Operational Parameter Study of an Aircraft Turning on the Ground

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Summary. Safety and economy are primary concerns in the study of ground manoeuvres for commercial aircraft, the ultimate goal being automation and optimisation of taxi operations. The application of mathematical and computer modeling to this problem is beneficial due to the relative costs compared with actual tests. As an example of utilising mathematical tools in the investigation of industrial problems we make use of a computer model of a passenger aircraft to perform a bifurcation analysis of turning solutions. In particular, we study how altering the longitudinal centre of gravity position of an aircraft affects its ground dynamics.

1 Introduction

During the daily service of passenger aircraft there are operational parameters that may vary considerably. Many of these parameters can have a significant effect on the ground handling properties of the aircraft. Important parameters include the loading of the aircraft in terms of passengers and fuel, runway and taxiway conditions, and wear on important components such as the tyres. In order to inform operational procedure it is important to understand how variation of these parameters affects the ground dynamics. Large costs associated with performing ground (flight) tests motivates the use of mathematical and computer modeling. In previous work a combination of flight test data and low-order computer bicycle models were used to study the ground handling properties of aircraft [1, 2], including the effect of tyre pressure on ground handling [3]. A previous study by the authors utilised a SimMechanics model to study the dynamics of aircraft on the ground under variation of thrust [4]. In this paper we use continuation analysis to perform a parameter study of a mathematical model of a passenger aircraft. Specifically, we investigate the effect that the aircraft's longitudinal centre of gravity position has on its ground handling.

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During taxiing to and from the airport terminal a passenger aircraft will undertake various turning manoeuvres. Turns are made by applying a steering angle to the wheel and tyres of the nose gear while thrust from the engines remains constant. Our approach is to study the ensuing dynamics in terms of turning circle solutions of the system; their stability dictates whether a particular manoeuvre can be made safely. Depending on the loading of passengers, luggage and fuel levels, the centre of gravity position of the aircraft can vary considerably in day-to-day use. It is therefore of interest to treat the centre of gravity position along the longitudinal axis of the aircraft as a system parameter and to investigate changes in the turning dynamics under its variation.

We use a fully parametrised mathematical model of a typical medium sized single aisle passenger aircraft implemented in Matlab. The aircraft is modeled as a tricycle with the airframe having three translational and three rotational degrees of freedom. The equations of motion were obtained via balancing forces and moments in each degree of freedom. Nonlinear effects are included in the tyre model, depending on tyre load and slip angle, and in the aerodynamic model, depending on velocity, angle of attack and slip angle of the airframe. The steering angle δ and the centre of gravity position CG are the free parameters in our analysis. The centre of gravity position is measured as the percentage along the mean aerodynamic chord (MAC), taken from the leading edge; negative values represent a position in front of the leading edge.

The tool used here is numerical continuation; specifically, we perform a bifurcation analysis with the software package AUTO [5]. Continuation analysis is a powerful tool used to study steady-state solutions of dynamical systems [6], which are tracked under the variation of system parameters; during computations solutions are monitored to detect bifurcations, which are qualitative changes in the dynamics [7, 8]. Identifying where bifurcations occur is important because they may form boundaries of safe behaviour. The use of continuation and bifurcation analysis to study ground manoeuvres is a computationally inexpensive way of analyzing the dynamics under variation of several parameters.

2 Bifurcation Analysis of Turning Solutions

We present a bifurcation analysis of aircraft turning solutions; the results are represented as one-parameter and two-parameter bifurcation diagrams. In our model fixed-radius turning circles correspond to steady-states of the system. The analysis focuses on how (steady-state) turning circle solutions change under variation of parameters. In the one-parameter study the CG position is kept fixed and the steering angle δ is varied; solutions are plotted against a state variable. In the two-parameter study we also vary CG and the results are represented as a surface of solutions that describe the dynamics over the entire range of δ and CG.

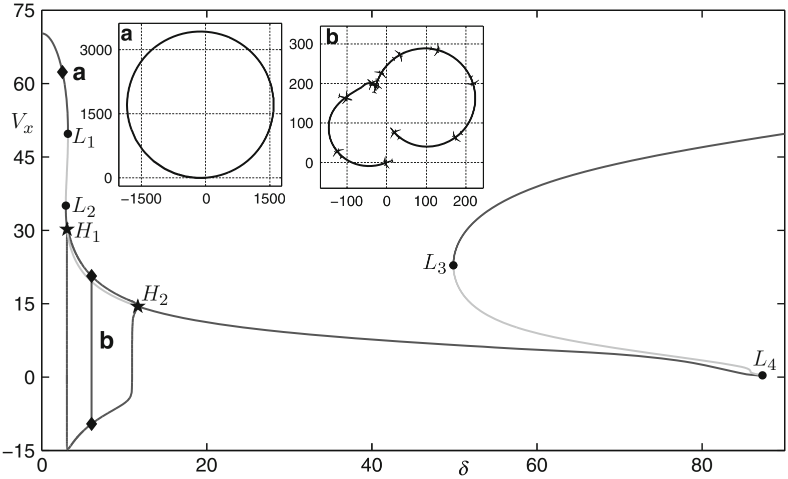

Fig. 1. One-parameter bifurcation diagram in δ for $CG = 35\%$ with a single branch of solutions; stable parts are *black* and unstable parts are *grey*. Changes in stability occur at the bifurcation points L_{1-4} and H_{1-2}. The maximum and minimum forward velocity of a branch of periodic solutions between H_1 and H_2 are plotted in *black*. *Insets* (**a**) and (**b**) show examples of the aircraft's motion

2.1 One-Parameter Study

Figure 1 shows a one-parameter bifurcation diagram in δ for $CG = 35\%$, where the forward velocity V_x of the aircraft is used as a measure of the solution. A single branch of solutions initiates in the top left of the figure and terminates in the top right; changes of stability occur at the limit point bifurcations L_{1-4} and Hopf bifurcations H_{1-2}. On the branch of solutions, stable parts are black and unstable parts are grey. Periodic solutions exist between H_1 and H_2 and their maximum and minimum velocities are plotted as black curves. Qualitatively different types of behaviour can be observed at the labeled points (a) and (b). The respective insets in Fig. 1 show a top down view of a CG-trace of the aircraft in the horizontal ground plane; in (b) markers are drawn to scale and show the aircraft's attitude along the CG-trace.

At the initial point where $\delta = 0$, the aircraft travels in a straight line with a constant velocity of $V_x = 70$ m/s due to constant thrust from the engines. As steering is applied ($\delta > 0$), the solutions represent fixed large radius turns. For example, at (a) the aircraft follows a stable turning circle of radius $r \approx 1.7$ km with a forward velocity of 63 m/s. This type of solution with a small steering angle, large radius turn persists from the initial point up to the bifurcation L_1; the radius of the turn decreases as L_1 is approached. At the bifurcation the turning moment generated by the nose gear tyres overcomes the stabilising aerodynamic force generated by the tail fin of the aircraft [4].

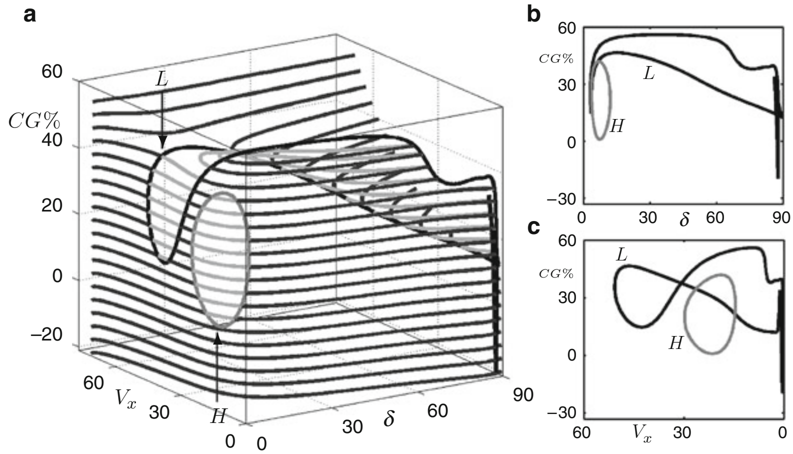

Fig. 2. Panel (**a**) shows a surface plot of solutions in (δ, V_x, CG)-space; stable solutions are *black* and unstable solutions are *grey*. The loci of limit point bifurcations L is the *thick black curve* and the locus of Hopf bifurcations H is the *thick grey curve*. Panels (**b**) and (**c**) show two-dimensional projections of the bifurcation curves onto the (δ, CG)-plane and (V_x, CG)-plane, respectively

When the steering angle is increased beyond L_1, the aircraft loses velocity rapidly over a transient period and starts to follow a solution in the region between the Hopf bifurcations H_1 and H_2. Hopf bifurcations are associated with the onset of periodic motion [8]. In this case, passing a Hopf bifurcation represents a change in which the aircraft attempts to follow a turning circle that is too tight and, therefore, there is a loss of lateral stability associated with the main landing gear tyres saturating. For example, at (b) the aircraft attempts to follow an unstable turning solution with radius $r \approx 125$ m but loses lateral stability, enters a spin and briefly travels backwards before coming to a halt. The aircraft then moves off under constant thrust, repeating the motion periodically relative to the unstable turning solution with a maximum and minimum velocity of 20 and -10 m/s, respectively. A detailed description of this undesirable behaviour that persists between H_1 and H_2 is given in [4]. Between H_2 and L_4 high steering angle, small radius turns can be observed, and between L_3 and the end point at the top right high steering angle, large radius turns can be observed for which the nose gear is almost perpendicular to the direction of motion and, hence, is effectively dragged along the ground.

2.2 Two-Parameter Bifurcation Study

One-parameter continuation runs, as in Sect. 2.1, were computed over a range of CG at discrete points. When plotted together in (δ, V_x, CG)-space the individual bifurcation curves form a surface of solutions. Two-parameter continuation was used to compute the loci of bifurcations continuously under the variation of both δ and CG. Combining the results from these two computations into a single plot is an effective way of representing the behaviour over the complete range of δ and CG in a single figure. Two-dimensional projections of bifurcation curves show certain features more clearly.

Figure 2a shows the resulting surface plot of solutions in (δ, V_x, CG)-space; again stable solutions are black and unstable solutions are grey. Changes in stability occur at bifurcation curves on the surface. The curve L of limit point bifurcations is represented by the thick black closed curves and the curve H of Hopf bifurcations by the thick grey closed curve. The one-parameter case discussed above represents a horizontal slice of Fig. 2a at $CG = 35\%$. The bifurcations in Fig. 1 lie on the locus curves in Fig. 2a, L_1, L_2, L_3 and L_4 on L, and H_1 and H_2 on H.

Figures 2b and 2c show two-dimensional projections of the bifurcation curves onto the (δ, CG)-parameter plane and the (V_x, CG)-plane, respectively. In the (δ, CG)-parameter plane bifurcation curves bound regions with different numbers of solutions, each with a specific stability. In the largest region, not bounded by any of the bifurcation curves, a single stable turning circle solution exists. In the region bounded by the Hopf bifurcation curve H a single unstable turning circle solution exists and the attracting solution is a periodic motion relative to this unstable turning circle, as was discussed in Sect. 2.1. In the region bounded by the limit point bifurcation curve L two stable and one unstable turning circle solutions exist. Figure 1 provides an example of traversing each region in the parameter δ. A hysteresis loop results when traversing the regions bounded by limit point curves in different directions. The same data plotted in the (V_x, CG)-plane reveals the relative positions of the bifurcation curves in terms of the forward velocity V_x.

Within the operational range of $CG \in (10\%, 40\%)$, the laterally unstable behaviour inside the region bounded by H in Fig. 2 persists. However, for $CG < 15\%$ (a forward position) no limit point bifurcations will be observed at low steering angles as seen clearly in Fig. 2b. This means that the region of laterally unstable dynamics could be approached more suddenly and at lower velocities. Taking values of CG outside of the operational range (an extreme forward or aft position) results in uniformly stable behaviour at low steering angles, where intersections with L and H are not possible. In Fig. 2b there is a region for small $\delta < 3°$ to the left of L and H for which no bifurcations occur. This bound does not change under variation of CG and could provide a limit for steering angles used in high-velocity turns.

3 Conclusions

A comprehensive bifurcation analysis of a mathematical model of a typical single aisle passenger aircraft was performed in terms of the steering angle and the aircraft's longitudinal centre of gravity (CG) position. A one-parameter study in the steering angle illustrated different types of solutions and their bifurcations. These results were extended to a two-parameter study by computing solution branches over a range of CG positions and tracking the loci of the bifurcations continuously in the parameter plane. Combining the results gives a complete account of the possible turning dynamics of the aircraft under variation of both parameters.

The results presented here reveal how changing an aircraft's CG position can affect its ground dynamics. Over the operational range of the CG position there is a region of laterally unstable dynamics existing between two Hopf bifurcations. Depending on the CG position, this unsafe region of dynamics can be approached in different ways at small steering angles. With an aft position the region can be approached at high velocity by passing a limit point bifurcation, but with a forward position the solutions can be approached more suddenly at a lower velocity by passing one of the Hopf bifurcations. Additionally, a steering angle of 3° was identified as an upper bound independent of CG position for making stable high-velocity turns.

Ongoing work focuses on the sensitivity of the results presented here to variation of the additional parameters, for example, the mass and thrust of the aircraft. However, there are many other parameters that are of interest, including the track-width of the main landing gears, runway conditions and tyre properties. Physical phenomena associated with changes in qualitative dynamics are also the subject of ongoing studies.

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