

---

## Minisymposium *Dynamical Systems Methods in Aerospace Engineering*

B. Krauskopf<sup>1</sup> and M.H. Lowenberg<sup>2</sup>

<sup>1</sup> Department of Engineering Mathematics, University of Bristol, UK

[B.Krauskopf@bristol.ac.uk](mailto:B.Krauskopf@bristol.ac.uk)

<sup>2</sup> Department of Aerospace Engineering, University of Bristol, UK

[M.Lowenberg@bristol.ac.uk](mailto:M.Lowenberg@bristol.ac.uk)

The 1990s and early 2000s saw substantial research into the application of nonlinear dynamical systems theory in the field of aerospace engineering. This focussed principally on the flight mechanics behaviour of aircraft operating at extremes of their flight envelopes, where aerodynamic and other phenomena are significantly nonlinear. Useful results were obtained, and some practical tools were developed, for example, for the analysis of underslung loads below a helicopter, and the analysis of flight control law robustness. In spite of these initial efforts, methods from nonlinear dynamics have not as yet entered the industrial mainstream. On the other hand, there is today a growing realisation that, for the industry to develop and improve its products, it must face the fact that many of its problems are indeed nonlinear in nature. Hence, the necessary advances require that this type of behaviour is properly accounted for.

The contributions for this minisymposium showcase examples of recent nonlinear studies of aeronautical applications which are indeed being integrated into industry. Specifically, these studies show how continuation methods and bifurcation analysis are incorporated into the investigation and evaluation of a variety of aircraft systems – both from a vibration and a rigid-body motion perspective. In each case, the approach brings a new extended capability in the understanding of nonlinear engineering systems and the papers show the promise offered by these techniques to the aerospace sector in both analysis and design.

Rezgui et al. focus on the stability of a rotor in autorotation, which is investigated via the bifurcation analysis of a periodically forced system. Despite the model being relatively simple and of low order, it yields multiple autorotation solutions. At the same time an experimental rig of the system is utilised to generate experimental bifurcation diagrams. The paper shows how a co-ordinated implementation of both the numerical and experimental systems, the latter used to tune the model and the former to help select test conditions that are meaningful and achievable, provides a low-order model that is able to generate a rich variety of verifiable results not previously achieved.

The subject discussed by Rankin et al. is the nonlinear analysis of an aircraft turning on the ground. A mathematical model of low order, developed and verified with direct input from industry, reveals regions of instability of turning that may be encountered in practice as certain parameters are varied. The paper shows the results in a graphical manner intended to be accessible and easy to interpret for engineers not well versed in nonlinear systems theory. Hence, nonlinear modelling and analysis techniques become engineering tools that can be used, for example, to evaluate new design concepts at an early stage.

Shimmy in aircraft landing gear systems is a common problem encountered in design and operations, yet the methods for understanding and hence eliminating shimmy are often inadequate. In the contribution by Krauskopf et al. bifurcation analysis is applied to a mathematical model of an aircraft nose landing gear with geometric nonlinearity, which takes the form of five coupled first-order ordinary differential equations. The results clearly show the parameter regions in which one or more shimmy modes exist. The practical use of such a stability map is illustrated by simulated take-off runs of a light and a heavy aircraft.

The final contribution by Coetzee discusses the potential for nonlinear dynamical systems theory from an aerospace industry perspective. Several recent examples of their industrial use are cited, but the focus is on opportunities for the application of nonlinear methods in landing gear and related systems. Apart from specific technical challenges, the paper also discusses the need for providing the necessary framework and management support for nonlinear modelling in an industrial context.

Taken together, the contributions to the minisymposium provide useful lessons to aid in the adoption of nonlinear methods within the aerospace industry. Engineers need to understand the value not only of the traditional complex linear models, but also the power of reduced-order nonlinear models to capture important behaviour. They need to learn how to generate such models, how to analyse them with advanced tools, and how to interpret the results properly. Certainly, software for continuation and bifurcation analysis needs to be made more user-friendly for this community, and graphical methods need to be developed for both inputting information and presenting results.

In conclusion, we hope that the discussions presented here will contribute to a growing recognition of the practical benefits of nonlinear modelling and analysis in the aerospace industry and beyond.