Geometric Nonlinearities of Aircraft Systems

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Summary. Nonlinearities due to geometric effects, in particular, via angular variables that are not small, are important for aircraft operation. Geometric nonlinearities have a strong effect on the dynamics of the aircraft system under consideration, and they are especially pronounced in aircraft ground operations. As a concrete example we consider here the effect of a non-zero rake angle on the dynamics of a nose landing gear. More specifically, we use tools from bifurcation theory to investigate the stability of the straight-rolling motion during a take-off run.

1 Introduction

Many systems of an aircraft operate in such a way that nonlinearities need to be taken into account to describe their dynamics correctly. Sources of nonlinearities include nonlinear properties of individual components (for example, tyres and dampers), range limits of control surfaces and, in particular, geometric nonlinearities due to the fact that angular variables are not small. As a specific example, we consider the role that geometric nonlinearities play in the phenomenon of shimmy oscillations in aircraft landing gears during high-speed straight-line rolling. Due to their implications for passenger comfort, safety and maintenance costs, shimmy oscillations are an unwanted type of dynamics. They may occur in any wheeled vehicle, including cars, pulled trailers, motorcycles and indeed aircraft; see the overview papers [1–3].

We consider here shimmy oscillations of the nose landing gear of a mid-size passenger aircraft, as sketched in Fig. 1. A nose landing gear consists of a strut, attached to the aircraft body, to which a wheel is mounted with an offset from the strut axis, called the caster length. The system's dynamics are dominated by the interplay between the two basic modes [4]: the torsional mode of rotation around the strut axis, and the lateral mode of deflection of the entire gear from side to side. These two modes are coupled via the nonlinear interaction of the elastic tyre with the ground. The overall landing gear system is characterised by geometric nonlinearities, because the torsional

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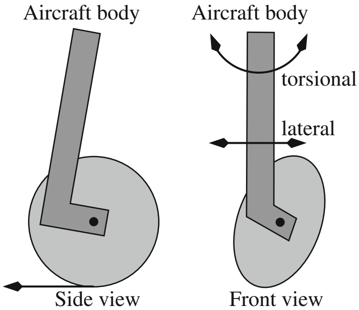

Fig. 1. A non-zero rake angle (of the strut with the vertical) of an aircraft nose landing gear results in a tilt of the tyre plane; the two main modes are the torsional mode of rotation around the steering axis, and the lateral mode of sideways motion of the gear around its attachment point

and the lateral mode may show dynamics of considerable amplitude during shimmy oscillations. An important feature of an aircraft nose landing gear is the presence of a non-zero rake angle of the steering axis with the vertical, typically in the range of $0°–10°$. A positive rake angle introduces additional geometric nonlinearities into the problem. First of all, it contributes to an overall effective caster length, which in turn enters the coupling between the two modes. Furthermore, steering results in a tilt of the wheel, meaning that the wheel plane is not perpendicular to the ground; see the front view in Fig. 1.

We model the nose landing gear by equations for the torsional mode ψ, the lateral mode δ and the lateral deformation λ of the tyre (for which we use the well-established stretched string model [5]). Overall we obtain a mathematical model in the form of five coupled nonlinear ordinary first-order differential equations. The model depends on a number of parameters, including the dimensions of the landing gear, stiffnesses and dampings of the two modes and parameters specifying the tyre forces. The values of these parameters were chosen to represent a midsize passenger aircraft (with a rake angle of $9°$); see [6] for details of the model and the specific values of the modelling parameters.

2 Bifurcation Analysis of Shimmy Oscillations

The landing gear moves at horizontal velocity V, subject to a vertical force F_z that is exerted by the aircraft body (which is modelled as a block of mass). It is therefore natural to study the dynamics of the nose landing gear in dependence on the operational parameters V and F_z. Figure 2 shows how the operational range of the (V, F_z)-plane is divided into regions of qualitatively different

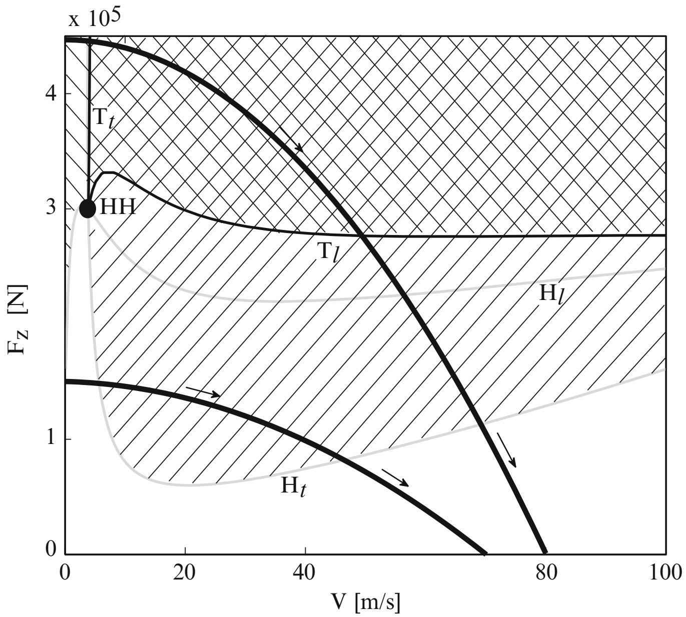

Fig. 2. Two-parameter bifurcation diagram in the (V, F_z)-plane, consisting of curves H_t and H_l of Hopf bifurcations (*grey*) and curves T_t and T_l of torus bifurcation (*black*). The straight-rolling solution is stable in the white region; torsional and lateral shimmy oscillations occur in the regions of right-slanted and left-slanted shading, respectively. The two *thick black curves* are two simulated take-off runs, of a light and a heavy aircraft, respectively

dynamics. The boundaries between regions are given by curves of bifurcations, which have been computed with the continuation software AUTO [7].

In the white region in Fig. 2 the straight-rolling motion is stable, that is, the nose landing gear does not show shimmy oscillations. Stability is lost when one of two Hopf bifurcation curves, H_t or H_l, is crossed. Specifically, crossing H_t corresponds to an undamping of the torsional mode. The ensuing torsional shimmy oscillations are stable in the region of right-slanted shading and they are characterised by oscillations of the landing gear around the strut axis. By contrast, crossing H_l corresponds to an undamping of the lateral mode, meaning that the gear shows lateral shimmy oscillations in the plane perpendicular to the direction of travel. This type of shimmy is stable in the region of left-slanted shading. The curves H_t and H_l intersect at a double-Hopf point HH, which gives rise to two curves, T_t and T_l, of torus (or Neimark-Sacker) bifurcations [8]. Crossing these two curves corresponds to the undamping of the second mode, which gives rise to the creation of an invariant torus. We find that the bifurcating torus is unstable throughout. As a result the curves T_t and T_l bound a large region where torsional and lateral shimmy oscillations

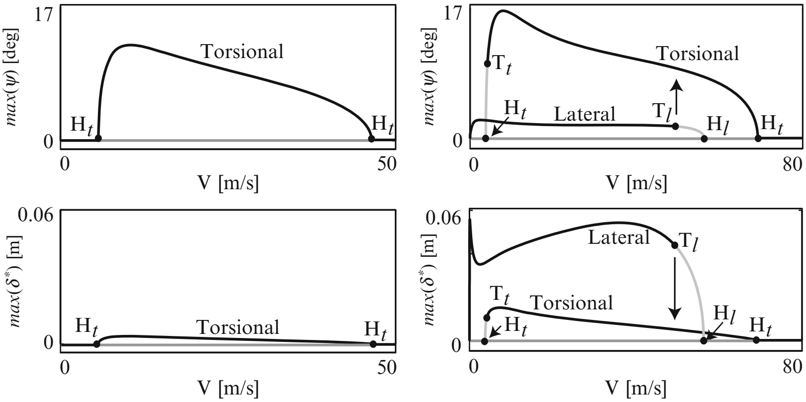

Fig. 3. One-parameter continuations along the two simulated take-off runs of a light (*left column*) and a heavy (*right column*) aircraft. The top panels show the maximum of the torsion angle ψ and the bottom panels the maximum of the lateral bending stroke δ^*; stable parts of branches are *black* and unstable parts *grey*

are both stable. In this region of bistability, it depends on the initial condition which type of shimmy the landing gear performs.

3 Shimmy Dynamics During Take-Off

Figure 2 gives a comprehensive picture of the behaviour of the aircraft over the relevant ranges of forward velocity V and downward force F_z. Each point in the (V, F_z)-plane corresponds to a type of dynamics and it typically lies in one of the regions that were identified. Hence, the bifurcation diagram in Fig. 2 illustrates the robustness of a typical choice of V and F_z with respect to small changes of their values. On the other hand, larger changes that result in a crossing of bifurcation curves lead to qualitative changes of the behaviour of the system.

To demonstrate how the information in Fig. 2 can be used in practice we consider the dynamics of the nose landing gear during take-off. During a take-off run the aircraft accelerates from zero velocity to its take-off speed, during which the vertical force F_z on the nose landing gear decreases from its maximal (static) value to zero. Hence, a take-off run corresponds to a one-dimensional curve in the (V, F_z)-plane. Two examples of take-off runs (chosen to feature shimmy oscillations), one for a light and one for a heavy aircraft, are shown as bold black curves in Fig. 2. Owing to the quadratic dependence of lift on velocity, they have been modelled as parabolas. One immediately notices that the two take-off runs are qualitatively different, because they intersect different regions of the (V, F_z)-plane. Notice further that the exact shape of these curves is not crucial, as long as the same regions are encountered in the same order.

Figure 3 shows the results of two one-parameter continuations with AUTO along the two take-off runs. Shown are the amplitudes of the torsion angle ψ and of the lateral bending stroke δ^* (the lateral stroke of the strut at ground level). The take-off run for the light aircraft case, shown in the left column of Fig. 3, starts at $F_z = 150\,\text{kN}$ and ends at a take-off speed of 70 m/s. The straight-rolling motion is stable, but then loses stability when the curve H_t is crossed in Fig. 2. The amplitude of the ensuing torsional shimmy oscillations increases rapidly up to a maximum of about 14°. It then decreases as the aircraft accelerates. Finally, at about 45 m/s the straight-rolling motion regains stability and the torsional shimmy oscillations disappear. Notice that the lateral bending stroke δ^* shows small amplitude oscillations during torsional shimmy; namely, it follows the torsional mode passively due to the coupling via the tyre [6].

The take-off run for the heavy aircraft case is shown in the right column of Fig. 3; it starts as a vertical force of $F_z = 450\,\text{kN}$ and ends at a take-off speed of 80 m/s. This take-off run is such that the straight-rolling motion is unstable from the very beginning. Instead at low speeds the nose landing gear performs lateral shimmy oscillations with a lateral stroke amplitude of around 5 cm; again due to the coupling via the tyre, the torsional mode follows this motion with small amplitude. The lateral shimmy oscillations are stable until the curves T_l is encountered in Fig. 2 at a velocity of about 50 m/s. This curve marks the boundary of the bistable region and the system switches to the branch of torsional shimmy oscillations, as is indicated by the arrows in Fig. 3 (right column). The torsional shimmy oscillations gradually decrease and finally disappear at around 70 m/s just prior to take-off. We remark that during the switching from lateral to torsional shimmy oscillations one may encounter quasiperiodic (two-frequency) shimmy oscillations as long transients; see [6] for more details.

4 Conclusions

We presented a study of aircraft nose gear shimmy as an example of how geometric nonlinearities influence the dynamics of aircraft systems. Specifically, we performed a bifurcation analysis of a mathematical model that describes the interaction of the torsional and lateral modes via the elastic tyre. Geometric nonlinearities arise from the fact that the amplitudes of the torsion angle and the bending stroke may be substantial – an effect that is further enhanced by the geometric nature of the coupling between the two modes via a non-zero rake angle. Torsional and lateral shimmy oscillations occur in large regions in the plane of velocity versus vertical force, including in a region of bistability. One-parameter continuations along take-off runs for a light and a heavy aircraft demonstrated how shimmy oscillations are encountered in practice when the different regions are crossed.

An obvious question is how the bifurcation diagram presented here depends on the parameters that specify the landing gear, especially on those that have a bearing on geometric nonlinearities. The influence of the rake angle has been considered in [6], where it was found that the region of torsional shimmy oscillations shrinks with an increase of the rake angle. Our present work focuses on the dependence of the bifurcation structure on other parameters, those that determine the geometry of the nose landing gear as well as those that specify tyre properties. The study of additional effects, for example, dynamics of vertical shock absorbers in the presence of a rough runway, can be addressed via an expansion of the model of the nose landing gear. Furthermore, we also intend to model and study the dynamics of main landing gears of different geometries (with different numbers of wheels). In the longer term, our goal is to couple the dynamics of individual gears via a flexible fuselage to obtain a realistic, yet tractable model to describe aircraft ground dynamics.

Acknowledgments

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