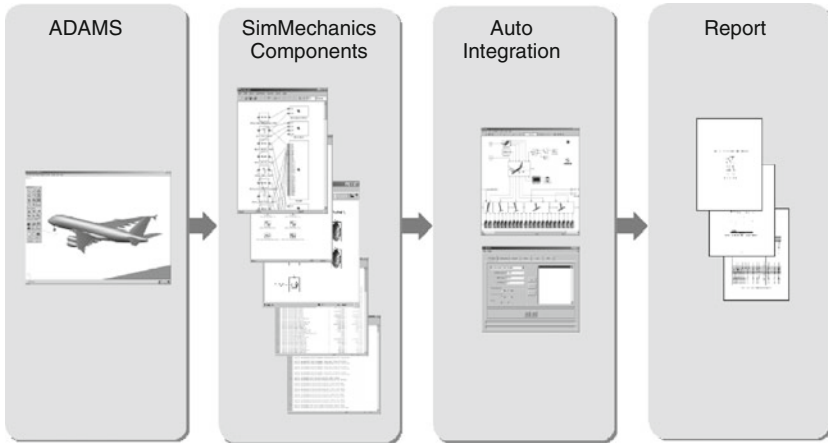
of a comprehensive map of all the types of shimmy under different operational conditions, as well as the development of preliminary rules to avoid shimmy already at the design phase of an aircraft.

## 4 Dynamics of a Braking Wheel

The longitudinal traction force of a braked wheel is a consequence of the relative difference between the vehicle velocity and the velocity of the wheel at the contact patch, which is also known as wheel slip [7]. It depends on the normal force on the wheel, as well as the friction coefficient between the wheel and the road surface. A free-rolling wheel is defined to have a slip-value of 0, while a locked wheel has a slip-value of 1 [8]. It is known that a hysteresis loop exists when a brake torque is applied [9]. This means that the brake torque where lockup occurs and where control is regained could be very different. Recent research on aircraft has also shown that the unstable point after which lockup occurs, does not necessarily occur at the peak value on the slip curve. In fact, braking is one of the most nonlinear processes in aircraft, and understanding it fully will require the use of advanced methods from dynamical systems theory.

## 5 Landing Gear Mechanisms

A mechanism is defined as a combination of parts, that are joined in a specific way, to perform a certain function. Figure 3 shows an example of a latch that contains several pinned arms and a spring. A relatively small force can be applied to the handle of the latch, yet the clamping force on a component could



**Fig. 4.** Suggested integrated environment for the nonlinear analysis of linked models of aircraft components

become significant. A point could also be reached when the handle “jumps” to a new position where no additional force is needed to hold the part in place. This jump indicates the presence of a fold bifurcation as shown in Fig. 3. The envelope of where this fold occurs can be calculated by varying the spring stiffness and applied force. A landing gear effectively is a mechanism quite similar to a latch. Importantly, the landing gear needs to reach a downlock solution at a certain applied force. Nonlinear dynamics methods are being used in ongoing research to map out the envelope of downlock solutions of different types of landing gears as a function of gear spring stiffness and applied force values.

## 6 Conclusions and Outlook

Several case studies have clearly demonstrated that methods from nonlinear dynamics allow engineers to discover, and explain, the rich dynamical behaviour that is observed during aircraft operations on a daily basis. Traditional linear methods are adequate for many engineering systems, but nonlinear effects need to be considered if a system is to be used to its full potential.

In spite of their huge potential, bifurcation theory methods are presently being used only by small pockets of engineers in the aviation industry. In fact, when one wants to introduce nonlinear dynamics into the engineers’ normal toolsets one encounters both societal and technological challenges. Primarily, the societal ones relate to management support and education. The technology needs to be supported by all tiers of management, and a strong business case needs to be made to gain this support. The technological challenge is

one of education and development of the right tools. Training is needed to familiarise engineers with the vocabulary and tools of dynamical systems theory, which are still largely unknown to the average engineer. Indeed, there is a need to learn how to formulate a problem in a way conducive to nonlinear analyses, and how to interpret the results. A level of intuition similar to that concerning, say, Bode diagrams, needs to be developed for the interpretation of bifurcation diagrams. At the same time more emphasis should be placed on the development of well-documented, industrial, integrated toolsets for nonlinear analyses. Whilst several software tools are freely available, they were developed primarily for research purposes. The overall goal is to develop an integrated and user-friendly environment where validated models can be studied with bifurcation software. Figure 4 shows an example of what such an environment may look like at a high level.

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