Spin Dynamics in Quantum Dots

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Summary. Leakage current of double quantum dot systems in the spin blockade regime has been attributed to hyperfine interactions. In this work electron transport through double quantum dots is analyzed in the spin blockade regime, in the presence of hyperfine interaction by means of rate equations. In agreement with experiment, current hysteresis as a function of magnetic field is found. This behavior comes from the interplay between dynamic nuclear spin polarization and the electronic energy states renormalization due to the Overhauser shifts induced by the nuclei.

1 Introduction

The Pauli exclusion principle can play an important role in current rectification [1, 2] in both molecular and semiconductor nano-structures transport. Spin blockade (SB), which occurs in double quantum dots (DQDs) over certain ranges of gate voltage, external magnetic field, and bias voltage is one important example. The interplay between Coulomb and spin blockade can be used to block current in one direction of bias voltage while allowing it to flow in the opposite one. Because of this property DQDs can function as externally controllable spin-Coulomb rectifiers that have potential application in spintronics, as spin memories and transistors. Spin relaxation processes [3–5], induced by spin-orbit (SO) scattering [6] or hyperfine (HF) interactions [7–12], produce a leakage current which limits the SB resistance. Spin-flip (sf) relaxation times in QDs are rather long however and the SB resistance is large.

In this paper we report on a model for transport through two weakly coupled vertical QD's in the spin blockade configuration [13, 14]. Recent experiments [1] show current leakage in the Spin Blockade regime which is attributed to hyperfine interaction between the nuclei spins and the electronic spins. On a spin blockade plateau, current flow between dots is possible only when each dot has one electron and their spins are opposite. A finite bias voltage allows an electron in the left dot to tunnel sequentially to the double

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occupied singlet state in the right dot and then to the collector. In this circumstance there is an approximately even chance that electrons in left and right dots will have the same spin when the left dot electron is refreshed from the source. When that happens, the Pauli exclusion principle prevents tunneling. Current flow stops until a spin flip takes place. The time averaged current is consequently strongly suppressed. The SB blockade regime is conveniently tuned by an external magnetic field (B). Fields applied in the plane of the quantum dots introduce a Zeeman energy splitting of the levels. Increasing the field allows to tune the relative energy between states with antiparallel spins and with parallel spins and bring them close to degeneracy. In this situation, it has been shown, both theoretically and experimentally that the current presents instabilities and hysteretic behavior [15]. We explain this behavior by accounting for the interplay between dynamic nuclear polarization and Overhauser shifts suffered by the electronic levels which are induced by the nuclei spins. Recent experiments by Koppens et al. show similar instabilities and bistable regions in the current as a function of magnetic field [16] through a lateral DQD, which likely have a similar explanation. Also, current hysteresis has been observed in InAs quantum dots by the group of Ensslin [17].

2 Electronic Transport Through Double Quantum Dots: Role of Hyperfine Interaction

2.1 Theoretical Model

We consider the Hamiltonian: $H = H_L + H_R + H_T^{LR} + H_{leads} + H_T^{l,D}$, where $H_L, (H_R)$ is the Hamiltonian for the isolated left (right) QD modeled as one-orbital Anderson impurity. H_T^{LR} and $H_T^{l,D}$ describe tunneling between QDs and between leads and QDs respectively, and H_{leads} is the Hamiltonian for the leads. In the presence of an external magnetic field and hyperfine interaction there is an additional contribution to the hamiltonian:

$$\hat{H} = g_e \mu_B \mathbf{S} \cdot \mathbf{B} + \frac{A}{N} \sum_{i=1}^{N} \left[S_z I_z^i + \frac{1}{2}(S_+ I_-^i + S_- I_+^i) \right] \tag{1}$$

where the average hyperfine coupling constant is $A \simeq 90\,\mu\text{eV}$ for GaAs and B is the external magnetic field. The basis considered consists on the eigenstates for the isolated quantum dots.

Rate equations for the occupation probabilities ρ_{ss} corresponding to the electronic states become:

$$\dot{\rho}(t)_{ss} = \sum_{m \neq s} W_{sm} \rho_{mm} - \sum_{k \neq s} W_{ks} \rho_{ss} \tag{2}$$

where W_{ij} are transition rates for the tunneling through the contact barriers and for the tunneling through the interdot barrier. We consider incoherent interdot tunnel and we account for both elastic and inelastic inter-dot tunneling [13, 14]. We consider as well transition rates which involve spin-flip coming from hyperfine interaction, as we will describe below. We calculate the electronic spin-flip scattering rate $W^{sf}_{i,j}$ using a microscopic model that accounts for HF interactions: The HF interaction can then be separated into mean-field and flip-flop contributions: $\hat{H} = \hat{H}_z + \hat{H}_{sf}$ where $\hat{H}_z = A\langle I_z\rangle S_z$ has an effective nuclear field $B_N = A\langle I_z\rangle/g_e\mu_B$ contribution which is added to the external magnetic field contribution to produce an effective Zeeman splitting of the levels and

$$\langle I_z\rangle = \frac{1}{N}\sum_{i=1}^{N}(I_z^i) = \left[\frac{N^{\uparrow} - N^{\downarrow}}{N^{\uparrow} + N^{\downarrow}}\right]|I_z| = P|I_z| \tag{3}$$

where $P = \left[\frac{N^{\uparrow}-N^{\downarrow}}{N^{\uparrow}+N^{\downarrow}}\right]$ is the nuclear spin polarization where $N^{\uparrow(\downarrow)}$ is the number of nuclei with spin up(down), in a QD. $\hat{H}_{sf} = (A/2N)\sum_i\left[S_+I_-^i + S_-I_+^i\right]$ is the flip-flop interaction responsible for mutual electronic and nuclear spin flips. Because of the mismatch between nuclear and electronic Zeeman energies transitions must be accompanied at low temperature by phonon emission. We approximate the spin-flip transition rate from parallel-spin to opposite-spin configurations by:

$$\frac{1}{\tau_{sf}} \simeq \frac{2\pi}{\hbar}|<\hat{H}_{sf}>|^2\frac{\gamma}{\Delta E^2 + \gamma^2} \tag{4}$$

where γ is the electronic state life-time broadening which is of the order of μeV, i.e., of the order of the phonon scattering rate [3, 4]. ΔE is the difference between the energy of a state with one electron in each dot with aligned spins $(|\downarrow,\downarrow\rangle/|\uparrow,\uparrow\rangle)$and the energy of a state with one electron in each dot with opposite spin orientation $(|\uparrow,\downarrow\rangle/|\downarrow,\uparrow\rangle)$ (see Fig. 1). The latter are *mixed* due to interdot tunneling with the intradot singlet state in the right QD $(|0,\downarrow,\uparrow\rangle)$. The energy of the *mixed* state with antiparallel spins is calculated perturbatively and depends mainly on the interdot tunneling (t) and the right and left dots level detuning.

In resonance, at $B \neq 0$, ΔE depends on the Zeeman energy due to the external field B and on the additional Zeeman splitting due to the magnetic field induced by the nuclei:

$$\Delta E = E_{(|\downarrow,\downarrow\rangle/|\uparrow,\uparrow\rangle)} - E_{(|\uparrow,\downarrow\rangle/|\downarrow,\uparrow\rangle)} = g_e\mu_B B + \frac{A}{2}P \tag{5}$$

The equations that describe the time evolution of the nuclear spin polarization for both dots include the flip-flop interaction and the nuclear spin relaxation time τ_{relax} (that we include phenomenologically and that is much longer than the electron-nuclei spin scattering time) become:

$$\dot{P}_L = W^{sf}_{6,3}\rho_3 - W^{sf}_{5,4}\rho_4 - \frac{P_L}{\tau_{relax}} \tag{6}$$

$$\dot{P}_R = W_{5,3}\rho_3 - W^{sf}_{6,4}\rho_4 - \frac{P_R}{\tau_{relax}} \tag{7}$$

Here, for instance:

$$|4\rangle \equiv |\downarrow,\downarrow\rangle \rightarrow |5\rangle \equiv |\uparrow,\downarrow\rangle \Rightarrow W^{sf}_{5,4} = \left[\frac{1}{\tau_{sf}}\right]_L \left[\frac{1+P_L}{2}\right], \tag{8}$$

where L and R mean left and right dot respectively. The system of time evolution equations for the electronic states occupations ρ_i and nuclear polarization of the left and right dot is self-consistently solved. From that we calculate the total current through the system which is the physical observable of interest.

For $B = 0$, experiments [1] show a weak peak in the current at low V_{DC} followed by a wide plateau, and then finally a very strong peak at $V_{DC} \geq 6$ MeV, in good agreement with the results plotted in Fig. 1. The leakage current observed in the plateau is due to the finite probability for electrons in the QD's to flip their spin by interaction with nuclei.

In Fig. 2 we show I/B (B in-plane) for V_{DC} near the center of the SB region. When sweeping up and down the magnetic field, we find current hysteretic behavior in agreement with experiment [15]. The source of this behavior is the interplay between the induced nuclei polarization due to HF interaction and the energy shift induced in the electronic states by the nuclear magnetic field which modifies the spin-flip rate. At small B, for the DC voltage that we have considered the $|\downarrow,\downarrow\rangle$ state has lower energy than the state with antiparallel spins, and then, at low temperatures spin-flip has low probability. Increasing B, the state $|\downarrow,\downarrow\rangle$ becomes higher in energy than the state with

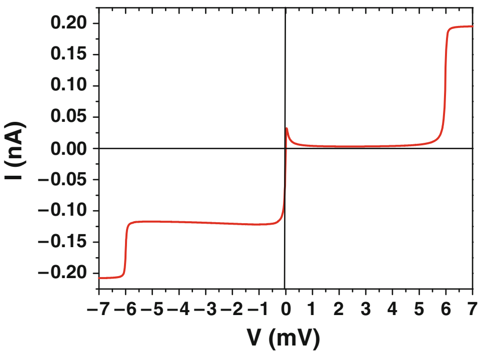

Fig. 1. Stationary I/V_{DC} (B = 0). At low V_{DC}, I takes place when one electron from the (1, 1) state with two electrons, one in each dot, with opposite spins tunnels to the singlet double occupied state in the right QD (0, 2). Once one electron tunnels from the emitter contact to the left dot with the same spin polarization as the electron in the right dot, the current drops abruptly due to spin blockade. A finite current leakage is observed due to spin flip induced by HF interaction. At $V_{DC} \geq 6$ MeV the chemical potential of the right lead crosses the (1, 1) state with parallel spins and the right QD becomes suddenly discharged producing a large peak in I

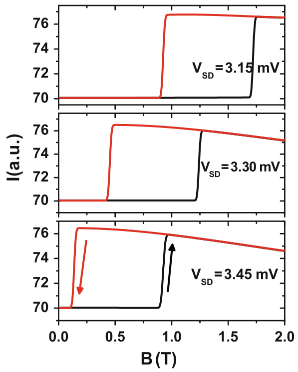

Fig. 2. I/B in the SB region for a DQD under in-plane B for different DC voltages. The current shows hysteresis reflecting strong non-linearities induced by the interplay of electron and nuclear spin dynamics. Electronic levels energies depend on the level detuning which depends on the source-drain voltage. This is the reason why the hysteresis region shifts with voltage

antiparallel spins and then, electrons have a finite spin-flip rate and relax to states with antiparallel spins, producing a small leakage current. In this case, as the electronic spin flips from down to up the nuclei spin flips from up to down and the effective field produced by the nuclei is aligned with the external field. This feed-back mechanism between the nuclei polarization and the electron state renormalization implies hysteresis in the electronic current as a function of B as observed experimentally [15].

In conclusion we have proposed a model which describes charge transport through double quantum dots in the spin-blockade regime including HF interactions. The interplay between electronic charge occupation and spin polarization of the nuclei is accounted for by solving coupled rate equations self-consistently. We interpret current features seen experimentally [15] at the SB regime as evidence for hyperfine interaction between electronic and nuclei spins in the double quantum dot structure. At the SB plateau, electronic spin flip from states with parallel spins to states with antiparallel spins states produces a nuclear field which shift the electronic levels. This shift modifies the spin-flip rate and therefore the nuclei polarization. This feed-back is responsible of the strongly non linear current behaviour.

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