Minisymposium *Asymptotic Properties of Complex Random Systems and Applications*

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There has been recent intense activity in the study of the asymptotic character of sequences of random processes arising e.g. in computer science, statistical physics and mathematical biology. These may model the emergence of certain graph properties; load-sharing among links or servers; the survival and extinction of species; co-operation and competition in a social context; spread of epidemics; DNA, RNA and amino-acid sequences. Under appropriate conditions, a sequence of processes converges to the solution of a differential equation, which may be interpreted as a functional law of large numbers. Such approximations are of great significance as a way to interpret the qualitative behaviour of a complicated, multi-faceted structure in terms of a considerably simpler one. Unfortunately, it is often difficult to prove their validity, especially when the random process has an unbounded number of components in the limit. We would hope that over the coming years, the intense interest in the field will produce a coherent and widely applicable theory. At present, it often appears that each new problem defies the existing theory in an interesting way. In organising the ECMI Minisymposium 'Asymptotic properties of complex random systems and applications', our motivation was to bring these problems into focus and highlight their importance in modelling of real-world situations. Our aim was thus to generate interest among the applied mathematics community, in the hope that interesting new insights and ideas may result. The following sections summarise the contents of the four talks given.

Andrew Barbour's talk. Andrew Barbour, from the University of Zürich, talked on 'Laws of large numbers for epidemic models with countably many types' (work joint with Malwina Luczak). We establish a quantitative law of large numbers for a large class of stochastic epidemic models. It was previously known that certain host-parasite systems can be approximated by systems of differential equations, but rates of convergence were not available. With such diseases, it is natural to distinguish hosts according to the number of parasites they carry. Since it is not usually possible to prescribe a fixed upper limit for the parasite load, this leads to models with countably infinitely

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many types, one for each possible number of parasites. This causes difficulty with many arguments which for finitely many types would be quite standard; in particular, proving limit results is a much more delicate issue. A further difficulty is that the operator driving the deterministic limit is non-Lipschitz.

Carl Graham's talk. Carl Graham, from École Polytechnique, lectured on 'A multiclass mean-field model with graph structure for TCP flows', based on his joint work with Philippe Robert. TCP is one of the core protocols used on the Web and other communication networks. Unlike previous studies of TCP window evolutions, the authors consider interaction between diverse kinds of TCP flows through the congestion they create along flow routes. Resources may consist of switches, buffers, links or processors. Flow characteristics include the route, utilisation of specific resources, and the round trip time (influenced by congestion). A Markovian multi-class mean-field interacting model for the window size evolution of a large number of TCP flows is analysed. In the limit as the numbers of flows in different classes become large while keeping the relative weight of each class fixed, the process converges to a deterministic function solving a non-linear differential equation. Also, the system is chaotic, i.e. different flows become approximately independent.

Petra Berenbrink's talk. Petra Berenbrink, from Simon Fraser University, gave a talk entitled 'Distributed selfish load-balancing', based on joint research with Tom Friedetzky, Iman Hajirasouliha, and Zengjian Hu. A congestion game model is considered with n identical resources and m players with weighted tasks. The system goal is to allocate every task to exactly one resource, and the goal of each selfish player is to be allocated to a resource with minimum total load. Agents migrate from overloaded to underloaded resources in a distributed setting, until the allocation becomes balanced. An allocation is a Nash Equilibrium if no player can benefit from changing their strategy. The authors analyse a simple, decentralised protocol converging to a Nash equilibrium, proving bounds on the rate in terms of n, m and Δ (maximum task weight). Proofs involve analysing a suitable potential function.

Ilkka Norros's talk. Ilkka Norros, from VTT, lectured on 'Features of power-law random graphs' (joint work with Hannu Reittu). A power-law random graph model is considered in a regime where the vertex degree has finite mean and infinite variance. Power-law graphs are commonly used to model inhomogeneous random networks, such as the Internet. These graphs have some remarkable features: e.g. with high probability there are subgraphs with arbitrary edge densities, and the typical distance between a pair of vertices in the giant component of a graphs of size N is $O(\log\log N)$. Also, the random graph has a robust structure in that the deletion of highest-degree vertices does not decrease the relative size of the giant, even though it does cause a moderate increase in distances between vertices.